A STRAIGHTFORWARD AZIMUTHAL ANGLE BIASING FOR VARIANCE REDUCTION OF MONTE CARLO PARTICLE TRANSPORT CALCULATIONS

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ABSTRACT

In this work, a variance reduction scheme was devised to sample azimuthal scattering angles with higher probabilities to lead the scattering particle towards a point in space. This is accomplished by means of an importance function which forces the particle to azimuthally scatter into the semicircle that contains the point-of-interest direction in its middle. Also, no biasing is applied in the polar angle selection. An increase in the efficiency of the calculation (fom) was verified as shown in a demonstration problem. However, the procedure may yield erratic estimates if not used properly.

I. INTRODUCTION

The utilization of variance reduction techniques in Monte Carlo simulation of the radiation transport phenomena is needed because of the great reductions in computational time spent in the majority of the calculations. These techniques are used in the source sampling distributions and in the random walk space, energy, and angle variables dependent functions. Source biasing is often the most efficient technique. Space, energy, and angle importance functions are easily implemented for the selection of source particles. Angular biasing of the scattering kernel has been investigated by several authors with different levels of success. This is partly because it is less effective than the biasing of the other variables, and also because of the increase in the amount information required. However, when source and all the other variables biasing are used together, the effectiveness of the angular biasing may become significant.

In this work it is investigated the biasing of the collision kernel, or more especifically, biasing of the azimuthal angle variable. The motivation for this work is to seek an easier means to implement an all situation angle biasing technique. In a previous work[1], a variance reduction scheme was devised to sample azimuthal scattering angles as a means to guide the scattering particle towards a preferred direction, which was accomplished by selecting a quadrant of scattering that would lead the particle towards a point in space. In this work, the particles are forced to scatter into the semi-circle that contains the point-detector direction in its center. The results show a significant improvement in the figure of merit for the outer detectors in the sample problem presented.

A brief presentation of the theorethical foundations is given in the next section. Also, two problems are presented to illustrate the performance of the technique.

II. ANALYSIS

The Boltzmann integral transport equation for the emergent particle density in multigroup notation and time-independent may be written as[2]

$$\chi_g(\vec{r},\vec{\Omega}) = S_g(\vec{r},\vec{\Omega}) + \sum_{g'=1}^{G} \iint \chi_{g'}(\vec{r}',\vec{\Omega}') K^{g'\to g}(\vec{r}',\vec{\Omega}' \to \vec{r},\vec{\Omega}) d\vec{r}' d\vec{\Omega}' \quad (1)$$

The transition kernel may be expressed by the product of two kernels, the transport and the collision kernels both multiplied by the non-absorption probability. The collision kernel is a joint probability distribution function for the random variables g and $\bar{\Omega}$. The g energy group of emergent particles is selected from a marginal p.d.f. and the $\bar{\Omega}$ directions are selected from the conditional p.d.f.

$$\frac{\Sigma_s^{g'\to g}(\bar{r},\bar{\Omega}'\to\bar{\Omega})}{\Sigma_s(\bar{r})} = h(\varphi)w(\mu) \tag{2}$$

The details of the angular distributions can be found elsewhere[2]. Angular biasing schemes usually apply importance sampling weighting functions on both the azimuthal and polar angles, i.e, a couple of biased values (φ,μ) is sampled[3]. In this work, only the azimuthal angle (φ), is biased, so that no penalty is applied in the polar angle selection.

To select an outgoing direction at a collision site, a scattering polar angle is selected from the distribution derived from the scattering laws, $w(\mu)$, and an azimuthal angle is selected from the uniform distribution, $h = 1/2\pi$. Because they are independent, either angle can be selected first.

Azimuthal Angle Biasing. When a particle goes from the source towards the detector the polar angle of scattering is more important than the azimuthal angle to establish how far the particle will travel. The azimuthal angle has to do with the particle going up and down, left or right. Therefore, without interfering in the polar angle selection, the azimuthal angle will be selected in the semicircle which contains the point-detector direction in its center, which means in vector's $\vec{b}$ direction, as can be seen in Fig. 1.

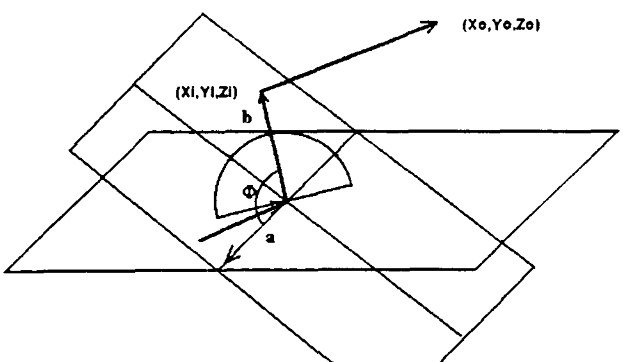

Figure 1. Geometrical Scheme of the Azimuthal Angular Biasing.

One major dificulty is to determine the intersection point (x_i, y_i, z_i), which has been shown in reference[1]. It can be seen in Figure.1 vectors $\vec{a}$ and $\vec{b}$:

$$\begin{aligned} \vec{a} &= (v,-u,0) \\ \vec{b} &= (x_i - x_c, y_i - y_c, z_i - z_c) \end{aligned} \tag{3}$$

Vector $\vec{b}$ is defined using the collision coordinates (x_c, y_c, z_c) and vector $\vec{a}$ is the vector product between the particle incoming direction cosines (u, v, w) and the horizontal plane direction vector. Therefore it is possible to use the scalar product $\vec{a}.\vec{b}$, for the calculation of the angle Φ, which will be used to sample the azimuthal angle φ in the interval of interest $\Phi - \pi/2 \le \varphi \le \Phi + \pi/2$.

Fig. 2 shows possible situations to illustrate the selection of azimuthal angles.

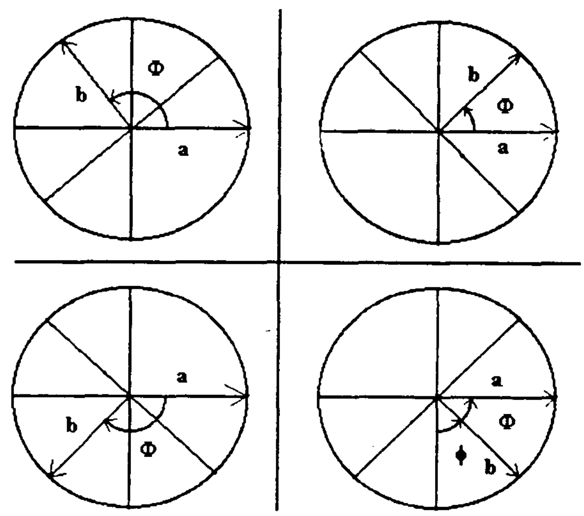

Figure 2. Examples of the Determination of Most Important Semicircle.

The analog sampling distribution for φ is given by

$$h(\varphi) = \frac{1}{2\pi} \tag{4}$$

To sample the interval of interest with probability C times more often, one can use the importance function

$$\begin{aligned} I(\varphi) &= C, & \Phi - \pi/2 \le \varphi \le \Phi + \pi/2 \\ I(\varphi) &= 1, & \Phi + \pi/2 \le \varphi \le \Phi + 3\pi/2 \end{aligned} \tag{5}$$

which is applied with formula

$$\tilde{h}(\varphi) = \frac{h(\varphi)I(\varphi)}{\int_0^{2\pi} \tilde{h}(\varphi)I(\varphi)d\varphi} \qquad (6)$$

Where the normalization factor is given by

$$J = \int_0^{2\pi} \tilde{h}(\varphi)i(\varphi)d\varphi \qquad (7)$$

or

$$J = \int_{\Phi-\pi/2}^{\Phi+\pi/2} \frac{C}{2\pi} d\varphi + \int_{\Phi+\pi/2}^{\Phi+3\pi/2} \frac{1}{2\pi} d\varphi = \frac{C+1}{2} \qquad (8)$$

The cumulative distribution function for $\tilde{h}$ in the interval $\Phi - \pi/2 \leq \varphi \leq \Phi + \pi/2$ is

$$\tilde{H}(\varphi) = \int_{\Phi-\pi/2}^{\varphi} \frac{C}{2\pi J} d\varphi = \frac{C}{2\pi J}(\varphi - \Phi + \pi/2) \qquad (9)$$

The value of this distribution for the point in the separation of the two hemispheres is

$$\tilde{H}(\varphi = \Phi + \pi/2) = \frac{C}{C+1} \qquad (10)$$

The cumulative distribution function for $\tilde{h}$ in the interval $\Phi + \pi/2 \leq \varphi \leq \Phi + 3\pi/2$

$$\tilde{H}(\varphi) = \int_{\Phi-\pi/2}^{\Phi+\pi/2} \frac{C}{2\pi J} d\varphi + \int_{\Phi+\pi/2}^{\varphi} \frac{1}{2\pi J} d\varphi \qquad (11)$$

or

$$\tilde{H}(\varphi) = \frac{1}{2\pi J}(\varphi - \Phi - \pi/2) + \frac{C}{C+1} \qquad (12)$$

The importance sampling procedure will be as follows:

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1) Pick a random number ξ,

2) If $\xi < C/(C+1)$ where $\tilde{H}(\varphi) = \xi$ is a random number in the interval $0 \leq \xi \leq 1$, then

$$\varphi = \frac{2\pi\xi J}{C} + \Phi - \pi/2$$

The particle's weight is modified by $J/I(\varphi) = J/C$.

3) If $\xi \geq C/(C+1)$, then

$$\varphi = 2\pi J(\xi - \frac{C}{C+1}) + \Phi + \pi/2$$

The particle's weight is modified by $J/I(\varphi) = J$.

Subroutine COLISN in the MORSE[2] code was modified to have COSETA = cos φ and SINETA = sin φ.

III. APPLICATION

Table 1 shows the results for a sodium cylinder with 30 cm radius and 102 cm height, immersed in vaccum. The neutron source is located at the bottom of the cylinder with an upward isotropic direction distribution. Energies from 15.0 down to 5.8E-04MeV were analyzed. The biasing consisted in selecting the semicircle of interest 2, 4, and 8 times more frequently than the complementary semicircle. Ten detectors are defined in the axis of the sodium cylinder 10 cm apart. The results are compared with an unbiased calculation (No bias). For $C = 4$ there was a decrease in the fractional standard deviation for the outermost detectors

TABLE 1. Sodium Cylinder Problem

Det	No bias	C=2	C=4	C=8
1	5.6694(-6)* (0.050)	5.2661(-6) (0.023)	5.2274(-6) (0.015)1	5.3364(-6) (0.052)
2	1.9154(-6) (0.014)	2.0543(-6) (0.0325)	1.8954(-6) (0.022)	1.9335(-6) (0.037)
3	8.0607(-7) (0.042)	7.6522(-7) (0.021)	7.6838(-7) (0.027)	7.4863(-7) (0.033)
4	3.2779(-7) (0.037)	3.4475(-7) (0.045)	3.4679(-7) (0.045)	3.6585(-7) (0.081)
5	1.4620(-7) (0.052)	1.5550(-7) (0.066)	1.4665(-7) (0.046)	1.4262(-7) (0.091)
6	6.0807(-8) (0.049)	6.2350(-8) (0.068)	6.2260(-8) (0.055)	6.0927(-8) (0.081)
7	2.5082(-8) (0.051)	2.8182(-8) (0.078)	2.7529(-8) (0.094)	2.5122(-8) (0.067)
8	1.0837(-8_) (0.077)	1.2877(-8) (0.090)	1.0121(-8) (0.050)	1.4180(-8) (0.315)
9	4.6048(-9) (0.104)	6.1090(-9) (0.192)	4.5863(-9) (0.074)	4.2953(-9) (0.082)
10	2.5002(-9) (0.242)	2.4649(-9) (0.148)	2.0698(-9) (0.135)	2.1871(-9) (0.243)

*Read as 5.6694E-06 neutrons/cm/s with fractional standard deviation of 0.050.

Table 2. shows the figure-of-merit, $1/\sigma^2 T$ [4], of the results presented in Table 1. For $C = 2$, no gain was noticed, but for $C = 4$ there was an increase in calculation efficiency for the three outer detectors. For $C = 8$ there was overbiasing as can be seen for Detector 8. The results are normalized by the unbiased calculation.

TABLE 2. MORSE Figure of Merit Comparison

Detetor	No bias	Bias C=2	Bias C=4	Bias C=8
1*	1	3.91	8.96	0.65
2	1	0.17	0.33	0.11
3	1	3.52	1.83	1.11
4	1	0.58	0.52	0.15
5	1	0.54	0.99	0.24
6	1	0.45	0.60	0.26
7	1	0.37	0.23	0.43
8	1	0.62	1.80	0.04
9	1	0.24	1.53	1.16
10	1	2.30	2.49	0.72

Table 3 shows the results for the sample problem which is distributed with the MORSE package. It consists of a point fission source in air. The results were labeled SXE for surface crossing estimator solutions, no bias, for no variance reduction, and with the angle biasing scheme for C equals 2 and 4. In this case there were minor gains for $C = 2$. However, it can be seen that in the case of $C = 4$, there were erratic estimates. The main reason is because there was no greater importance in any of the semicircles, the overweight particles were equally probable to contribute.

TABLE 3. Results for the point fission source in air

Dist.	SXE	No bias	C=2	C=4
100*	1.538(-9) (0.015)	1.5377(-9) (0.015)	1.5531(-9) (0.024)	1.4744(-9) (0.023)
200	4.189(-10) (0.016)	3.9634(-10) (0.036)	4.0161(-10) (0.029)	3.5340(-10) (0.033)
300	1.585(-10) (0.010)	1.4794(-10) (0.031)	1.8664(-10) (0.098)	1.5222(-10) (0.100)
600	1.481(-11) (0.015)	1.1645(-11) (0.091)	1.2805(-11) (0.085)	1.2066(-11) (0.133)
700	6.811(-12) (0.014)	6.8825(-12) (0.174)	4.9176(-12) (0.076)	4.3932(-12) (0.125)
900	1.735(-12) (0.021)	1.1884(-12) (0.186)	1.1182(-12) (0.119)	8.3255(-13) (0.203)
1200	2.252(-13) (0.034)	1.3781(-13) (0.578)	1.7148(-13) (0.377)	3.5681(-13) (0.809)

*Read as 1.558E-09 neutrons/cm/s with fractional standard deviation of 0.009 for the detector located at 100 meters from the source.

IV. CONCLUSIONS

In the process of angular biasing, the weight penalty can be high enough to offset the variance reduction attempted gain. Although the presented angular biasing scheme seems to be safe for small values of C, more study is necessary to verify its efficiency for other types of problem.

Future work. As future work it will be investigated the behavior of the technique when used together with space and energy biasing.

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