Analysis of Multi-Parametric Coincidence Measurements with n Detectors

Frederico Antonio Genezini[1], *Guilherme Soares Zahn*[2], *Marcus Paulo Raele*[2], *Cibele Bugno Zamboni*[2] *and Manoel Tiago Freitas da Cruz*[3]

[1] Centro Regional de Ciências Nucleares - CRCN-CNEN/PE

[2] Instituto de Pesquisas Energéticas e Nucleares, Caixa Postal 11049, 05422-970, São Paulo, SP, Brazil

[3] Instituto de Física da Universidade de São Paulo, Caixa Postal 66318, 05315-970, São Paulo, SP, Brazil

Abstract. This work describes a method for calculating transition intensities from coincidence measurements. In order to use all the available statistics, the spectra obtained for all the detector pairs are summed up, and the resulting bi-dimensional spectrum is used to calculate the transition intensities. This method is then tested in the calculation of intensities for well-known transitions from both ^{152}Eu and ^{133}Ba standard sources.

Keywords. intensity calculation, multi-detector array, gamma-gamma coincidence

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1 Introduction

The use of multi-detector setups for data acquisition in coincidence measurements of γ-ray spectroscopy has become very frequent. While most multi-detector experiments aim to analyze angular correlation parameters, those facilities are useful for non-angular coincidence measurements as well, as in the verification of gamma-gamma coincidence relations and the determination of transition intensities. In these cases, the use of a multi-parametric coincidence system, where n detectors are associated as $L = n(n-1)/2$ independent pairs, represents an improvement, due to the reduction of the acquisition time, when compared to setups with only two detectors.

2 Theoretical Basis

Assume that there are two photons, γ_a and γ_b, emitted by a nucleus in a rapid succession (it is said that they belong to a *cascade*, $\gamma_a\gamma_b$), forming the angle ϕ_{ab} between their directions of emission. Two infinitesimal detectors, 1 and 2, subtending the solid-angle elements $d\Omega_a$ and $d\Omega_b$ respectively, are positioned at the direction of emission of the photons, at some distance from the radioactive source. The radioactive source is point-like and is positioned at the origin of the coordinate system. The number of detected coincidences is given by

$$dN(\phi_{ab}) = Sy_{ab}\left[\frac{d\Omega_a}{4\pi}\frac{d\Omega_b}{4\pi}W(\phi_{ab})\right]\epsilon^1(E_a,\Omega_a)\,\epsilon^2(E_b,\Omega_b) \tag{1}$$

where ϕ_{ab} is the angle between the solid-angle elements $d\Omega_a \equiv (d\vartheta_a, d\varphi_a)$ and $d\Omega_b \equiv (d\vartheta_b, d\varphi_b)$ with ϑ,φ representing angles in spherical coordinates. The term between brackets is the emission probability of the photons γ_a and γ_b inside the corresponding solid-angle elements. ϵ^1 and ϵ^2 are the intrinsic photo-peak efficiencies of detectors 1 and 2, respectively. Finally, S is the number of disintegrations occurred during the measurement time and y_{ab} is the emission probability of the $\gamma_a\gamma_b$ cascade (per disintegration). $W(\phi_{ab})$ is expressed as

$$W(\phi_{ab}) = 1 + A_{22}P_2(\cos\phi_{ab}) + A_{44}P_4(\cos\phi_{ab}) \tag{2}$$

where P_k are the Legendre polynomials of order k, with coefficients A_{kk}. It is seen that this is the angular correlation function, truncated due to the experimental sensitivity, normalized and not affected by solid angle effects.

Passing now to detectors of finite size, the number of detected coincidences becomes

$$N_{ab}(\theta^{12}) = \int dN(\phi_{ab}) = \iint \frac{d\Omega_a}{4\pi}\frac{d\Omega_b}{4\pi}W(\phi_{ab})\epsilon^1(E_a,\Omega_a)\epsilon^2(E_b,\Omega_b) \tag{3}$$

Here, θ^{12} is a reference angle, between the symmetry axes of detectors 1 and 2. Substituting $W(\phi_{ab})$ by (2), we have:

$$N_{ab}(\theta^{12}) = Sy_{ab}\sum_{k\ \mathrm{even}=0}^{4} A_{kk} \times \iint \frac{d\Omega_a}{4\pi}\frac{d\Omega_b}{4\pi}\epsilon^1(E_a,\Omega_a)\epsilon^2(E_b,\Omega_b)P_k(\cos\phi_{ab}) \tag{4}$$

with $A_{00} = 1$. Factoring the zeroth-order term, we produce:

$$N_{ab}(\theta^{12}) = \alpha\left[1 + \sum_{k\ \mathrm{even}=2}^{4} A_{kk}F_k(\theta^{12})\right] = \alpha\omega_{ab}(\theta^{12}) \tag{5}$$

with

$$\begin{aligned}\alpha =& Sy_{ab}\int \frac{d\Omega_a}{4\pi}\varepsilon^1(E_a,\Omega_a)\int \frac{d\Omega_b}{4\pi}\varepsilon^2(E_b,\Omega_b)\\ =& Sy_{ab}\varepsilon^1(E_a)\varepsilon^2(E_b)\end{aligned} \tag{6}$$

and

$$\omega_{ab}(\theta) = 1 + A_{22}Q_{22}P_2(\cos\theta) + A_{44}Q_{44}P_4(\cos\theta) \tag{7}$$

where one should note that now $\varepsilon^{i(j)}(E_{a(b)})$ is the absolute photo-peak efficiency of the i(j)-th detector at the energy $E_{a(b)}$. Q_{kk} are the solid angle correction coefficients that account for the attenuation of the angular correlation function. From equations (5)-(7), we can see that the number of detected coincidences is proportional to the product of the absolute efficiencies times the angular correlation function. Generalizing for several detectors and rewriting y_{ab}, the total number of coincidences for cascade $\gamma_a\gamma_b$ may be written as:

$$N_{ab}^{tot} = \sum_{(ij)=1}^{L} N_{ab}(\theta^{ij}) = SI_a br_b \sum_{(ij)=1}^{L} \varepsilon^i(E_a)\varepsilon^j(E_b)\omega_{ab}(\theta^{ij}) \tag{8}$$

where ij is the detector pair, θ^{ij} is the angle between the symmetry axes of the detectors, I_a is the emission probability of the first photon of the cascade, γ_a, and br_b is the normalized branching ratio of the second photon of the cascade, γ_b.

In order to eliminate the influence on the intensity calculations from both the source activity and the real time of acquisition (which is hard to determine in multiparametric measurements), two $\gamma\gamma$ cascades with a common transition were used. The ratio of the total numbers of detected coincidences for these cascades, by all detector pairs, is:

$$\frac{N_{ab}^{tot}}{N_{cb}^{tot}} = \frac{SI_a br_b \sum_{(ij)=1}^{L} \varepsilon^i(E_a)\varepsilon^j(E_b)\omega_{ab}(\theta^{ij})}{SI_c br_b \sum_{(ij)=1}^{L} \varepsilon^i(E_c)\varepsilon^j(E_b)\omega_{cb}(\theta^{ij})} \tag{9}$$

Now, in order to allow the analysis of very weak cascades, it would be interesting to use the spectrum summed over all angles. For that purpose, the following approximation shall be made, which relies on two basic assumptions: **a.** that the detector efficiencies are not very dissimilar from each other; and **b.** that the sum of $\omega(\theta)$ over the six detector pairs is approximately constant for all cascades:

$$\frac{N_{ab}^{tot}}{N_{cb}^{tot}} = \frac{I_a \sum_{(ij)=1}^{L} \varepsilon^i(E_a)\varepsilon^j(E_b)\omega_{ab}(\theta^{ij})}{I_c \sum_{(ij)=1}^{L} \varepsilon^i(E_c)\varepsilon^j(E_b)\omega_{cb}(\theta^{ij})} \approx \frac{I_a \sum_{(ij)=1}^{L} \varepsilon^i(E_a)\varepsilon^j(E_b)}{I_c \sum_{(ij)=1}^{L} \varepsilon^i(E_c)\varepsilon^j(E_b)} \tag{10}$$

and now the intensity of the transition γ_a may be written as:

$$I_a = I_c \frac{N^{tot}_{ab} \sum_{(ij)=1}^{L} \varepsilon^i(E_c)\varepsilon^j(E_b)}{N^{tot}_{cb} \sum_{(ij)=1}^{L} \varepsilon^i(E_a)\varepsilon^j(E_b)} \tag{11}$$

3 Experimental Validation

In order to give experimental support for the approximation made in eq.10, the intensities of both the 779- and 1090-keV transitions from ^{152}Eu, as well as the 356-keV transition from ^{133}Ba, were calculated using the forementioned methodology, and the results compared to the values found in reference [2].

3.1 Data Acquisition

The multi-parametric system used in this test, located at the Linear Accelerator Laboratory of the Instituto de Física da Universidade de São Paulo, is composed of four Ge detectors with active volumes ranging from 50 to 190 cm^3. The signals from these detectors are inserted into a usual fast-slow electronics, where the decisory functions are performed by a custom CAMAC module. If all logic conditions are met, a CAMAC controller, directly coupled to the bus of an IBM-compatible personal computer (PC) through an ISA-bus lengthener, manages the data transfer to the PC. Both devices, the controller and the lengthener, were designed and built in-house [1], and allow a high data throughput. The data were recorded on an event-by-event mode on hard disk. The whole system can be seen in Figure 1.

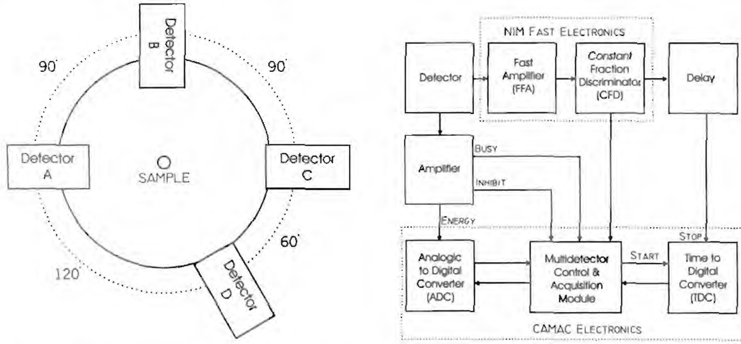

Figure 1 Experimental setups used in this work.
(left) Placement of the four detectors around the sample;
(right) Simplified scheme of the electronic setup.

In the present measurement, the system was exposed to the radiation from a 3.6μCi ^{152}Eu standard source for a total of 60 h; a standard ^{133}Ba source of approximately the same activity was also counted for the same time.

3.2 Data Reduction

The summation procedure mentioned above must be carefully done, and two steps are required. In the first step, a data set taken with a particular detector pair is time-gated in order to produce a pair of two-dimensional spectra, one containing accidental coincidences only (AC) and other with the total (true + accidental) coincidences (TC). In the second step, the AC spectra from all pairs are relocated in energy, in order to correct for differences in the energy calibrations of the detectors, and then summed together to produce a single two-dimensional AC spectrum; the same procedure is applied to the TC spectra, and the two "all-in-one" two-dimensional spectra are analyzed as if they were obtained from a single detector pair.

4 Results and Discussion

For this method to be applicable, the decay scheme of the nucleus under study must contain pairs of cascades with a common transition. Four cascades from ^{152}Eu have been used in this test: 344.3-778.9 keV and 344.3-411.1 keV, to determine the intensity of the 778.9 keV transition, and 344.3-1089.8 keV and 344.3-1299.2 keV to determine the intensity of the 1089.8 keV transition. Both decay schemes are shown in Figure 2. The results are presented in Table 1.

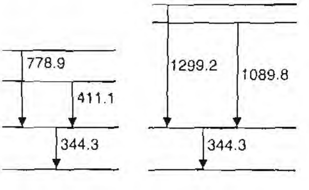

Figure 2 Simplified decay schemes showing the transitions from ^{152}Eu used in this work.

In addition to those, the 81.0-356.0 keV and the 81.0-302.9 keV cascades from ^{133}Ba were also used to determine the intensity of the 356.0 keV transition; the decay scheme for these transitions is shown in Figure 3, and the results are shown in Table 2.

As shown in Table 1, the intensity values obtained for the ^{152}Eu transitions by this method are statistically compatible with the corresponding values from the literature, while Table 2 shows that the result obtained for the 356 keV ^{133}Ba transition is roughly within 2 standard deviations from the value found in [2]. These results show that the proposed method is suitable for the obtention of transition intensities from multi-detector coincidence experiments.

Table 1 Comparison of the intensity results obtained for the ^{152}Eu transitions by the present method to the values found in [2].

Transition	778.9 keV	1089.8 keV
Peak Area	2.385(7) × 10^6	1.973(22) × 10^5
[cascade 1]	[344.3 - 778.9] keV	[344.3 - 1089.8] keV
Peak Area	6.91(4) × 10^5	1.669(19) × 10^5
[cascade 2]	[344.3 - 411.1] keV	[344.3 - 1299.2] keV
I_γ (this work)	13.1 (3)	1.69 (3)
I_γ ref [2]	12.98 (25)	1.711 (9)

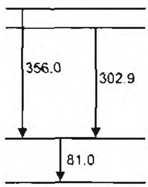

Figure 3 Simplified decay scheme showing the transitions from ^{133}Ba used in this work.

Table 2 Comparison of the intensity results obtained for the ^{133}Ba transition by the present method to the value found in [2].

Transition	356.0 keV
Peak Area	3.448(10) × 10^6
[cascade 1]	[81.0 - 356.0] keV
Peak Area	1.179(6) × 10^6
[cascade 2]	[81.0 - 302.9] keV
I_γ (this work)	0.605 (5)
I_γ ref [2]	0.6205 (19)

Bibliography

[1] D. Bärg-Filho, R. C. Neves, and V. R. Vanin. Proceedings of the XX Brazilian Workshop on Nuclear Physics, S. R. Souza *et al.* eds. (World Scientific, Singapore, 1998), p. 424.

[2] R. B. Firestone and V. S. Shirley, **Table of Isotopes** John Wiley & Sons, New York, 1996.