COLLAPSE OF A RING-STIFFENED CYLINDRICAL SHELL UNDER EXTERNAL HYDROSTATIC PRESSURE - COMPARISON BETWEEN FEA AND ANALYTICAL FORMULAE

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ABSTRACT: In this paper the collapse of a ring-stiffened cylindrical shell under external hydrostatic pressure is evaluated using nonlinear FEA, with the ANSYS program, and analytical formulae. Some conclusions and comments are addressed from the comparison between the results of both approaches.

1.0 INTRODUCTION

Ring-stiffened cylindrical shells subjected to external hydrostatic pressure are of considerable interest to the various navies of the world and some civilian design codes [1], [2] are, in fact, based to a large extent on the various military investigations.

The following principal failure modes can occur: local inter frame collapse of the shell between ring frames, and overall collapse of the shell-and-rings combination between rigid sections such as flat heads.

The local inter frame collapse of the shell between ring frames is an interaction between elastic-plastic buckling (called lobar buckling) and axisymmetric membrane shell yielding at midbay.

Lobar buckling is an elastic-plastic instability of the shell between ring frames and is characterized by inward and outward lobes, which may or may not develop around the entire periphery of the cylindrical shell. The failure may occur in one or more ring spaces. This mode of failure indicates that the rings have greater resistance to buckling than the shell between them.

Axisymmetric shell yielding is initiated by elastic yielding at the extreme fibers at both the outer surface of the shell midway between ring frames and the inner surface of the shell at the stiffeners. The yield leads to elastic-plastic collapse that is characterized by an accordion type of pleat extending around the periphery of the cylinder. This failure may occur in one or more spaces between the rings.

Overall collapse of the shell-and-rings combination occurs between rigid sections such as flat heads (reinforced membrane) or heavy stiffeners. It is associated with rings which are weak to resist widely spaced hard sections and an out-of-circularity bending leading to premature yielding. Sideways tripping of the rings, which precipitates out-of-circularity bending, may cause overall collapse. This failure mode is also characterized by inward and outward lobes, but the lobes are fewer (usually just two or three) than the number of lobes in lobar buckling.

Other than the physical and geometric properties of the ring-stiffened cylindrical shells, there are several factors that influence the failure modes and the respective collapse pressures. These factors must be considered in the design and are called "imperfections" related to the geometry of the structure, boundary conditions, materials and loads (including residual stresses). The presence of imperfections can cause the shell to reach yield and collapse at pressure lower than that one for a geometrically perfect ring-stiffened cylinder.

In this paper we investigate the collapse of an imperfect ring-stiffened cylindrical shell under external hydrostatic pressure using a combination of analytical and numerical

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methods. Analytical formulae are used to predict the pressures that cause the different modes of failure described. Nonlinear finite element analysis (FEA) with the ANSYS program are used to predict the elastic-plastic collapse pressures. It is important to notice that in FEA the geometric nonlinearities are also considered. Some comments and conclusions are addressed from the comparison between the FEA and analytical results.

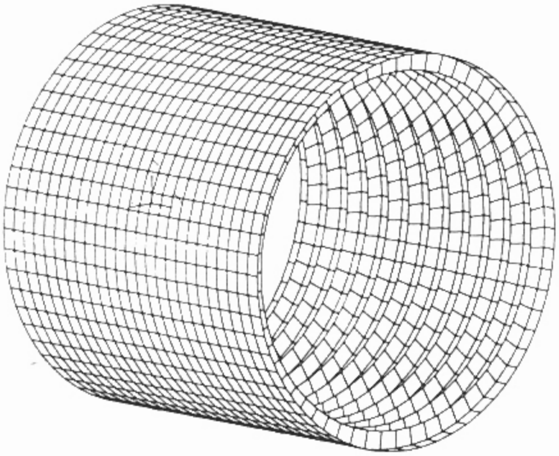
2.0 THE RING-STIFFENED CYLINDRICAL SHELL UNDER ANALYSIS

Following the analytical approach described in item 3 of this paper a ring-stiffened cylindrical shell was designed. In Figure 1 are shown its general dimensions and the adopted model (due to the symmetry of the geometry and the loading only one half of the cylinder was modeled). The flat head is supposed on the plane X = 0 and the symmetry is defined in the model extreme section (that one on the plane X=9265/2).

3.0 ANALYTICAL METHODS

According to [1], the collapse pressures corresponding to local interframe failure of the shell between ring-stiffeners can be predicted using the stresses in the shell and in the stiffeners obtained from [3] and formulae shown in [4] and [5] for lobar buckling and axisymmetric shell yielding, respectively. Corrections for symmetric buckling can be done following the prescriptions of [6].

The local modes of failure are less sensitive to imperfections than the overall mode and the simultaneous occurrence of all failure modes has been argued by theoreticians as being the only criterion to consider for optimum design. Thus, it is usual to design so that the collapse is precipitated by local inter ring failure of the shell.



shell mean radius = 2500 mm
shell thickness = 24 mm

ring thickness = 24 mm
ring height = 223 mm

number of ring stiffeners = 24
distance between ring stiffeners = 9265/25
distance between rigid sections = 9265 mm

FIGURE 1: GENERAL DIMENSIONS AND THE DETAILED FINITE ELEMENT MODEL OF HALF OF THE RING-STIFFENED CYLINDRICAL SHELL

The stiffening is usually designed so that local interframe collapse leads to overall collapse if the pressure is maintained. In the overall mode, the collapse pressures are greatly reduced by out-of-circularities which create an eccentric load path for the compressive hoop force producing bending moments in the rings.

The collapse pressure corresponding to the overall mode of an imperfect ring-stiffened cylindrical shell between hard sections (such as flat heads) can be predicted using the formulae shown in [7],[8]. In the formulations the ring-stiffeners are assumed equally spaced and the imperfections expressed as sinusoidal shapes having a maximum amplitude of the out-of-circularity in the central ring and half wave and n waves in the axial and circumferential directions of the shell, respectively.

The collapse pressures obtained from the analytical formulae, corresponding to the three modes of failure, are shown in Table 1. The amplitude of the sinusoidal shape imperfection, as described above, used in the calculation of the collapse pressures of the overall failure modes is an out-of-circularity in the central ring of the structure of 0.3 % of the inner radius of the shell.

It can be seen that the collapse pressures of all three modes have about the same value if we take the minimum value for the overall mode (n=3 waves in the circumferential direction of the shell).

4.0 FEA WITH ANSYS

The shell and the ring frames are modeled using quadrilateral shell finite elements with 4 nodes and 6 degrees of freedom per node of the ANSYS element library.

The material properties used in FEA are: 205000 MPa as the Young's modulus; 0.3 as the Poisson's ratio; and the stress-strain data of Table 2. From these data, the yielding stress is 337.30 Mpa.

TABLE 1: COLLAPSE PRESSURES FROM ANALYTICAL FORMULAE

Mode of Failure	Collapse pressure (MPa)
Lobar buckling	4.27
Axisymmetric shell yielding	4.28
Overall mode *(n=2 waves)*	5.05
Overall mode *(n=3 waves)*	4.35
Overall mode *(n=4 waves)*	4.40

TABLE 2: STRESS-STRAIN DATA

Stress (MPa)	254.20	321.77	344.92	356.30	399.86
Strain (‰)	1.24	2.70	4.70	8.50	46.50

First, to evaluate the local interframe elastic-plastic collapse of the shell between ring frames and to do a sensitivity check of the mesh size several analysis with reduced finite element models were performed, with no shape imperfection taken into account [10]. They are:
(i) 5 degrees of the structure and 10 ring-stiffeners (Figure 2); (ii) 5 degrees of the structure and 3 ring-stiffeners (Figure 3); (iii) 5 degrees of the structure and 10 ring-stiffeners (Figure 4); (iv) 5 degrees of the structure and 3 ring-stiffeners (Figure 5).

Then, other analyses were done with more complete finite element models with the same mesh size shown in the models of the Figures 4 and 5. They are:
(i) 90 degrees of the structure and 3 ring-stiffeners (Figure 6); (ii) 180 degrees of the structure and 3 ring-stiffeners (Figure 7).

Finally, a detailed model of the structure were built with the same mesh size shown in the models of the Figures 4 to 7. Taking into account the longitudinal symmetry of the structure and of the loads only half structure is modeled as shown in Figure 1.

The FEA with ANSYS were nonlinear static analysis. The loads were the external hydrostatic pressure and the corresponding forces in the boundaries of the models. The stress-strain data of Table 2 were input in a multilinear plasticity model. The elastic-plastic collapse pressure in each analysis was obtained from the nonconvergence of the finite element solution and from the asymptotic behavior of a selected displacement in a load-displacement plot.

The same shape imperfections with the same amplitude used in the analytical formulations were introduced only in the detailed finite element model with n= 2, 3 and 4 waves.

5.0 RESULTS

Due to symmetry, from the reduced finite element models, without shape imperfections, it was possible to evaluate only the local axisymmetric shell yielding. The corresponding elastic-plastic collapse pressures obtained are shown in Table 3.

TABLE 3: COLLAPSE PRESSURES FROM FEA - REDUCED MODELS

Model	Collapse Pressure (MPa)
5 degrees, 3 or 10 ring-stiffeners *(Figures 2 and 3)*	4.56
5 degrees, 3 or 10 ring-stiffeners *(Figures 4 and 5)*	4.37
90 or 180 degrees, 3 ring-stiffeners *(Figures 6 and 7)*	4.29

The elastic-plastic collapse pressures obtained from the complete finite element model with shape imperfections are shown in Table 4.

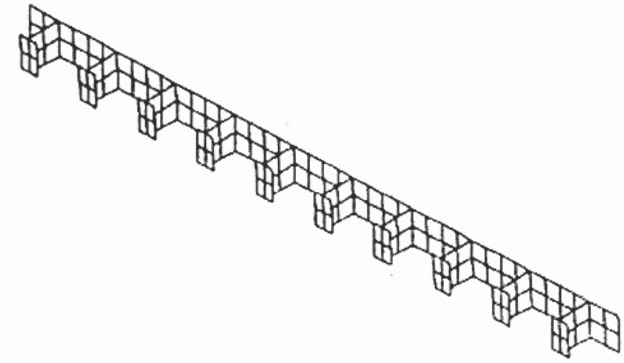


FIGURE 2: REDUCED FINITE ELEMENT MODEL
5 DEGREES, 10 RING-STIFFENERS

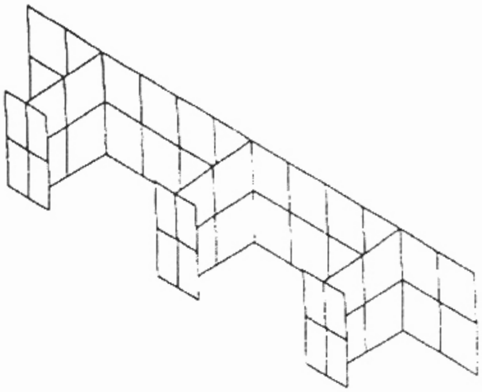


FIGURE 3: REDUCED FINITE ELEMENT MODEL
5 DEGREES, 3 RING -STIFFENERS

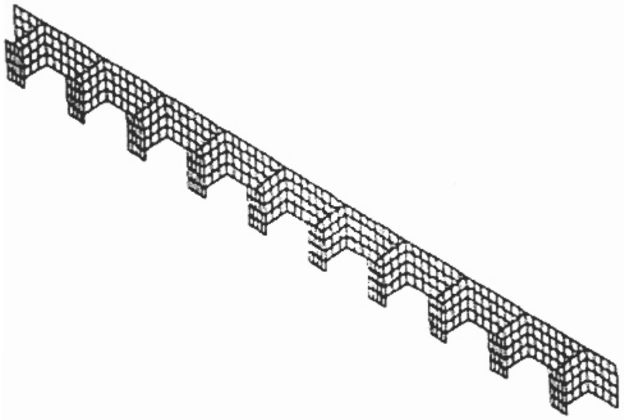


FIGURE 4: REDUCED FINITE ELEMENT MODEL
5 DEGREES, 10 RING-STIFFENERS

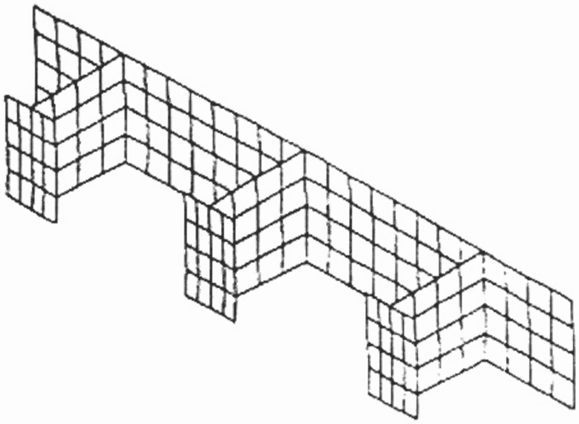


FIGURE 5: REDUCED FINITE ELEMENT MODEL
5 DEGREES, 10 RING-STIFFENERS

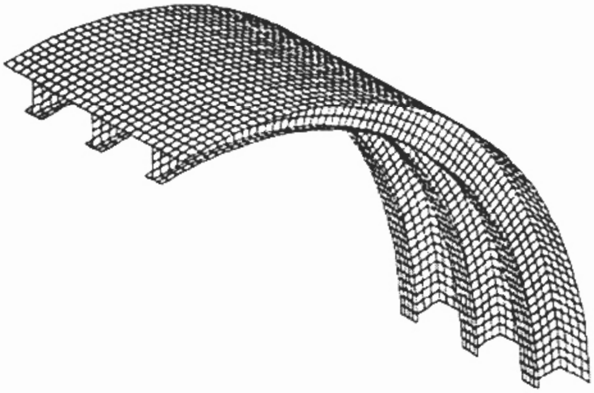


FIGURE 6: REDUCED FINITE ELEMENT MODEL
90 DEGREES, 3 RING-STIFFENERS

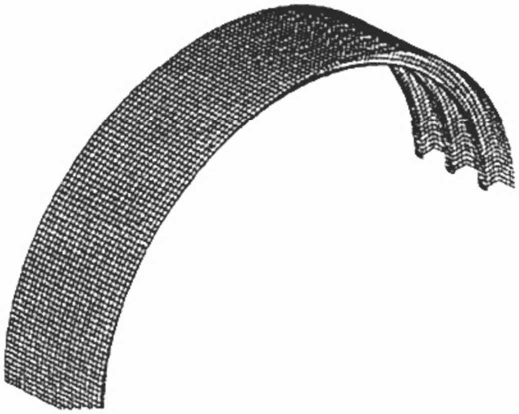


FIGURE 7: REDUCED FINITE ELEMENT MODEL
180 DEGREES, 3 RING-STIFFENERS

In Table 4, the collapse pressures correspond to failure of the shell in the central position. This can be seen in Figure 8 for n = 3 waves and the same occurs for n = 2 and 4 waves. If it is possible to maintain the applied hydrostatic pressure, the shell failure precipitates the overall collapse with the yielding of the rings.

In Figure 8, it can be seen that the stresses in the central frames are near the yielding stress of the material.

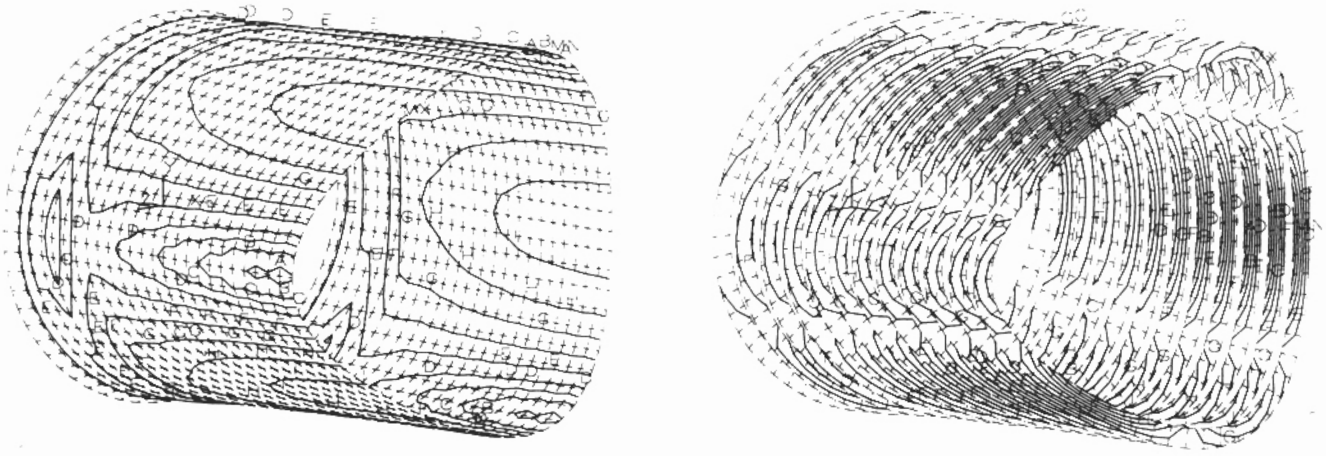
In the detailed 3-D finite element model it was necessary to reinforce the shell near the flat head (rigid sections on extremes) in order to force that the failure occurs in the center of the shell.

Figure 9 shows the displacement field, just before the collapse, for the structure with n = 2 waves, without and with a reinforcement of 25 % of the thickness of the shell that extends over 2.5 ring spaces from the discontinuity.

The reinforcement of the shell near discontinuities is a recommended design practice [9] to avoid weak points in the structure.

TABLE 4: COLLAPSE PRESSURES FROM FEA - DETAILED MODEL

n (number of waves)	Collapse pressure (MPa)
2	5.11
3	4.75
4	5.08



A = 227 MPa to I = 357 MPa
(a) Shell

A = 114 MPa to I = 341 MPa
(b) Ring frames

FIGURE 8: VON MISES STRESS INTENSITIES (MPa)

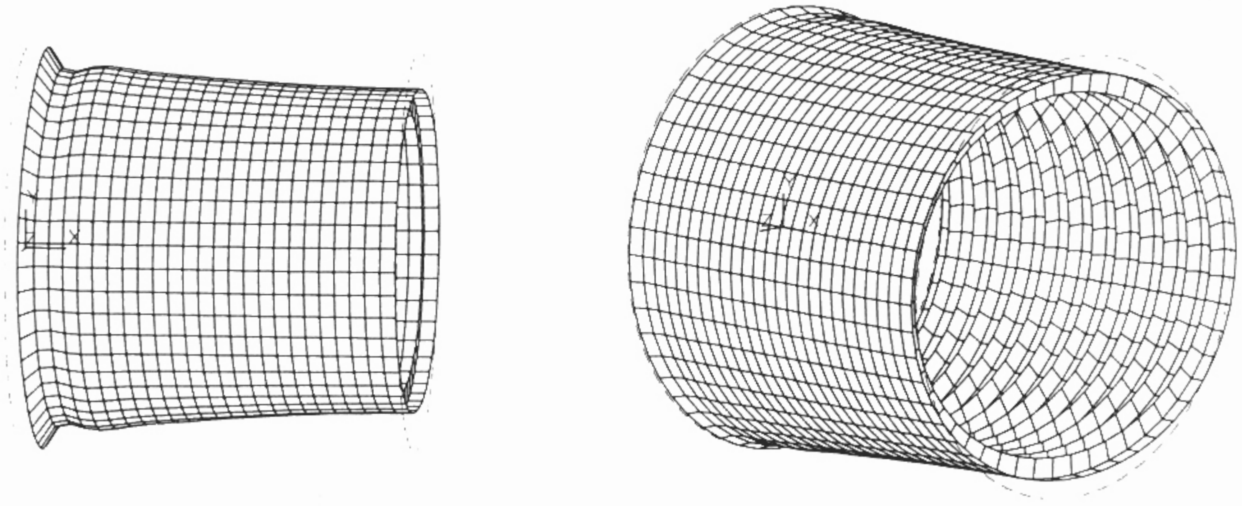
6.0 CONCLUSIONS

For the shell under study, the elastic-plastic FEA collapse pressures are in good agreement with the collapse pressures obtained from analytical formulae.

The analytical formulae give a good estimate of the collapse pressures but some used assumptions such as equally spaced ring-stiffeners, perfect boundary conditions, and sinusoidal shape imperfections introduce limitations in the application of them to all situations with the required confidence.

Thus, the formulae can be used to give the basic dimensions of this type of structure, in the first design phase. Further studies, mainly with nonlinear FEA, are necessary to conclude the design considering rings not equally spaced, the inclusion of reinforcement near discontinuities and a more precise definition of imperfections.

To perform the necessary nonlinear FEA the ANSYS program is an excellent tool because its options, the easy way to use it, and the quality of its results.



(a) without reinforcement

(b) with reinforcement

FIGURE 9: DISPLACEMENTS OF THE SHELL (*n = 2 waves*)

7.0 REFERENCES

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