2007 International Nuclear Atlantic Conference - INAC 2007
Santos, SP, Brazil, September 30 to October 5, 2007
ASSOCIAÇÃO BRASILEIRA DE ENERGIA NUCLEAR - ABEN
ISBN: 978-85-99141-02-1

THE ANALYTICAL BENCHMARK SOLUTION OF SPATIAL DIFFUSION KINETICS IN SOURCE DRIVEN SYSTEMS FOR HOMOGENEOUS MEDIA

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ABSTRACT

This paper describes a closed form solution obtained by the expansion method for the general time dependent diffusion model with delayed emission for source transients in homogeneous media. In particular, starting from simple models, and increasing the complexity, numerical results were obtained for different types of source transients. Thus, first an analytical solution of the one group without precursors was solved, followed by considering one precursors family. The general case of G-groups with R families of precursor although having a closed form solution, cannot be solved analytically, since there are no explicit formulae for the eigenvalues, and numerical methods must be used to solve such problem.
To illustrate the general solution, the multi-group (three groups) time-dependent without precursors was also solved and the results inter compared with results obtained by the previous one group models for a given fast homogeneous media, and different types of source transients. The results are being compared with the obtained by numerical methods.

1. INTRODUCTION

A Coordinate Research Project (CRP) under the auspices of IAEA is on-going on Analytical and Experimental Benchmark Analyses of Accelerator Driver Systems(ADS)[1,2].One of the works tasks concerns the development of ADS kinetics analytic benchmarks suitable for the assessment of models, numerical methods and codes for the time dependent analysis of source-driven multiplying systems. Within this framework, a collaboration is established between Politecnico di Torino (Italy), IPEN and IEN (Brazil.)

The study of time-dependent problems for neutron multiplying media is usually carried out in high performance computers by means of numerical codes .The computational tools use algorithms which reduce the model equations to algebraic problems that are then numerically solved, with regards to the full simulation tool, is necessary to consider the adequateness of the model in describing the physical situation. Also verification procedure has to be carried out, to verify that the software has been coded in such a way that the required operations are correctly performed (software verification) .A validation process will guarantee that the model adopted can capture the physical phenomena the interest (validation model) and that the numerical schemes and techniques can assure the required accuracy of the numerical results (validation of numerical method).

The above steps constitute a benchmarking process and can be conducted through different analytical, numerical and experimental approaches. In this paper an analytical solution is being briefly analyzed and some results are presented, considering highly simplified and idealized configuration. An analytical benchmark can be defined as a closed-form solution to some references problems that are important for the physical situation of interest [3].

A general rule for the validation of codes must be considered: one numerical technique that gives bad results for a simple system cannot give trustworthy results for more complicated systems [4]. In the field of the neutron kinetics, the analytical principle of benchmarking already was used successfully for the validation of numerical codes [5], and more recently an attempt was made for the source-driven systems [6]. Furthermore, a study on the convergence pattern and accuracy of the numerical finite difference scheme is on going. The preliminary assessment of the CINESP code [7] is among the objectives of the present research.

2. PHYSICAL PROBLEM

2.1. Physical Model

The diffusion model is used to study the neutron kinetic, considering homogeneous medium, a slab reactor, with zero boundary flux condition and the cross sections being typically for a fast system.

The general multigroup diffusion equation with delayed emissions can be written as follows:

$$\begin{cases} \dfrac{1}{u_g}\dfrac{\partial \phi_g(r,t)}{\partial t} = [D_g \nabla^2 - \Sigma_g]\phi_g(r,t) + \sum_{g'}^{G}[\Sigma_{g\to g'} + \chi_{P,g}(1-\beta)\nu\Sigma_{f\to g'}]\phi_{g'}(r,t) \\ + \chi_{D,g}\sum_{j=1}^{R}\lambda_j C_j(r,t) + S(r,t), \qquad , \ \mathrm{g} = 1,\ldots\ldots,\mathrm{G \ and \ j} = 1,\ldots,\mathrm{R} \\ \dfrac{\partial C_j(r,t)}{\partial t} = -\lambda_j C_j(r,t) + \beta_j \sum_{g'=1}^{G} \nu\Sigma_{f,g'}\phi_{g'}(r,t), \end{cases} \tag{1}$$

.

With the initial conditions:

$$\begin{cases} \Phi_g(r,0) = \Phi_{g,0}(r), \\ C_j(r,0) = C_{j,0}(r). \end{cases} \tag{2}$$

The boundary condition assumed to be:

$$\Phi_g(r_b, t) = 0, \tag{3}$$

where r_b is the boundary position.

The unknown functions, the precursor concentrations and also the source term are expanded in terms of eigenfunctions, that are solution of the Helmholtz problem, and if we introduce a matrix approach the vectors are expressed as a superposition of the Helmholtz eigenfunctions, givens by:

$$\overline{X}_k(t) = \sum_{l=1}^{G+R} a_{lk}(t)\overline{U}_{kl}(t), \qquad . \tag{4}$$

The vectors solution can be expanded in terms of the orthogonal eigenvectors solution of the eigenvalue problem for the characteristic matrix associated to the balance equation is a M (G x R) x (G x R) matrix, the eigenvalues are solution of the characteristic polynomial. These eigenvectors satisfies the ortogonalization condition, due to fact that one speed approximation guarantees self adjoint operators [8].The general solution for the vectorial problem is given by:

$$\overline{\Xi}(x,t) = \sum_{n=1}^{\infty} \sum_{l=1}^{G+R} a_{lk}(t)\overline{U}_{lk}\varphi_n(x), \qquad . \tag{5}$$

Where the solution vector has as components the flux for each energy group, and $\overline{U}_{lk}$ is the eigenvector solution for the characteristic matrix and $\varphi_n(x)$ is the eigenfunction solution of the spatial problem.

2.2. Solution of the Diffusion Problems

In this work three cases are studied, first for one speed neutrons approximation, without precursors that gives the solution:

$$\Phi(x,t) = \sum_{n=1}^{\infty} \left\{ A_n(0)e^{\alpha^n t} + \int S_n(t')e^{\alpha^n(t-t')}dt' \right\} \varphi_n(x), \tag{6}$$

The time constants α_n are depending of the k_{eff} and the neutron lifetime for each Helmoltz harmonic [9]. The second case is studied considering one family of precursors, this follows:

$$\Phi(x,t) = \sum_{n=1}^{\infty} \sum_{1}^{2} \left\{ a_{km}(0) e^{w_{jk}t} + \int_0^t s_{km}(t) e^{w_{jk}(t-t')} dt' \right\} \varphi_n(x), \tag{7}$$

The constants w_{jn} are the solution of the eigenproblems, and are identical for the direct and adjoint problems, and were found through the solution of the algebraic Inhour equation.

For three energy groups, similar results can be derived by generalizing the expansion method used previously and representing the vectors of the n-th spatial components flux in three components by means of the three eigenvectors, and three row eigenvector associated to the adjoint problem, for the same eigenvalues, that in this case are obtained using the explicit solution of the cubic equations [10], thus we obtain:

$$\overline{\Xi} = \sum_{n=1}^{\infty} \left[a_{n1}(t)\overline{U}_{n1} + a_{n2}(t)\overline{U}_{n2} + a_{n3}(t)\overline{U}_{n3} \right] \varphi_n(x), \tag{8}$$

Where the vector solution is given by: $\overline{\Xi} = (\Phi_1, \Phi_2, \Phi_3)^T$, and the components of this vector are the fluxes for each energy group.

3. RESULTS

The solution for the three cases allows to study the behavior of the neutron population following different space and time source shapes, with total unitary injection of neutrons into the system. The time distribution is here always considered as a pulse of 10e-6 s.
The figures 1, 2, 3 and 4 are presented to illustrate the transients following a spatial delta function or spatially rectangular injection of neutrons. For the three cases studied were used the physical constants in the Table 1 or Table 2[4].

The effective multiplication factor of the system is 0.98, in particular for the case 1, was obtained H= 22.89416192. For the case 2 was assumed β =0.0045, λ=0.08 and was obtained the same subcritical dimension. To the case 3 the subcritical dimension for effective multiplication constant equal 0.98 using cross sections for Table 2 has been calculated as H=23.09695823 [4].

Table 1. Cross sections for the homogeneous 1D system in one-group diffusion

One Energy Group	
1/v	0.90621e-7
Σ*tot*	3.45987e-1
Σ*a*	1.58430e-2
*ν*Σ*f*	3.33029e-2
Σ*s*	3.30178e-1

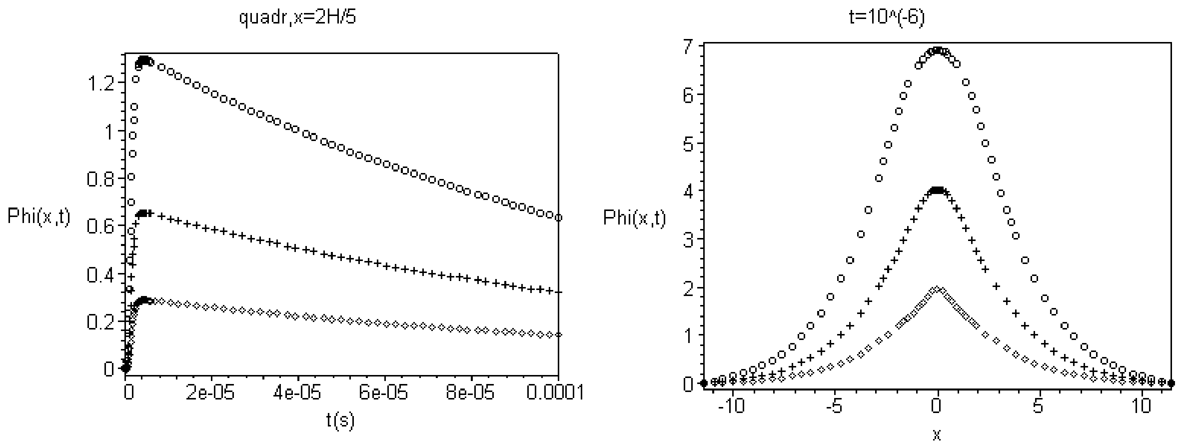

Figure 1. Dispersion of the fluxes in case 1, for a fixed time and position for different width pulses: "°" H/10, "×" H/20 and "◊" Dirac pulse.

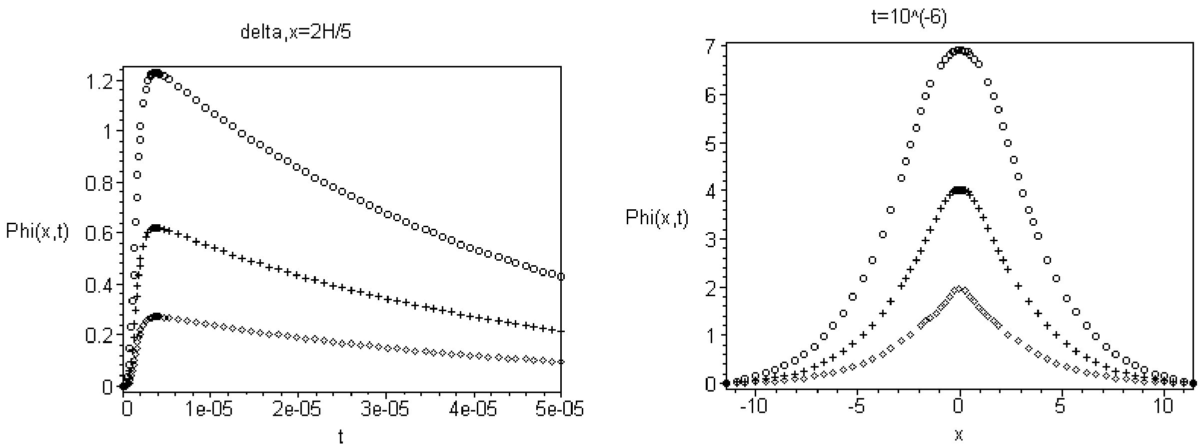

Figure 2. Dispersion of the fluxes in case 2, for a fixed time and position for different width pulses: "°" H/10, "×" H/20 and "◊" Dirac pulse.

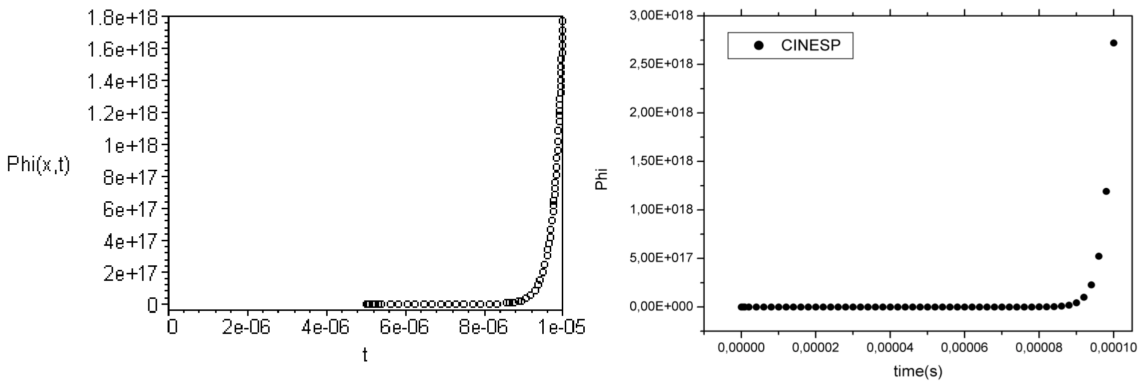

Figure 3. Dispersion of the fluxes in case 3, for a fixed position (2H/5) for a pulse of width H/10 for analytical results "°"and for numerical results "•", using CINESP.

A modified model was proposed, considering fission only in the group 3, and the spectrum of neutron produced by fission as zero in the first-energy group. Applying theses conditions in the general model (see eq.1), was calculated the flux distribution with slab subcritical dimension associated to keff =0.98 equal to H=14.06414306 (Figure 4).

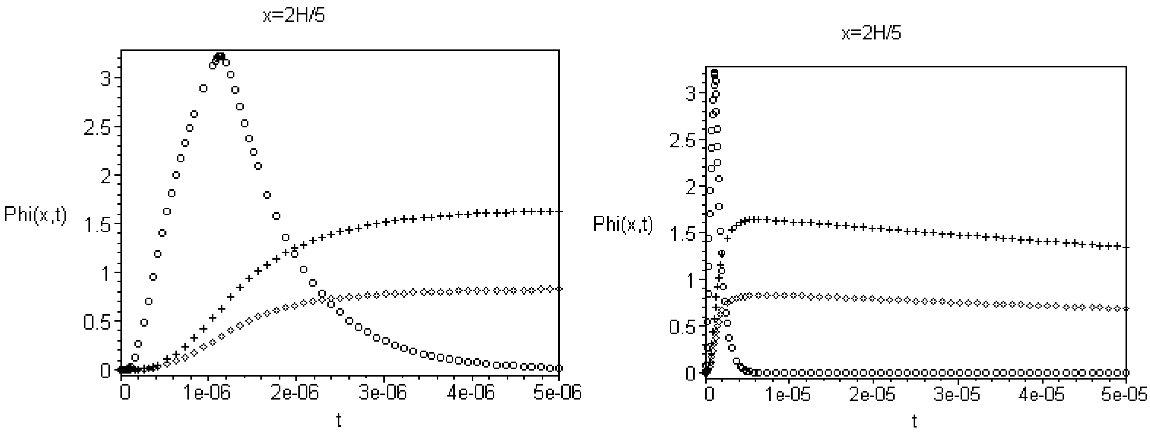

Figure 4. Dispersion of the fluxes for case 3 modified, for a fixed position for the three energy groups: "°" group 1, "×" group 2 and "◊" group 3, in different time periods.

Table 2. Cross sections for the homogeneous 1D system in three-group diffusion

		g =1	g=2	g=3
1/vg	[s.cm-1]	5.35994e-8	1.47756e-7	1.33310e-6
$\Sigma tot, g$	[cm-1]	2.83110e-1	3.65360e-1	6.65500e-1
$\Sigma a, g$	[cm-1]	1.40280e-2	1.63760e-2	6.98540e-2
$\nu\Sigma f, g$	[cm-1]	3.45310e-2	3.28690e-2	1.20020e-1
χ, g	-	0.7112	0.2886	0.0002
$\Sigma g \rightarrow g$	[cm-1]	2.36040e-1	3.48954e-1	5.95640e-1
$\Sigma g \rightarrow g+1$	[cm-1]	3.31803e-2	3.45000e-5	-
$\Sigma g \rightarrow g+2$	[cm-1]	1.16620e-5	-	-

4. CONCLUSIONS

The work presented some basic aspects in the neutron kinetics of subcritical multiplying systems. Some results for different physically problems were presented. Particularly, the study of the delayed neutrons showed that it can be disregard once the time scale of interest in faster systems is much shorter than the time delay of the precursor emission process, as we can see in Figure 2.

The solution methodology showed to be correct once the results for one group has presented physically acceptable results that agreed very well with known results[9].The analytical results for three energy groups using the cross sections from Table 2 presented physically unexpected results, once the system is subcritical. However, using the same cross sections similar results were obtained by numerical calculation using CINESP (Figure 3).We believe that the unexpected result showed in Figure 3 is due to the eigenvalues associated to the characteristic matrix for the three energy groups problem . These results are preliminary and will be more discussed in another work.

ACKNOWLEDGMENTS

The work is performed under the auspices of CNEN and support of the International Atomic Energy Agency (RC 13388).The authors are grateful to Prof. Piero Ravetto, by the nice reactor course lectured at the University of São Paulo.

REFERENCES

1. H.Ait-Abderrahim, A.Stanculescu, "IAEA Coordinate Research Project on Analytical and Experimental Benchmark Analyses of Accelerator Driven Systems," *Proceedings of ANS Topical Meeting on Reactor Physics, PHYSOR 2006*.Vancouver, 10-14 September, Paper C074 (2006).
2. J.R.Maiorino et al. "The Participation of IPEN in the IAEA Coordinated Research Projects on Accelerator Driven Systems (ADS)," *International Nuclear Atlantic Conference, INAC*,Santos, SP, Brazil (September 2007).
3. B.D.Ganapol, "The Analytical Benchmark Library for Nuclear Engineering: PC Version," Idaho National Engineering Laboratory, Idaho Falls (1992).
4. S. Dulla et al. , "Analytical Benchmarks for the Kinetics of Accelerator-Driven Systems," *Joint International Topical Meeting on Mathematics & Computation and Supercomputing in Nuclear Applications(M&C+SNA 2007)*, Monterey, California, April 15-19,2007,on CD-ROM,American Nuclear Society, LaGrange Park,IL(2007).
5. S.E. Corno, G.Manzo, P. Ravetto, R.Ricchena, "Analytical Methods in Local Reactor Dynamics and Validation of the Quasi-Static Approximation," *EUR 6390 En and European Applied Research Reports*, **1**(4), pp.831-945 (1979).
6. E.H.Mund, B.D.Ganapol, P.Ravetto, M.M.Rostagno, "Multigroup Diffusion Kinetics Benchmark of an ADS System in Slab Geometry," *Proceedings of Conference on the New Frontiers of Nuclear Technology: Reactor Physics, Safety and High-Performance Computing,* PHYSOR '02, Seoul, October, Paper 7B-03 (2002).
7. R.S.Santos, "Dynamics of nuclear cores based on one and two dimensional multigroup diffusion theory," *Proceedings of ENFIR-Brazilian Meeting on Reactor Physics,* Aguas de Lindoia, Brazil, 7-11 August, pp.143-148(1995).
8. P. Zweifel, *Reactor Physics*, McGraw-Hill, Tokyo, Japan (1973).
9. S. Dulla, P. Ravetto, M.M. Rostagno, "Some Features of Spatial Neutron Kinetics for Multiplying Systems," *Nuclear Science and Engineering*, **149**, pp.34-89 (2005).
10. W. H. Beyer, *CRC Standard Mathematical Tables*, CRC Press, Ohio, USA, 25 ed (1978).