Influence of Periodic Magnetic Field Profiles on the Focusing of Electron Beams

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Abstract: *This paper presents the resuls of the influence of several periodic magnetic field profiles apllied on a typical electron beam existent in linear microwave tubes, such as traveling-wave tubes, from the well known trajectory equation. It was verified that a field profile like a square wave is the most indicated for the perfect focusing.*

Keywords: Magnetic focusing systems; microwave devices; periodic permanent magnet; traveling-wave tubes.

Introduction

It is well known the properties of periodic magnetic fields to confine electron beams. In linear microwave vacuum devices design, it is desirable a mathematical expression to describe the electric charge path from the anode toward the collector taking into account the axial and radial forces. The most common one is the Mathieu's equation, which considers that $\vec{B}$ presents a co-sinusoidal periodicity. However, the electron beam behaviour with other magnetic field profiles usually is not reported, so that is relevant to verify the existence of a more interesting situation. Thus, this paper discusses the influence of a sinusoidal, triangular and square wave profiles, using a co-sinusoidal function for comparisons.

Simulations

A commercial mathematical software has been used to carry out the beam trajectory simulations, considering the Brillouin focusing system, using the following equation

$$\frac{d^2\rho}{dz^2} = \left[\frac{\eta I_0}{2\pi\varepsilon_0\left(2\eta V_0\right)^{3/2}}\right]\frac{1}{\rho} - \left(\frac{\eta B_z^2}{8V_0}\right)\rho \qquad (1)$$

where η is the electron charge-to-mass ratio, I_0 the beam current, V_0 the beam voltage, ρ the electron radial position, and B_z the axial magnetic flux density. In (1), it was used the B_z profile for each case evaluated, graphically shown in Fig. 1, obtained from $B_z = B_0 cos(k)$ and $B_z = B_0 sin(k)$, or

$$B_z = B_0\left\{\frac{2.55}{\pi}\left[\frac{cos(k)}{1^2} + \frac{cos(3k)}{3^2} + \frac{cos(5k)}{5^2} + \ldots\right]\right\} \qquad (2)$$

$$B_z = B_0\left\{\left[\frac{4}{\pi}\left(\frac{sin(k)}{1} + \frac{sin(3k)}{3} + \frac{sin(5k)}{5} + \ldots\right)\right]\right\} \qquad (3)$$

where (2) and (3) refer to the triangular and square field profiles, respectively, obtained from Fourier series, B_0 is the peak axial magnetic flux density, $k = (2\pi z)/L$, z is the axial distance, and L is the wave period. Table 1 describes the parameters of the considered electron beam.

Results and discussion

Figure 2 presents the results of a comparison of the best electron beam paths for the four cases evaluated. It can be noticed that the outermost electron radial motion, analyzing the influence of either sinusoidal or co-sinusoidal magnetic flux density profiles, is exactly the opposite when it was used the same B_0 value, as reported in Tab. 2. Meanwhile, the ripple in module is the same, as expected, which kept around 3.5%. Furthermore, the beam edge dynamics agrees with previous reported researches [1]. The values of the magnetic field and the space charge parameters are not equal (the former is exactly a half compared to the latter), related in Tab. 2, because the trajectory equation that contains the sinusoidal function was not simplified, being solved to $sin^2\,(2\pi z)/L$.

An interesting result was obtained with a square "magnetic wave". As it can be verified, the ripple for this situation is inexistent, since it is less than 0.5% along the analyzed normalized axial distance. Although the field profile is periodic, it is possible that the square waveform itself acts on the beam in a similar way like a field with constant profile, as a solenoid, and because of this the peak magnetic flux density is lower than the previous situations evaluated. Similarly to the situation above discussed, $\alpha < \beta$.

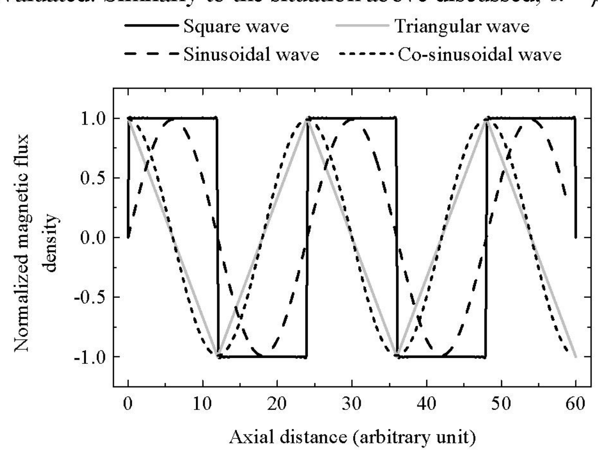

Figure 1. The magnetic field profiles evaluated. The magnitude for each one is not necessarily the same.

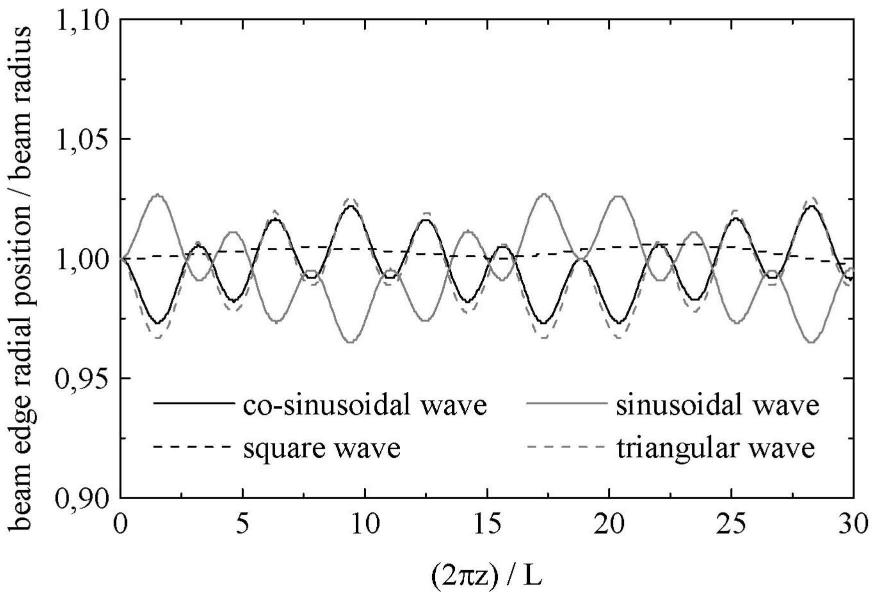

Figure 2. Electron beam paths confined by different periodic magnetic field profiles.

Table 1. Parameters of the electron beam used for simulations.

Parameter	Symbol	Value
Electric current (*A*)	I_0	8.6
Beam voltage (*kV*)	V_0	50.0
Beam radius (*mm*)	*a*	1.8
Period of field (*mm*)	*L*	24.0
Perveance (μPerv)	*K*	0.77

Table 2. Peak magnetic flux density utilized (B_0), value of the magnetic field (α) and the space charge (β) parameters for each case.

Waveform	B_0 (mT)	α (no unit)	β (no unit)
Co-sinusoidal	139	~ 0.05	~ 0.05
Sinusoidal	139	~ 0.11	~ 0.05
Square	98	~ 0.09	~ 0.05
Triangular	170	~ 0.11	~ 0.05

The last profile tested to confine the electron beam was the triangular waveform. Once again, the ripple of the outermost electron was kept small, around 3.5%. At this point, it is relevant to point out the likeness of the beam dynamics between the triangular and the co-sinusoidal profiles which can be previously noticed when the waveforms are analyzed in Fig. 1.

Conclusions

In this paper it was investigated the influence of the periodic magnetic field profile on the focusing of electron beams. Sinusoidal and co-sinusoidal magnetic field profiles act on electric charges in opposite way, generating a ripple which, in module, presents the same magnitude. In these situations, the peak magnetic density flux used was the same. Referring to the square wave profile, B_0 is the lowest compared to other situations analyzed and, because of this, it would be interesting for experimental application, if it could be possible. Triangular profile acts on the beam almost the same way the co-sinusoidal one, although B_0 is the biggest among the evaluated cases. As could be seen, for sinusoidal, triangular and square field profiles, the magnetic field parameter does not need be equal to the space charge parameter to obtain an homogeneous electrons flow, remembering that none of the equations were simplified (specially the Fourier series).

References

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