PRESSURE VESSEL COMPONENTS ANALYSIS BASED ON LEFM

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ABSTRACT

Linear Elastic Fracture Mechanics (LEFM) has important applications in pressure vessel components analysis related to design and plant operating procedures. The design procedures assume the presence of an undetected crack or the formation of one during life. In plant operating procedures, LEFM analysis justifies the existence of defects where the defect has been found by in-service inspections (ISI). It gives guidelines for crack growth analysis and for precluding the brittle fracture. In this paper, code LEFM formulations used in pressure vessel components evaluation are presented and discussed.

INTRODUCTION

Linear Elastic Fracture Mechanics (LEFM) has four major applications: design and plant operating procedures, assessment of defects found by non destructive examination (NDE), prediction of remaining life, and assessment of failure: In the pressure vessel and piping (PVP) industry, the most important guidelines are in ASME Code Section III, Nuclear Power Plant Components, Appendix G (Section III) [1] for design and Section XI [2] for assessment of defects found during in-service inspection (ISI) and for establishing operating procedures.

It is known that present developments in the mechanics of fracture of pressure vessels and piping are mainly in plastic and dynamic domains. However, a linear analysis remains necessary for fatigue growth calculation and for possibly embrittled parts.

LEFM technology is adequately developed, and has advantages as a design tool, for any pressure vessel. The design application assumes the presence of an undetected crack or the formation of one during life.

The Section III guidelines are quite limited. The ASME Code Section XI allows LEFM analyses to justify the existence of defects where the defect has been found by in-service inspection. It gives guidelines for crack growth analysis and for precluding brittle fracture. The Section XI approach is considered the best available procedure for defect evaluation, but there are areas where updating to current technology is appropriate.

There are currently no published guidelines for plant life predictions. This application is based on the presence of a hypothesized crack (similar to the design application) or on a defect found by ISI. Because better materials data and operating experience are available, life prediction methods should be more accurate than design methods. This is especially true when they are performed for economic decisions as opposed to safety evaluations.

The failure assessment application requires an accurate application of FM technology. The goal is to understand whether the cause of failure was because of faulty construction, misoperation, or a combination of factors. Thus, accuracy in the analysis is emphasized over conservatism.

BASIC CONCEPTS OF LEFM AND STRESS INTENSITY FACTORS CALCULATION

Crack Shapes. Cracks can occur as through-thickness cracks or surface cracks, or they can be fully embedded in the material. The majority of the cracks of interest to the pressure vessel designer can be represented by an elliptical shape. These include semi-elliptical surface cracks, quarter-elliptical corner cracks, and fully elliptical embedded cracks. The aspect ratio of the ellipse can reduce to 1: 1 or increase to 5: 1, or even 10: 1; thus, they can vary from circular to continuous cracks.

Crack Tip Stress Intensity Factor - K. LEFM can be divided into two major parts: brittle fracture evaluations and fatigue crack growth analyses. Both require knowledge of the stress intensity factor K along the crack front. The determination of this K value at a point on the crack front is central to all LEFM analyses. Formulations for calculating K are discussed in the next section. There are two aspects that are important in the calculation of K: the location along the crack front and the proximity of this point to the surface of the structure.

K Locations - Degrees of Freedom. A value of K can be calculated at every point on the crack front or as an average value of K over a portion of the crack front. The locations on the crack front where K is to be calculated are called degrees of freedom (DOF). For most loading conditions, the maximum K value occurs at !he end of the major or minor axes. Therefore, the DOFs of interest are those that define what occurs at the ends of the major and minor axes.

Calculating K at every point on the crack front can usually be done because most formulations calculate K as a function of the angle around the crack front. For crack growth calculations, the crack must be reshaped into an ellipse before the next crack growth iteration. For fracture evaluations, the maximum K value on the crack front is needed. This normally occurs at or near ends of the axes. Therefore, reasonable accuracy can be obtained by calculating K at the ends of the major and minor axes or calculating average values representative of these locations.

Closeness to Surface. A defect is considered embedded when its surface ligament is greater than 0.4 times a (the through-thickness crack dimension). As the crack grows, the ligament decreases until the criterion is no longer met. The crack must then be considered as a surface defect with the through-thickness dimension equal to the actual through-thickness dimension plus the ligament dimension. The surface length dimension for the new crack can be the length of the major axis of the embedded crack. However, half the surface length must be greater than the depth.

As a surface crack grows in the through-thickness direction, being t the thickness, its a/t ratio increases. When a/t reaches 0.8, the remaining ligament equals 0.2t or 0.25a. Some K formulations have been developed for even smaller ligaments, but it is reasonable to change the crack to a through-thickness crack when the ligament is less than 0.25a. For thin ligaments, the level of plasticity is very high; because the K formulations are linear elastic, K values are unconservative for high plasticity levels. Also, for high a/t values, the crack growth rates are extremely high - few cycles cause the crack to break the remaining ligament. Thus, there is no significant conservatism in changing the crack geometry to a through-thickness crack at a/t=0.8.

Plastic Zone Correction. The K formulations are based on LEFM conditions; i.e., there is no plastic zone at the crack tip. Because pressure vessels are constructed using ductile materials, there is a plastic zone at the crack front that affects the stress distribution and thus the K value. This is acceptable for LEFM provided an adjustment is made in the calculation of K. The K formulations discussed in the next section generally do not include the plastic zone correction, but they can be superposed on the formulation by adjusting the crack size.

$$a = a_a + r_y \qquad (1)$$

where a_a is the actual crack size

The value of the plastic zone (r_y) is defined by

$$r_y = \alpha\,(K_I / S_y)^2 \qquad (2)$$

In brittle fracture evaluations, the value of α is $1/6\pi$ for plane strain conditions and $1/2\pi$ for plane stress.

The plastic zone is normally included in fracture assessments ($K_I < K_{Ia}$), but is often not included in crack growth calculations. It is more accurate to include a reduced plastic zone in crack growth, where the reduced size is one-fourth of the values defined in Eq. (2). Thus, for plane strain in crack growth analyses, $\alpha = 1/24\pi$.

The plastic zone should be small; i.e., less than 10% of the actual crack length. As the plastic zone becomes greater than. 0.1a, the plasticity makes the K calculation inaccurate and unconservative.

METHODS FOR K CALCULATION

Formulations of K can be divided into three approaches: handbook solutions, finite element, and influence functions. These three categories are used to simplify the discussion of the various approaches and are not intended as precise categories.

Handbook Solutions. Handbook solutions [3] are generally presented in closed, tabular, or graphic form. Most are quite accurate, but for limited geometries. The solutions are presented in relatively simple equations or relationships, but are often based on complex, closed-form equations. Handbook graphs usually provide the magnification factor or flaw shape parameters as a function of crack depth and component geometry.

Finite Element. Finite element (FE) solutions [4] are considered by many FM practitioners as the standard of accuracy for any complex geometry and stress distribution. There are a number of techniques for translating FE results into K values. The more common techniques are the stress, energy, and displacement methods. Depending on conditions, such as solution technique, geometry, and so on, the different techniques can give results that vary by as much as 10%; however, these differences are the exception, rather than the rule. The K calculations by FE require modeling crack geometry. Therefore the use of the finite element method (FEM) can be accurate, but considerably more expensive than many other methods.

Although expensive, FEM for calculating K is good for single-condition applications and for generating influence functions. However, it is too expensive and timeconsuming for repetitive applications such as crack growth. FEM is excellent for development work and often for failure assessment, but other non-FEM approaches are better for design, crack evaluations, and life prediction analyses.

Influence Functions. For a given geometry, influence functions [5] reduce the calculation of K to a simple operations. They can be developed by various methods; the most popular have been finite element and boundary integral equations (BIE). Influence factions once developed, are readily programmable and quite efficient. They are all based on the calculation of stress in the absence of the crack.

The basic idea of BIE influence factions (BIE/IF) is that K is calculated based on a unit load at a single location. The total K is obtained by integrating K from multiple locations. It is especially powerful when used for bivariate stress distributions that act over the crack, but that are determined (e.g., by FEM) in the absence of a crack.

Pressure Vessel Formulations. There are general LEFM formulations for two and three problems and for shells. Applications for pressure vessels can be derived from those general formulations such as long axial and circumferential cracks, semi-elliptical cracks in meridional planes, cracks in nozzle corners, part circumferential cracks, and shells.

The following paragraphs present a few of the more commonly used code K formulations.

ASME Code Section III.. Section III gives a simple approximation of K for a 3:1 semi-elliptical surface crack that is t/4 deep. It is intended for use with constant-thickness cylinders. For mechanical loads, K is calculated by

$$K = (M_m \sigma_m + M_b \sigma_b) \quad (3)$$

where σ_m and σ_b are the effective membrane and bending stress distributions, respectively. The term M_m is given in a figure as a function of vessel thickness and the stress-to-yield-strength ratio. Bending M_b is given as (2/3) M_m. For thermal loads, $K = M_t \Delta T_w$, where M_t is given in a figure as a function of thickness, and where ΔT_w represents the thermal gradient.

ASME Code Section XI. Section XI gives formulations for both surface and embedded elliptical cracks for membrane and bending stress distributions. K is calculated by

$$K = (M_m \sigma_m + M_b \sigma_b) \sqrt{(a\pi/Q)} \quad (4)$$

The term Q includes both aspect ratio and plastic zone. It is defined in a figure as a function of the aspect ratio and stress-to-yield-strength ratio. The analytical expression for Q is given by

$$Q = [1 + 4.593\ (a/l)^{1.65}] - [0.167\ (\sigma/S_y)^2] \quad (5)$$

where the first term accounts for the aspect ratio effect and the second term for the plastic zone.

For embedded flaws, M_m and M_b values are given for both ends of the minor axis, but K values cannot be calculated at the ends of the major axis because only continuous flaw solutions are available. The M_m and M_b factors are a function of closeness to the surface for both locations. They are not a function of the aspect ratio. Values are given in a graph.

CRACK GROWTH

The LEFM fatigue crack growth procedure (da/dN, crack growth versus number of cycles) is a natural extension of the traditional S-N (stress versus number of cycles to failure) approach. The S-N approach predicts the generation of a crack and its propagation in a 0.3 inches specimen. The da/dN procedure extends the propagation phase to thick sections. As discussed in the following paragraphs, the crack growth formulations are a nonlinear relationship.

Paris Law. The fundamental equation for crack growth is

$$da/dN = C\ \Delta K^n \quad (6)$$

This is known as the Paris Law. It describes the crack growth rate (da/dN) as a function of ΔK. The values of C and n are obtained from test data. A value of K_{max} and K_{min} must be determined for each transient.

Limitations of the Paris Law. The Paris Law does not account for the actual distribution of test data, because the data are not linear over the full range and there is a mean-stress effect. Most test data show a sigmoidal relation, but the Paris Law is applicable only to log-linear relations. This requires an upper and lower threshold when using the Paris Law. At the upper end (higher ΔK), the linear relationship becomes unconservative. At the lower end, the linear relationship becomes conservative.

PRECLUDING BRITTLE FRACTURE

In general, one precludes brittle fracture by ensuring that $K_I < \alpha K_{Ic}$., where α is the desired design margin. The crack size is tracked from the postulated or inspected initial size to the final size that results from crack growth. The $K_I < \alpha K_{Ic}$ must be satisfied at each crack size. This is a simple relationship, except that K_{Ic} is a function of temperature; thus the temperature must be known as a function of crack depth, so that K_{Ic} is known for every crack size. This is true for all four applications: design, flaw justification, life prediction, and failure assessment.

K_{Ic} Versus Temperature. To meet the preceding criteria, one must evaluate multiple crack sizes and sometimes multiple locations on the crack (normally the deepest point or surface point is controlling). As a result, both the stress and temperature distributions are needed through the thickness of the part, and they may be needed for multiple load cases where the temperatures can be changing from the ductile to brittle fracture regimes. Normally, FEM results can match stresses and temperatures to load cases and link them to each other by grid location. Therefore, the input to an evaluation program includes stress and temperature as a function of location on the crack front for all load cases within the brittle fracture regime. Once K values are obtained, they are matched to K_{Ic} at the time and location.

Section III. Section III presents the equation:

$$K_{Ia} > 2\,K_I - pr + K_I - th \quad (7)$$

where K_{Ia} is the plane strain fracture toughness , based on dynamic loads and crack arrest, pr and th are the primary loads and the thermal loads, respectively.

Thus, it applies a design margin of 2 on primary loads and a design margin of 1 on thermal- or deformation-controlled loads. Section III uses K_{Ia} (crack arrest) rather than K_{Ic} as the limit. The crack size is set at $a = t/4$; it is a 1-DOF problem, and a crack growth analysis is not performed. To perform the evaluation, one must know the stress and temperature distributions over the crack, as a function of time. For example, for heat-up and cool-down, the temperature and stress distributions change with time; therefore K_{Ia} and K_I are changing with time.

Section XI. For ISI flaw justifications, the initial defect size is known and its growth to a final crack size is calculated. Section XI gives two acceptance criteria for evaluating normal operating conditions and two for evaluating emergency and faulted conditions.

Normal Operating Conditions. For normal operating conditions, brittle fracture is precluded using K_{Ia} or the critical crack size, a_c. Using K_{Ia} the design margin is $\sqrt{10}$; i.e., $K_{Ia}/K_I > \sqrt{10}$. The design margin must be met for all crack sizes from a_i to a_f. As an alternate, K_I must be less than K_{Ia} for all flaws from a_i (the detected flaw) to a_f, where a_f is the flaw at the end of the crack growth calculations. This criteria requires determining K_I for multiple crack sizes. Even though K_I is directly related to $\sqrt{a}$, the stress gradient is often decreasing so that the maximum K_I need not occur at the maximum crack size. Also, K_{Ia} is a function of temperature. If the temperature increases with depth, K_{Ia} also increases with depth. Therefore, the critical crack size need not be the largest crack size.

Emergency and Faulted Conditions. For emergency and faulted conditions, one is ensuring safe shutdown. The criteria is $a_f < .1\ a_c$, where a_c is defined as the crack size where unstable crack growth ($K_I > K_{Ic}$) will not arrest ($K_I < K_{Ia}$) before the crack reaches 0.75t. Multiple crack sizes are evaluated, and K_I, K_{Ic}, and K_{Ia} profiles must be established for the full thickness at the crack location.

CONCLUSIONS

It was presented code LEFM formulations applicable to pressure vessels and piping based on the prescriptions of ASME Code Sections III and XI for crack growth analysis and to preclude brittle fracture. Despite the simplicity of the formulations, care must be taken to apply the code rules.

REFERENCES

[1] ASME, **ASME Boiler and Pressure Vessel Code , Section III**, ASME, NY, 1995.

[2] ASME, **ASME Boiler and Pressure Vessel Code, Section, XI**, ASME, NY, 1995.

[3] Rooke, D. P. and D. J. Cartwright, **Compedium of Stress Intensity Factors**, HMSO, London, 1975.

[4] Banks-Sills, L. and Boutman, Y., **Reappraisal of the Quarter-Point Quadrilateral Element in Linear Elastic Fracture Mechanics**, Int. J. Fract. Mech., vol. 25, n. 3, pp. 25-36, 1984.

[5] Heliot, J., Labbens, R., Robisson, F., **Influence Functions for Stress Intensity Factors at a Nozzle Corner**, Fracture Mechanics, ASTM STP 743, pp. 493-521, 1981.