STRESS INTENSITY FACTORS FOR NOZZLE CORNER CRACKS

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ABSTRACT

The location where the nozzle inside radius blends with the inside surface of the vessel is both a high-stress location and a location of complex bivariate stress distribution. Cracks that are located in this region and that are oriented with the hoop stress normal to it are often called nozzle corner cracks.

There are pressure vessel formulations to calculate stress intensity factors for nozzle corner cracks that are adequate for some geometric relations and loads (e. g., for relatively large cracks under internal pressure). Many times, general stress intensity factor solutions are not currently available. This paper examines and suggests ways in which stress intensity factors for such nozzle corner cracks could be estimated using different approaches considering various geometric and loading situations.

INTRODUCTION

The location where the nozzle inside radius blends with the inside surface of the vessel is both a high-stress location and a location of complex bivariate stress distribution. Cracks that are located in this region and that are oriented with the hoop stress normal to it are often called nozzle corner cracks. (see Figure 1).

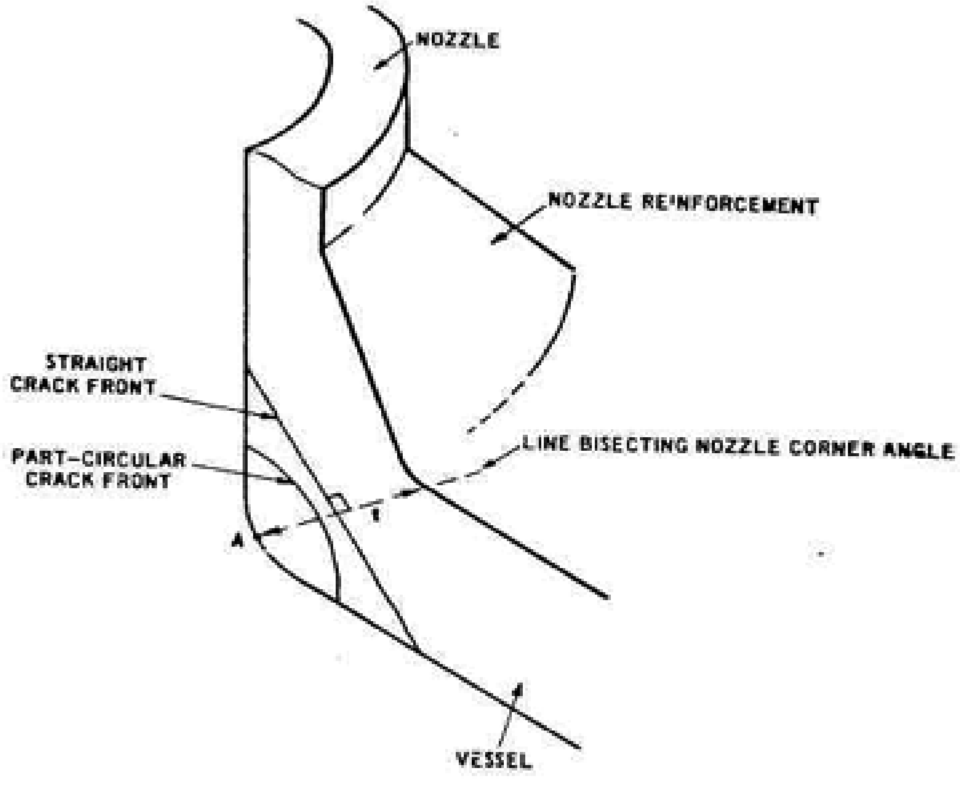

Figure 1. Nozzle Corner Crack

In pressure vessels integrity evaluations, particular attention is given to the possibility of fracture at this point. If the crack is present, a failure assessment requires the knowledge of the stress intensity factor at the crack tip, K. The stress intensity factor is dependent on the stress distribution in the region of the crack, the geometry of the crack and the stiffness of the structure. For simple geometries and loading systems, solutions are available in standard texts [1],[2], but they are not necessarily applicable to the geometry and constraints of a nozzle corner.

There are pressure vessel formulations to calculate stress intensity factors for nozzle corner cracks that are adequate for some geometric relations and loads (e. g., for relatively large cracks under internal pressure [3]). Many times, general stress intensity factor solutions for such nozzle corner cracks could be estimated using relatively simple techniques. Also, alternative more expensive methods such as the finite element method are briefly indicated for complex geometric and loading situations.

EVALUATION OF THE STRESS INTENSITY FACTORS FOR NOZZLE CORNER CRACKS

Several approaches have been applied to the problem of the evaluation of stress intensity factors for nozzle corner cracks. They can be separated as experimental techniques, numerical methods such as the finite element method, and simple methods. Simple methods may be defined as "enginnering approaches" where the solutions are obtained with some assumptions to reduce and to simplify the general problem.

Several methods are described as follows, in a sequence of increasing complexity to develop and to use them.

Method of the Welding Research Council Bulletin # 175. In this reference [3], a simple method to calculate the stress intensity factor for a nozzle corner crack subjected to a pressure load is presented. The nozzle in a pressure vessel is considered as a hole in the shell. By further assuming a large shell radius, it can be considered as a hole in a flat plate. According [4], this results in:

$$(K_I / \sigma)\,(\pi a)^{1/2} = F(a/r) \tag{1}$$

where σ is the applied stress, a is the crack length, and r is hole radius. Values of F(a/r) are given for one crack and for two diametrically opposite cracks under uniaxial tension and equal biaxial tension. It is assumed that the cracks extends completely through the plate thickness.

Equation (1) can be adapted to the pressure vessel case where a 2:1 stress biaxility exists by assuming that the values of F(a/r) are halfway between the uniaxial and the equibiaxial values. The shell hoop stress is used as the value of stress in the equation (1). The applicable curve F(a/r) versus a/r is shown in Figure 2 (from [3]).

This method is used in both Appendices G of the ASME Boiler and Pressure Vessel Code, Sections III [5] and Section XI [6].

Approximated Methods Based on Weight Functions. The weight function method [7],[8] enables the stress intensity factor for a crack be determined from the stress distribution present in the uncracked body. If the weight function , W(a,x), is known for a crack length a and a region characterized by the coordinate x, then, for extended defects the stress intensity factor K is

$$K = \int_0^a W(a,x)\,\sigma(x)\,dx \tag{2}$$

where $\sigma(x)$ is the uncracked body stress. By expressing $\sigma(x)$ as a polynomial in x, using the principle of linear superposition K can be written as:

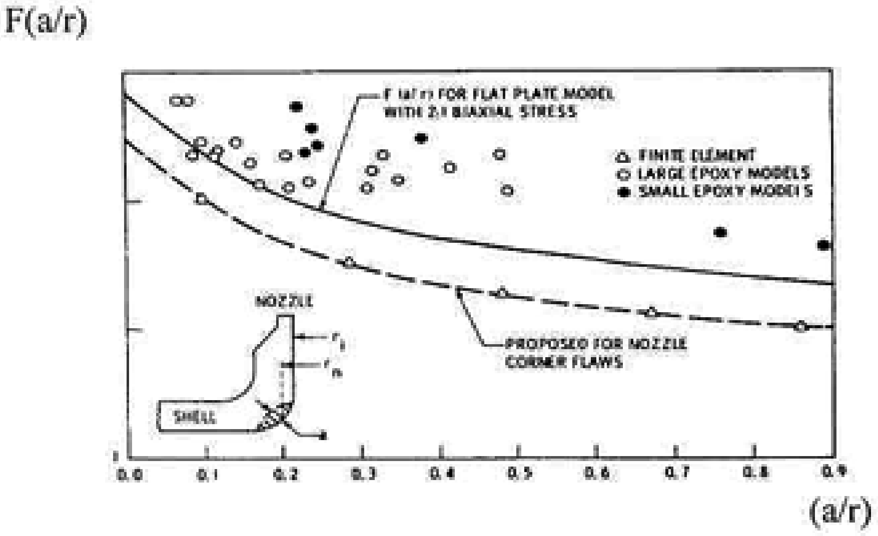

Figure 2. Estimates of stress intensity factors for nozzle corner cracks [3]

$$K = \sum_{n=0} A_n K_n \tag{3}$$

where

$$\sigma(x) = \sum_{n=0} A_n x^n \tag{4}$$

and

$$K_n = \int_0^a W(a,x)\, x^n\, dx \tag{5}$$

These techniques can be used to derive simple expressions for stress intensity factors, provided that the stress distribution is one dimensional or radially symmetric. Introducing the so-called averaged weight functions it is possible to reduce the two dimensional problem on the crack to a one dimensional problem [8]. The application of this approach to nozzle corner cracks problems is shown in the references [7] and [8].

The BIE/IF Method. Elasticity problems can be solved by a Boundary Integral Equation (BIE) method [9]. In this method, the elasticity equations are solved by the determination of two kernel tensors on the boundary, depending only on the geometry. These kernels allow the calculation of the displacements and stresses on the boundary by an integral equation. The displacements and stresses can then be calculated at any point in the body as functions of the kernels and the tractions and displacements on the boundary.

A three-dimensional problem is therefore reduced to a two dimensional problem on the boundary. Thus, problems on any crack can be treated this way and applications to nozzle corner cracks in the form of influence functions related to polynomial applied stresses (equivalent to weight functions for mathematical considerations).

The approach were the influence functions are developed by BIE method is called BIE/IF method [10],[11]. The basic idea of BIE/IF is that K is calculated based on a unit load at a single point location. The total K is obtained by iterating K from multiple locations. It is specially powerful when used for bivariate stress distributions that act over the crack, but that are determined (e. g., by the Finite Element Method, FEM) in the absence of a crack.

An application of this approach is shown in the BIGIF program [11]. The formulation approximates a semielliptical crack and maps the stress distribution to account for the difference between the modeled crack and the actual crack.

FEM. The application of the FEM to the nozzle corner crack problems is shown in several references such as [12],[13].

FEM can give accurate results for any complex geometry and stress distribution. Despite of the resulting accuracy, the use of FEM is, in general, considerably more expensive than many other methods.

There are a number of techniques for translating FEM results in K values. The more common techniques are the stress, energy, and displacement methods. In the most of FEM commercial programs there are features to model cracked geometries and to compute the K values.

CONCLUSIONS

Depending on the problem complexity, the required accuracy of the solution, and the purpose of the integrity evaluation, one of the above described methods gives the best relation between the quality of the results and the costs to obtain them.

In the design application, where the presence of cracks is assumed, the use of simple methods seems more adequate.

For evaluations during operation, where the cracks have been detected by inspection, more refined methods may be used to crack growth analysis or to demonstrate that no brittle failure will occur.

More complex techniques may be used when more realistic evaluations are required. This is the case of life prevision of components or the case of failure assessments.

It is important to notice that FEM has been used more than the other approaches such as the weight functions method or the BIE/IF methods. The reasons for this are: FEM commercial programs are more common than Fracture Mechanics programs and the use of FEM has become more and more inexpensive with the hardware and software evolutions.

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