Pierce Model for TWT Gain Analysis and Experimental Measurements

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Abstract— This work reports experimental and theoretical results from research into X-band traveling-wave tube amplifier design. A suitable microwave slow-wave circuit is investigated through the solution of dispersion equation. For this end a numerical solution for dispersion equation, following the Pierce approach, has been carried on and a mean gain of 20 dB has been obtained. The cathode perveance measured is 0.2 μPerveance and the life time of the cathode is also investigated and reported in this work. The dependence of the traveling-wave tube gain as function of the microwave frequency at X-band is also investigated.

Key words— Traveling wave tube, microwave electron devices, microwave tubes, thermoionic cathode.

I. Introduction

It is well known that the traveling-wave tube amplifier (TWT) has a property of a relatively constant gain over an octave of frequency or band of operation. The use of TWT is very attractive when compared with the use the other microwave tubes like klystrons and magnetrons. TWT are largely used in telecommunication links, satellites communication and radar systems. The TWT features come from of its particular mechanism of interaction of electron beam with the slow-wave structure. A general suitable slow-wave structure is obtained from a helix tape. As a helix tape is not a tuned circuit, TWT is considered a broad band device. In addition, the slow-wave structure is able to reduce the phase velocity of the electromagnetic making it equal to the velocity of the electron beam. This effect increases the interaction parameter between the electromagnetic field and the electron beam, so these TWT features provide devices with gains of 30 to 50 dB over a broad band in frequency.

In this work we describe the design and performance of a X-band traveling wave tube amplifier that has been developed is our laboratory at Centro Tecnológico da Marinha em São Paulo, (CTMSP). It is investigated a suitable microwave slow-wave circuit through the solution of the dispersion equation. The cathode life time and cathode perveance is also investigated and reported The dependence of the TWT gain as function of the microwave frequency at X-band was also investigated.

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This work has been supported in part by Fundação Amparo à Pesquisa do Estado de São Paulo (FAPESP), proc. number 99/06087-1.

This paper is organized as follows: In Section II, a TWT theory amplification following a linearized model is outlined. In Section III, a solution of dispersion equation TWT helix in presented. In Section IV, the major cathode termoionic features are discussed. In Section V, the general results of TWT performance are shown. Conclusions are found in Section VI.

II. TWT Amplification Theory

The interaction between the electromagnetic field sustained by TWT helix and the electron beam that moves away from the cathode region toward to the collector can be adequately described by Lorentz force as the resultant force in the motion equation [1]-[3],

$$\left[\frac{\partial}{\partial t} + (\vec{v}_t \cdot \nabla)\right]\rho \vec{v}_t = n_e e\left[\vec{E} + (\vec{v}_t \times \vec{B})\right]. \quad (1)$$

where $\vec{v}_t$ is the electron beam velocity, ρ_t is the charge density, n_e is the electron number density, m_e is the electron mass, e is the electron charge. The electric and magnetic fields are denoted by $\vec{E}$ and $\vec{B}$, respectively. Equation (1) can be linearized by using the perturbation theory, i. e., the time dependent quantities can be considered as a small variation around the dc values. These values are represented by subscript 0. So for this purpose one can write,

$$\vec{v}_t(\vec{r},t) = \vec{v}_0 + \vec{v}(\vec{r},t), \quad (2)$$

$$\rho_t(\vec{r},t) = \rho_0 + \rho(\vec{r},t), \quad (3)$$

$$\vec{B}_t(\vec{r},t) = \vec{B}_0 + \vec{B}(\vec{r},t), \quad (4)$$

$$\vec{J}_t(\vec{r},t) = \vec{J}_0 + \vec{J}(\vec{r},t), \quad (5)$$

where $\vec{J}_t$ is the total beam density current. Under this hypothesis, the first-order linearized motion beam equation (1) becomes

$$\left[\frac{\partial}{\partial t} + (\vec{v}_0 \cdot \nabla)\right]\vec{v} = -\frac{e}{m_e}\left[\vec{E} + (\vec{v} \times \vec{B}_0)\right]. \quad (6)$$

For the cylindric beam under consideration, $\vec{B}_0$, the magnetic field produced by the external solenoid, with the purpose of avoiding the radial electron beam dispersion, can be considered as large enough so the transverse components of $\bar{v}_t$ must vanish, and the term $\bar{v}\times\bar{B}_0$ in the (6) will also vanish. Thus $\bar{v}$ has a component in the z direction only. In addition, under the assumption of a time dependence expressed in terms of $\exp(j\omega t)$ where $\omega = 2\pi f$ is the angular frequency, (6) gives,

$$j\omega v_z + v_0 \frac{\partial v_z}{\partial z} = -\frac{e}{m_e} E_z. \qquad (7)$$

If $\bar{v}$ has only a z component, the current density $\bar{J}_t = \rho_t \bar{v}_t$, with its two parts $\bar{J}_0 = \rho_0 \bar{v}_0$ and $\bar{J} = \rho\bar{v}_0 - \rho_0\bar{v}$ having only a z component also. Furthermore $\bar{J}$ and ρ are related by the continuity equation that under the time dependence expressed in terms of $\exp(j\omega t)$ becomes

$$\frac{\partial J_z}{\partial z} + j\omega\rho = 0. \qquad (8)$$

Using (7) and (8) one can to express J_z as a E_z function. Since we are looking for a wave solution, we may assume that all small quantities have a z dependence in terms of $\exp(-j\beta z)$, so one finds that

$$J_z = -j\frac{\omega_p^2}{\omega}\frac{\beta_0\varepsilon_0}{(\beta_0-\beta)^2} E_z. \qquad (9)$$

where $\omega_p = (n_e e / m_e \varepsilon_0)^{1/2}$ is the plasma frequency, $\beta_0 = \omega / v_0$ is the dc propagation constant for the electron beam and ε_0 is the electric permittivity of vacuum.

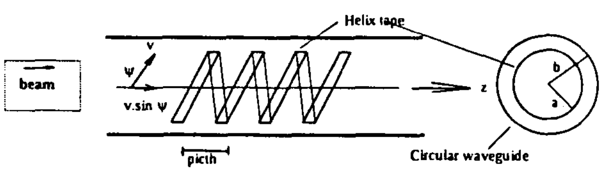

Figure 1 - A tape helix. p is the helix pitch. ψ is the pitch angle. a is the tape helix radius and b is the inner radius of circular waveguide.

The electric and magnetic fields in a slow-wave structure can be suitably described by the the magnetic vector potential $\bar{A}$. The magnetic vector potential satisfies the non-homogenic vetorial Helmholtz equation,

$$\nabla^2 \bar{A} + k_o^2 \bar{A} = -\mu_o \bar{J} \qquad (10)$$

where $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ and μ_o is the magnetic permeability of vacuum. The vector potential has only a z component due to the fact that the density beam current has only a z component. Considering the z dependence expressed in terms of $\exp(-j\beta z)$, (10) becomes

$$\frac{\partial^2}{\partial\rho^2}A_z + \frac{1}{\rho}\frac{\partial}{\partial\rho}A_z + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}A_z + (k_o^2 - \beta^2)A_z = -\mu_o J_z \qquad (11)$$

for the helix inner region, $0 \le \rho \le a$, where ρ is the radial coordinate, and a is the radius tape helix. The helix is considered as thickness. b is the inner radius of the circular waveguide. If (9) is used to solve J_z, then (10) can be written as

$$\frac{\partial^2}{\partial\rho^2}A_z + \frac{1}{\rho}\frac{\partial}{\partial\rho}A_z + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}A_z + p^2 A_z = 0 \qquad (12)$$

where

$$p^2 = (k_o^2 - \beta^2)\left[1 - \left(\frac{\omega_p}{\omega}\right)^2\left(\frac{\beta_0}{\beta_0 - \beta}\right)^2\right] \qquad (13)$$

For the region between the helix outer region and the circular wave guide, $a \le \rho \le b$, $J_z = 0$, so that the Helmholtz equation for A_z is written as

$$\frac{\partial^2}{\partial\rho^2}A_z + \frac{1}{\rho}\frac{\partial}{\partial\rho}A_z + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}A_z + (k_o^2 - \beta^2)A_z = 0 \qquad (14)$$

The same equation is valid to the inner region of the tape helix without beam.

Equations (11) and (12) have to be solved with suitable boundary conditions. The helix is considered as a sheath helix that is an approximate model of a tape helix. The tape helix, see Fig. 1, consists of a tape wound into a helical structure. The pitch is denoted by p, and the pitch angle by ψ. If the spacing between turns and the tape width are made to approach zero, the resultant structure becomes electrically smooth. At the boundary surface $(\rho = a)$ the boundary conditions for the electric field may be considered to be that the conductivity in the direction parallel to the tape is infinite $(\sigma_{//} = \infty)$ whereas that in the direction perpendicular to the tape is zero $(\sigma_\perp = 0)$. The use of these boundary conditions allows us obtain a solution for the electromagnetic field guided by the helix. This anisotropic conducting cylinder model of a tape helix is called the sheath helix. The field solution shows that the sheath helix supports a slow wave with a phase velocity $v_p = c\sin\psi$, where c is the light velocity.

Without electron beam in the inner region of helix, the field solution for the helix consists of both TE and TM modes since these are coupled together by the boundary conditions at $\rho = a$. Along the direction of the tape, the tangential electric field must vanish, since $\sigma_{//} = \infty$; thus

$$E_{z1}\cos\Psi + E_{\varphi 1}\,\mathrm{sen}\,\Psi = E_{z2}\cos\Psi + E_{\varphi 2}\,\mathrm{sen}\,\Psi \qquad (15)$$

where the subscripts 1 and 2 refer to the field components in the two regions $0 \le \rho \le a$ and $a \le \rho \le b$. The component of the electric field on the cylindrical surface ($\rho = a$) that is perpendicular to the tape, must be continuous since $\sigma_{\perp} = 0$ in this direction. Hence

$$E_{z1}\cos\Psi - E_{\varphi 1}\,\mathrm{sen}\,\Psi = E_{z2}\cos\Psi - E_{\varphi 2}\,\mathrm{sen}\,\Psi \qquad (16)$$

The component of $\vec{H}$ tangential to the tape must also be continuous since no current flows perpendicular to the tape, so a third boundary condition is

$$H_{\varphi 1}\cos\Psi + H_{z1}\,\mathrm{sen}\,\Psi = H_{\varphi 2}\cos\Psi + H_{z2}\,\mathrm{sen}\,\Psi \qquad (17)$$

Suitable expansions for the A_z in the two regions can be written as

$$A_z(\rho,\varphi,z) = \sum_{n=-\infty}^{\infty} a_n e^{-jn\varphi} I_n(h\rho)e^{-j\beta z} \text{ to } 0 \le \rho \le a \qquad (18)$$

$$A_z(\rho,\varphi,z) = \sum_{n=-\infty}^{\infty} b_n e^{-jn\varphi} K_n(h\rho)e^{-j\beta z} \text{ to } a \le \rho \le b \qquad (19)$$

where $I_n(h\rho)$ and $K_n(h\rho)$ are the modificated Bessel's functions [10]. The electric and magnetic fields components for the TE and TM mode can be obtained from (17) and (18). In the presence of an electron beam that propagates in direction of the helix axis, and for an electron beam with axially confined flow, where only a z component of velocity is permitted, the TE modes, $0 \le \rho \le a$ region, are not affected by the electron beam since these have $E_z = 0$, and then they are the same the ones that exist without a beam. For the $a \le \rho \le b$, region the TE and TM modes are the same modes in view that in this region there is no beam. On the other hand, the TM mode in $0 \le \rho \le a$ will have its eigenvalues given by (13). In view of that fact, we are trying to find a wave solution that corresponds to a growing wave, in which the eigenvalue p^2 will turn out negative and it will be replaced by $-g^2$, so

$$g^2 = (\beta^2 - k_o^2)\left[1 - \left(\frac{\omega_p}{\omega}\right)^2\left(\frac{\beta_0}{\beta_0 - \beta}\right)^2\right] \qquad (20)$$

For the sheath-helix model it is possible to find a solution for a field that satisfies the boundary conditions (16) through (18) for each integer n. For the present, we are interested in the solution $n = 0$, which has circular symmetry. The solution for A_z is proportional to $I_0(h\rho)$ and since E_z is proportional to A_z, we can choose

$$E_z = a_o I_o(g\rho)e^{-j\beta z} \qquad (21)$$

where a_o is an amplitude constant. The field components E_ρ and H_φ can be found from Maxwell's equation.

$$E_z = \frac{j\beta}{\beta^2 - k_0^2}\frac{\partial E_z}{\partial \rho} \text{ and } H_\varphi = \frac{k_o}{\beta} Y_o E_\rho$$

Thus the expressions for the fields in the regions $0 \le \rho \le a$ and $a \le \rho \le b$ for the axially symmetric case are:

For the TM modes

i) $0 \le \rho \le a$

$$E_z = a_o I_o(h\rho)e^{-j\beta z} \qquad (22)$$

$$E_\rho = \frac{j\beta g}{h} a_o I_1(g\rho)e^{-j\beta z} \qquad (23)$$

$$H_\varphi = \frac{-j\omega\mu_o}{h} a_o I_1(g\rho)e^{-j\beta z} \qquad (24)$$

ii) $a \le \rho \le b$

$$E_z = b_o K_o(h\rho)e^{-j\beta z} \qquad (25)$$

$$E_\rho = \frac{j\beta}{h} b_o K_1(h\rho)e^{-j\beta z} \qquad (26)$$

$$H_\varphi = \frac{-j\omega\mu_o}{h} b_o K_1(h\rho)e^{-j\beta z} \qquad (27)$$

For TE modes

i) $0 \le \rho \le a$

$$H_z = c_o I_o(h\rho)e^{-j\beta z} \qquad (28)$$

$$H_\rho = \frac{j\beta}{h} c_o I_1(h\rho)e^{-j\beta z} \qquad (29)$$

$$E_\varphi = \frac{-j\omega\mu_o}{h} c_o I_1(h\rho)e^{-j\beta z} \qquad (30)$$

ii) $a \le \rho \le b$

$$H_z = d_o K_o(h\rho)e^{-j\beta z} \qquad (31)$$

$$H_\rho = \frac{j\beta}{h} d_o K_1(h\rho)e^{-j\beta z} \qquad (32)$$

$$E_\varphi = \frac{-j\omega\mu_o}{h} d_o K_1(h\rho)e^{-j\beta z} \qquad (33)$$

where the eigenvalue is $h^2 = \beta^2 - k_o^2$.

The boundary conditions at $\rho = a$ for the sheath helix model are given by (15) through (17). These boundary conditions when used with (22) through (33) together will result in the following homogeneous equation system

$$\begin{pmatrix} I_o(ga)\cos\psi & 0 & -\frac{j\omega\mu_o}{h} I_1(ga)\cos\psi & 0 \\ 0 & K_o(ha)\sin\psi & 0 & \frac{j\omega\mu_o}{h} K_1(ha)\cos\psi \\ I_o(ga)\cos\psi & -K_o(ha)\cos\psi & -\frac{j\omega\mu_o}{h} I_1(ga)\sin\psi & \frac{j\omega\mu_o}{h} K_1(ha)\sin\psi \\ \frac{j\omega\varepsilon_o g}{h^2} I_1(ga)\cos\psi & \frac{j\omega\varepsilon_o g}{h} K_1(ha)\cos\psi & I_o(ha)\sin\psi & -K_o(ha)\sin\psi \end{pmatrix} \begin{pmatrix} a_0 \\ b_0 \\ c_0 \\ d_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (34)$$

For a nontrivial solution the determinant (34) must vanish. So, equating the determinant to zero gives the following equation dispersion

$$(ga)\frac{I_1(ga)}{I_o(ga)} = \frac{(ha)^3\tan^2\psi}{(k_0 a)^2}\left[\frac{I_o(ha)}{I_1(ha)} + \frac{K_o(ha)}{K_1(ha)}\right] - (ha)\frac{K_1(ha)}{K_o(ha)} \qquad (35)$$

where the eigenvalues g and h are related by

$$(ga)^2 = (ha)^2\left[1 - \left(\frac{\omega_p}{\omega}\right)^2\left(\frac{\beta_0}{\beta_0 - \beta}\right)^2\right]. \qquad (36)$$

and

$$(ha)^2 = (\beta a)^2 - (k_0 a)^2. \qquad (37)$$

III. DISPERSION EQUATION SOLUTION

In this section is presented a solution of the dispersion equation for the helix structure. In order to solve the transcendental dispersion equation (35), the following approach is adopted. Since we are dealing with a slow-wave system, the square of propagation constant β^2 will be large when compared with k_0^2, so $h^2 \approx \beta^2$. Additionally we assume that $\beta = \beta_0(1+\delta)$, where δ is a small complex quantity. With this hypothesis, we can obtain a suitable complex propagation constant that corresponds to a growing wave. We choose δ as

$$\delta_1 = \frac{1}{4}\left(\frac{\omega_p}{\omega}\right)^{\frac{2}{3}}(1 + j\sqrt{3}), \qquad (38)$$

$$\delta_2 = \frac{1}{4}\left(\frac{\omega_p}{\omega}\right)^{\frac{2}{3}}(1 - j\sqrt{3}), \qquad (39)$$

so that $\delta^2 = \delta_1\delta_2 = \frac{1}{4}(\omega_p/\omega)^{\frac{4}{3}}$. It can be seen that with this chosen δ^2 is a small quantity, because $\omega_p << \omega$. Therefore (36) can be written as

$$(ga)^2 = (ha)^2\left[1 - 4\left(\frac{\omega_p}{\omega}\right)^{\frac{4}{3}}\right]. \qquad (40)$$

so that (35) can now be solved by iterative methods. The solution of (36) is shown in Fig.2.

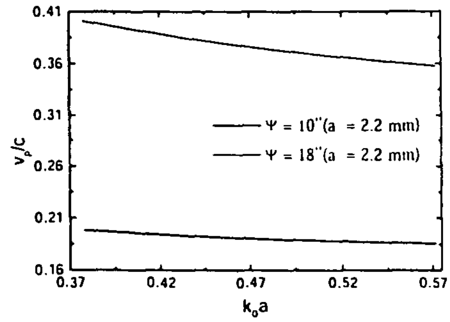

Figure 2. Dispersion equation solution

A third root for δ can be seen to be $\delta_3 = (1/2)(\omega_p/\omega)^{\frac{2}{3}}$, and the correspondent propagation constants are

$$\beta_1 = \beta_0\left[1 + \frac{1}{4}\left(\frac{\omega_p}{\omega}\right)^{\frac{2}{3}}(1 - j\sqrt{3})\right]. \qquad (41)$$

$$\beta_2 = \beta_0\left[1 + \frac{1}{4}\left(\frac{\omega_p}{\omega}\right)^{\frac{2}{3}}(1 + j\sqrt{3})\right]. \qquad (42)$$

$$\beta_3 = \beta_0\left[1 - \frac{1}{2}\left(\frac{\omega_p}{\omega}\right)^{\frac{2}{3}}\right]. \qquad (43)$$

and the growth constant α_g is, by (37), given by

$$\alpha_g = \beta_0\frac{\sqrt{3}}{4}\left(\frac{\omega_p}{\omega}\right)^{\frac{2}{3}}. \qquad (44)$$

IV. TWT CATHODE

The approach used to design the TWT cathode is based on Pierce [4]. A numerical code based on Poisson equation and integration of electron motion equation was developed to carry on the electron path. In Fig. 3 is shown the perveance cathode curve of the TWT cathode [9].

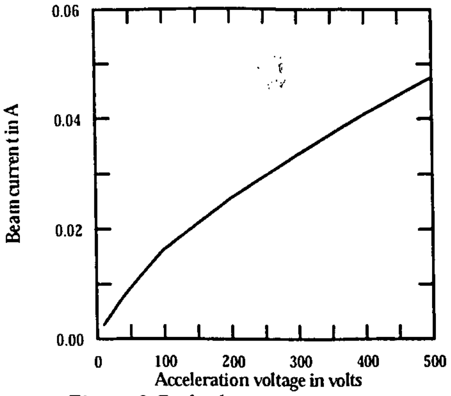

Figure 3 Cathode perveance curve.

Further details for the cathode design, cathode oxide and ultra high vacuum experimental techniques can be found elsewhere [7] [8].

V. RESULTS

Table I shows the main features of the TWT developed in our laboratory. The TWT was operated for hundreds of hours in a Marconi radar mainframe type 910 and during this time all the parameters have been constants [9].

TABLE I
MAIN FEATURES OF TWT

Quantity	Value
Helix length	290 mm
Helix radius	2.2 mm
Helix pitch	5 mm
Pitch angle	18°
Circular waveguide radius	4.8 mm
Mean beam current	120 mA
Tipical anode voltage	30 kV
Duty cycle	2%
Mean heater power	50 W
Gain	20 dB
Frequency	x-band
Grid bias	500 V

Figure 4 shows the theoretical gain feature of TWT as a function of frequency, where it is assumed a 0.6 as plasma frequency reduction factor [11].

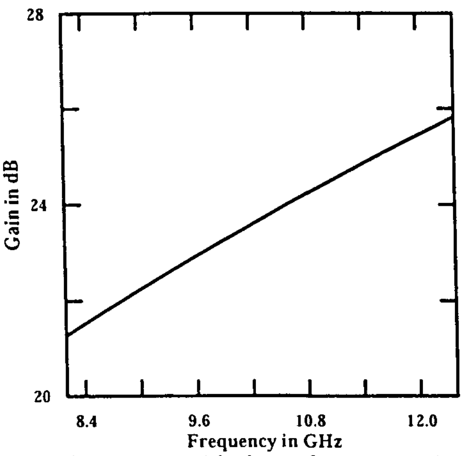

Figure 4. Theoretical gain curve with plasma frequency reduction factor 0.6 [11].

Figure 5 shows the TWT with the cathode disassembled.

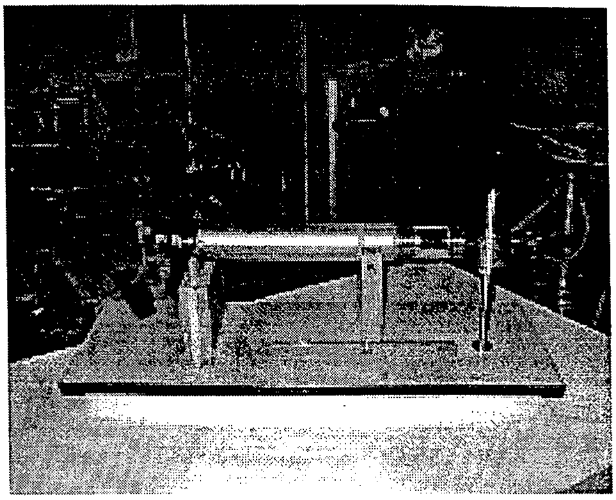

Figure 5 TWT with cathode disassembled.

VI. CONCLUSIONS

This paper presents the experimental and theoretical results from a research on X-band traveling wave tube amplifier designed and operated at CTMSP laboratory. It is investigated a suitable microwave slow-wave circuit through a solution of the dispersion equation. An iterative numerical solution for dispersion equation, following the Pierce approach, has been carried out and a mean gain of 20 dB has been obtained. The cathode perveance measured is 0.2 μPerveance and until now the life time of the cathode is over 1000 operation hours.

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IX SIMPÓSIO BRASILEIRO DE MICROONDAS E OPTOELETRÔNICA

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