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A Noise Analysis Approach for Measuring the Decay Constants and the Relative Abundance of Delayed Neutrons in a Zero Power Critical Facility

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This experimental work introduces a novice approach for the determination of the decay constants and the relative abundance of the delayed neutrons. The experimental procedure consists in obtaining the CPSD from the signals of two compensated ionization chambers in the frequency range from 0.005 to 5.0 Hz approximately. Assuming the point kinetic model, the theoretical expression for the CPSD can be written and the experimental CPSD can be fitted employing a least-square approach. The fitting parameters are β_i and λ_i. A preliminary experiment was made in the IPEN/MB-01 reactor. The least-square approach was able to fit all the betas for a six group model. However, it was not possible to fit the first decay constant (the group of longest half-live) accurately although the other five were achieved. The analysis reveals that the relative abundances of ENDF/B-IV and –VI are in a fair good agreement and that it is not only possible to perform successfully such measurements but also the decay constants and the relative abundance can be measured with a very good level of accuracy. ENDF/B-VI heavily underestimates the reactivity for small negative periods because its first decay constant is overestimated.

KEYWORDS: delayed neutrons, zero power reactor noise, kinetics parameters measurements

I. Introduction

Although comprising less than 1% of the neutrons emitted by fission, the delayed neutrons play a fundamental role in the reactor physics field. The control and accident analysis of a nuclear reactor as well as the conversion of period into reactivity requires the knowledge of the abundance (β_i) and the decay constants (λ_i) of the delayed neutrons. In a nuclear chain reaction there are many fission products (approximately 250) which can be considered potential delayed neutron emitters. However, an experimental characterization of all these emitters is very difficult due their low yield and/or low half-lives and also due to their very complex transmutation chains. Yet, it is possible to measure its aggregate behavior and generate a few groups model where the decay constants and abundance are mean values of various emitters with similar decay constants.

There are several experimental ways to determine these delayed neutrons parameters[1] and they are generally classified as "in-pile" and "out-of-pile" experiments. The purpose here is to introduce a new in-pile experiment based on the measurement of the fluctuations of the neutron population. In this technique the Cross Power Spectral Density (CPSD) between the signals of two neutron detectors is measured in a very low frequency range and the result is least-square fitted assuming a point kinetic model. The parameters of the fit are β_i or λ_i. This technique is interesting because it does not disturb the reactor, which is always maintained in a critical state, there is neither contamination of the results due to the harmonic excitation or residual multiplication and nor dependence on the efficiency and positioning of the detectors.

The experiments were realized in the IPEN/MB-01 critical facility which consists of a 28 x 26 array of UO_2 fuel rods, 4,3% enriched and clad by stainless steel (type 304) inside of a light water tank[2]. For the present work the original array was changed as described in the experimental procedure.

II. Theory

The global dynamic behavior of a nuclear reactor can be described adequately using the point reactor model through the well known point kinetic equations.

Considering six groups of delayed neutrons and assuming that the neutron flux and the delayed neutron precursor concentration are composed of a steady part and a fluctuating part, the zero-power transfer function can be obtained as[3,4]:

$$H(f) = \frac{\Lambda}{i\omega\Lambda - \rho + \sum_{j=1}^{6} \frac{i\omega\beta_j}{i\omega + \lambda_j}} \qquad (1)$$

where $\omega = 2\pi f$ is the angular frequency and $i = \sqrt{-1}$.

Now a generic CPSD which depends upon the electronics transfer functions, the detector currents and the reactor parameters can be written as[5]:

$$\Phi_{nk} = 2D\frac{\gamma I_n I_k}{P\Lambda^2}|W(f)|^2|H(f)|^2 \qquad (2)$$

where

D = 0.795 is the Diven factor

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$\gamma = 3.2E\text{-}11$ is the energy released per fission (in Joules)
I_j is the current from detector j (n or k) in Amperes
P is the reactor power in Watts
W(f) is the transfer function of the associated electronics.

III. Experimental Procedure

In order to obtain the experimental CPSD, the experimental setup was assembled as follow: two compensated ionization chambers CC54A from Merlin-Gerin operating in current mode, two Keitthley 614 electrometers to read and convert the currents from the CC54A chambers into voltage signals and two filter-amplifiers IPEN 036.ZZ (low frequency cut-off of 1.0 mHz) to cut off the DC component of the voltage signals and to amplify the AC component which is composed of the correlated and uncorrelated noise. The AC signals are then sent to the HP3562A dynamic signal analyzer (DSA) which has 800 frequency lines to perform the CPSD between the two signals. In making the CPSD the uncorrelated noise is eliminated or, at least, well minimized.

The transfer function of the electronic equipments can now be obtained. The electrometers do not alter the frequency of the signals from the ionization chambers since they just convert current into voltage. Thus, the transfer function of the electrometers is a constant given by:

$$E = \frac{2}{\text{electrometer scale}} \quad \frac{[\text{Volts}]}{[\text{Ampere}]}$$

The number 2 above is the output voltage when the current reading is at the full scale of the electrometer.

On the other hand, the filter-amplifiers change the frequency composition of the signal since they cut off the DC component. However this occurs in a very low frequency (around 1.0 mHz) and for our purposes the transfer function of the filters can also be considered a constant given by the gain G of the filter-amplifiers. For the IPEN 036.ZZ, G = 1, 10, 30, 100, 300, 1000, 3000 and 10000.

With the transfer functions of the electronic equipment as given above the theoretical CPSD for the system reactor-detectors-electronics is:

$$\Phi_{nk} = 2D\frac{\gamma I_n I_k}{P\Lambda^2}E_n E_k \cdot G_n G_k \cdot \left|H(f)\right|^2 . \qquad (3)$$

A dimensional analysis of eq. (3) shows that Φ_{nk} has units of V^2/Hz.

The experimental CPSD was obtained with the reactor as close to critical as possible and at a thermal power of 20 W. At this power level the average currents from the ionization chambers were 262×10^{-9}A and 280×10^{-9}A for the CC54A1 and CC54A2 respectively. With the electrometers scales in 2000×10^{-9} A their respective transfer functions are $E_n = E_k = 2/2000\times10^{-9} = 1.0\times10^{6}$ V/A. The gain of the filter-amplifiers was set to 30 so $G_n = G_k = 30$. The HP3562A DSA was set as follow: differential input mode, DC coupling, Hanning windowing, linear average and linear resolution.

In the course of the data acquisition, the control rods were "frozen" in order to avoid the interference of its movement in the low frequency region. Eventually the power level may begin to change and in this case, the data acquisition is stopped and the power is restored either manually or with one of the control rods returning to the automatic mode. The core configuration of the IPEN/MB-01 reactor has also been changed to get a lower reactivity and consequently a better manual control over the rod that correct the power. This new configuration as well as the detectors positioning are shown in **Fig. 1**.

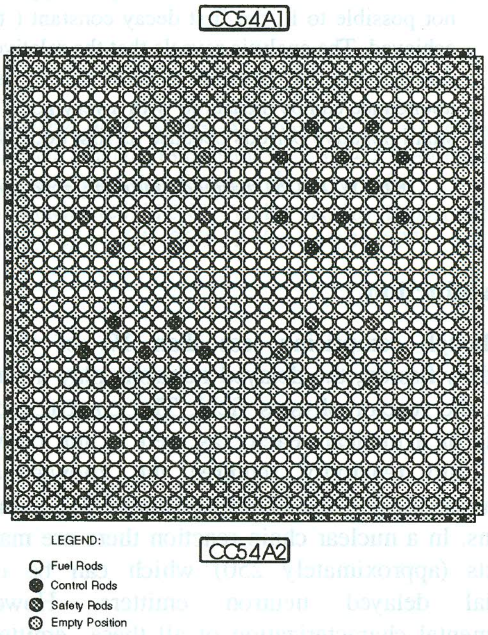

Fig. 1 Core configuration for the CPSD measurements and the detectors position.

The data acquisition by the dynamic signal analyzer was done in 5 frequency intervals in order to obtain the entire CPSD including the plateau. In the plateau region ($\omega >> \lambda_i$) one can obtain the absolute power of the reactor but it is not the interest of this work. The signals have been acquired up to the plateau and beyond that only for illustration purposes. The five steps of data acquisition were:

- from 0 to 0.640 Hz
- from 0 to 2.0 Hz
- from 0.640 to 16.64 Hz
- from 16.64 to 32.64 Hz
- from 16.64 to 56.64 Hz

In all acquisitions the experimental conditions were identical and it was performed 200 averages. The frequency resolution for each step is given by Δf/800 where Δf is the frequency interval and the record length is the inverse of frequency resolution. The resulting curve is shown in **Fig. 2**.

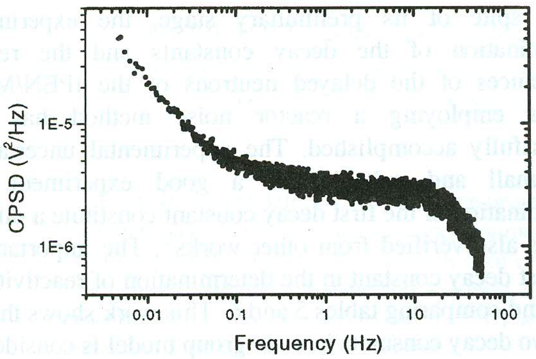

Fig. 2 Experimental CPSD containing 4442 data points and 200 averages obtained with all the five frequency interval added together.

Figure 2 shows that the plateau is in the interval 1.0 to 10 Hz approximately and the region of interest for the present work is about 0.0048 to 5.0 Hz because it is the region where the delayed neutrons dominate the CPSD; i.e , $\omega << \lambda_i$. This experimental CPSD was fitted assuming the theoretical CPSD given by Equation (5). The fitting parameters are β_i and λ_i .

IV. Results and Discussion

The least-squares fitting approach considered in this work assumes only six groups of delayed neutrons. In all cases, the neutron generation time was fixed at 30μs. Due to the preliminary stage of the experiment, this work considered a simplified approach in the least-square fitting process; i.e. the analyses let the relative abundances to be fixed and the decay constants to be fitted and vice-versa. To start the process, the delayed neutron data from two nuclear data library; namely ENDF/B-VI and ENDF/B-IV were considered and properly weighted by CITATION⁶⁾ in order to reflect the kinetic characteristic of the IPEN/MB-01 reactor.

Considering the fitting of the decay constant first, the relative abundances of ENDF/B-VI and -IV were kept fixed individually. Convergence was achieved only if the decay constant of the first group (longest-lived precursor) was kept fixed. In all cases, the results are very similar to those of the nuclear data libraries with exception to the last two decay constant which achieve higher values (more specifically 2.8860 and 12.9544 s^{-1}). On the other hand by specifying the decay constants of the aforementioned libraries convergence was obtained for the relative abundances. The results are shown in **Tables 1** and **2**. Exp.1 and Exp.2 means the results obtained for the relative abundance when the decay constants are those of ENDF/B-VI and -IV respectively, while Exp.3 and Exp.4 are respectively the results for the same conditions but with the last two decay constants replaced by 2.8860 and 12.9544 s^{-1} ,i.e. the values found in the fitting of the decay constants. These tables show that the relative abundances are sensitive to the choice of the decay constants and with exception to the 5th value of table 1 the uncertainties are in a very good level. Table 2 shows that when the last two decay constants are specified respectively to 2.8860 and 12.9544, the uncertainties come into a very good level.

Table 1 Results of Least-Squares Fit.

Exp-1 (λ's from ENDF/B-VI)	Exp-2 (λ's from ENDF/B-IV)
0.0333 ±0.0007	0.0365 ±0.0006
0.1993 ±0.0045	0.1880 ±0.0036
0.2357 ±0.0091	0.2320 ±0.0077
0.3490 ±0.0142	0.3528 ±0.0106
0.0022 ±0.0116	0.0492 ±0.0092
0.1806 ±0.0073	0.1415 ±0.0066

Table 2 Results of Least-Squares Fit.

Exp-3 (λ's from ENDF/B-VI)	Exp-4 (λ's from ENDF/B-IV)
0.0328 ±0.0004	0.0361 ±0.0004
0.1986 ± 0.0029	0.1883 ±0.0029
0.2220 ± 0.0065	0.2188 ±0.0063
0.3628 ± 0.0075	0.3719 ±0.0073
0.1438 ± 0.0057	0.1458 ±0.0056
0.0400 ± 0.0050	0.0391 ±0.0050

A comparison in terms of the relative abundances predicted by ENDF/B-VI and – IV with the experimental values is shown in **Table 3**.

Table 3 Comparisons of the calculated relative abundances with the in-pile experiments.

ENDF/B-VI C/Exp-1	ENDF/B-IV C/Exp-2	ENDF/B-VI C/Exp-3	ENDF/B-IV C/Exp-4
0.9969	0.9890	1.0121	1.0000
0.8886	1.0760	0.8917	1.0743
0.7344	0.8038	0.7797	0.8523
1.1031	1.1544	1.0611	1.0951
74.7727	2.7886	1.1439	0.9404
0.3709	0.2155	1.6750	0.7801

In general there appears to be qualitative agreement between theory and experiment, but in the case of Exp-1 and Exp-2 the disagreement for the last two relative abundance is quite severe. The agreement is improved as shown in table 3 in the case of Exp.3 and Exp.4. This aspect might indicate that the decay constants of ENDF/B-VI and –IV for the last two groups might be underestimated. In fact, the out-of-pile experiments, from which the kinetic parameters of these libraries were derived, rely on a fast removal system to remove the fissile sample from the irradiation position to the counting systems. Since the last two decay constants are quite large, the removal time of such experiments might not be fast enough to resolve them.

One of the most important benefits arising from the measurements of the delayed neutron parameters is the establishment of a more accurate relationship between true reactivity of the reactor system and an asymptotic period. This relationship, of-course, is the inhour equation. **Tables 4** and 5 show the comparisons of the reactivities in units of $ that would be obtained from the inhour equation considering the kinetic parameters of ENDF/B-VI and -IV, and those of Exp-1 trough Exp-4. In spite of the discrepancies from the relative abundances, the reactivities of tables 3 and 4 for the same decay constant set are in a fair good agreement. In contrast to that comparing table 3 and 4 the disagreement for small negative period is very noticeable. By analyzing the decay constant of ENDF/B-VI and –IV one may note that there is a slight difference in the first group. The first decay constant has a special meaning because it gives rise to the limit of the asymptotic period for large negative reactivities. These value for ENDF/B-IV (0.01272 s^{-1}) matches better the value found by several other authors[7,8]. Replaceing the first decay constant of ENDF/B-VI by that of ENDF/B-IV the results come into a better agreement as shown in **Table 6**.

Table 4 Comparison of Reactivity Calculated from Inhour Equation

Period (s)	ENDF/B-VI	Exp-1	Exp-3
1	0.7545	0.7516	0.7417
10	0.3510	0.3762	0.3714
100	0.0827	0.0904	0.0893
200	0.0464	0.0507	0.0501
-200	-0.0667	-0.0723	-0.0715
-100	-0.2093	-0.2225	-0.2199
-90	-0.2940	-0.3095	-0.3059
-80	-0.6674	-0.6866	-0.6780

Table 5 Comparison of Reactivity Calculated from Inhour Equation

Period (s)	ENDF/B-IV	Exp-2	Exp-4
1	0.7768	0.7486	0.7420
10	0.3814	0.3756	0.3723
100	0.0929	0.0912	0.0905
200	0.0524	0.0514	0.0510
-200	-0.0768	-0.0754	-0.0748
-100	-0.2579	-0.2549	-0.2530
-90	-0.3941	-0.3916	-0.3885
-80	-2.2213	-2.2381	-2.2179

Table 6 Comparison of Reactivity Calculated from Inhour Equation with the first Decay constant of ENDF/B-VI Replaced by that of ENDF/B-IV.

Period (s)	ENDF/B-VI λ_1=0.01272	Exp-1	Exp-3
1	0.7545	0.7516	0.7843
10	0.3512	0.3764	0.3809
100	0.0831	0.0908	0.0907
200	0.0467	0.0510	0.0509
-200	-0.0683	-0.0738	-0.0735
-100	-0.2307	-0.2440	-0.2422
-90	-0.3548	-0.3704	-0.3671
-80	-2.035	-2.0574	-2.0316

V. Conclusions

In spite of its preliminary stage, the experimental determination of the decay constants and the relative abundances of the delayed neutrons of the IPEN/MB-01 reactor employing a reactor noise method has been successfully accomplished. The experimental uncertainties are small and suitable for a good experiment. The determination of the first decay constant constitute a difficult task as also verified from other works[1]. The importance of the first decay constant in the determination of reactivity can be found comparing tables 5 and 6. This work shows that the last two decay constants in a six group model is considerable higher than that verified in the out-of-pile experiments. This aspect might indicate that the out-of-pile experiments may have difficult in the determination of such decay constants since in general they rely on a fast removal system which might not be fast enough. The relative abundances in general depends on the choice of the decay constants and this work shows that in order to have a better agreement between theory and experiment the last two decay constants have to be replaced by those obtained from this in-core experiment (higher values). The reactivities obtained from the inhour equation show little sensitivity to the relative abundances, and for the same decay constants set (ENDF/B-IV or –VI) a good agreement is found between theory and experiment. However, ENDF/B-VI heavily underestimates the reactivity for small negative periods. The main reason is its first decay constant that is a little bit higher than the value found by several other authors.

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