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Comparing Methods for the Numerical Integration of Signals in Accelerometry

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Summary: *The main objective of this work is to review the numerical integration methods of acceleration signals to obtain velocity signals used in vibration analysis of mechanical equipment. The acceleration signals are acquired from piezoelectric sensors positioned at the equipment. These signals are random continuous functions that are digitized by a data acquisition system generating a vector of discrete values. Several well established numerical integration schemes (Trapezoidal, Runge-Kutta, Open and Closed integration formulas) were tested as well as a integration method based on Fourier Transform. This last method proved to be the most adequate to integrate random signals since the gain in the low frequency range, typical of acceleration integration problems, was the lowest among the tested methods. The testing were conducted using analytical functions as well as real acceleration and velocity signals obtained experimentally.*

Keywords: *vibration, acceleration signals, velocity signals, methods of integration*

1. Introduction

The monitoring and diagnosis of industrial equipment and systems using vibration analysis requires generally the integration of acceleration signals sampled from accelerometers to generate velocity signals. The industrial standards and common practice indicate the velocity as the most appropriate variable to be used in vibration analysis for a wide range of applications. Sensors which generate directly signals that are proportional to the velocity are rarely used since they are difficult to fabricate and to use. The most commonly used sensors industry wise are based on piezoelectric elements actuated by inertial mass generating acceleration signals of the measured point.

Obtaining velocity from acceleration signals can be done either by electronic integration or by numerical digital integration. Electronic integration is performed during the data acquisition before the signal conversion to digital format, by electronic circuitry contained in the signal conditioning equipment. This technique is well consolidated and present adequate results requiring however conditioners which time response are compatible with the sample rate and the desired frequency range.

The objective of the present work is to assess the existing numerical integration methods for acceleration signals including a not commonly used method based on the Fourier Transform (Brigham, 1974). The classical methods evaluated include: Trapezoidal, Simpson, Runge Kutta, Open and Close Integration Formulas (Carnahan et all, 1969 e Pearson, 1974).

This work discuss the general problem of integrating signals and particularly, it address the numerical integration problem. It is discussed the numeric noise introduced in the high frequency range of the signal caused by truncation errors as well as the intensification of the noise already present in the signal, in the low frequency range. The influence of some parameters like the number of samples and the sampling frequency upon the velocity signal amplitudes and frequencies are also studied. The results are presented both for analytical signals as well as for real signals acquired from a experimental setup for simulation of defects in bearings.

2. Numerical Integration Formulas

Well established integration formulas are presented below without derivation, being the reader advised to reference numerical methods text books such as the one by Carnahan (1969) for details. Another method based on a Fourier transform property which can be used to integrate time dependent functions is also presented. The general problem statement is described as follows.

Given a time dependent continuous function a(t) we wish to calculate its integral v(t) pointwise for v(0)=v0 for:

$$v(t) = \int_0^t a(t)dt \qquad (1)$$

For digitized signals, the vector a_i (i=1,2...N) obtained from accelerometers, which is the case being studied, the function is random and continuous, representing the oscillatory vibration at the measured point. This peculiar characteristic of the function being integrated presents some special challenge to the problem.

It is presented below the integral rules for the four numerical methods. In these formulas, i=1,2....N, where N is the total number of sampled points and Δt is the time interval between two sampled points and equal to the inverse of the sampling frequency.

a) The Trapezoidal Rule (or the Euler Method) is calculated by:

$$v_{i+1} = v_i + 0.5*\Delta t\,(a_{i+1} + a_i) \qquad (2)$$

b) The Runge-Kutta fourth order method with Kutta's coefficients is:

$$v_{i+1} = v_i + \Delta t(a_{i+1} + 4*a_i + a_{i-1})/3 \qquad (3)$$

One should notice that in this case where the function to be integrated is time dependent only, this formula reduces to Simpson's rule. Although this formula is included, its results are not presented since it presents similar results compared to Trapezoidal rule.

c) The Open Integration Formula with order of the interpolating polynomial equal to 3, is done by:

$$v_{i+1} = v_i + \Delta t(55*a_i - 59*a_{i-1} + 37*a_{i-2} - 9*a_{i-3})/24 \qquad (4)$$

d) The Closed Integration Formula with order of the interpolating polynomial equal to 3, is done by:

$$v_{i+1} = v_i + \Delta t(9*a_{i+1} + 19*a_i - 5*a_{i-1} + a_{i-2})/24 \qquad (5)$$

One should notice that Runge-Kutta's method was derived using the Taylor's Series expansion, thus implying the existence and continuity of the derivatives up to the order of the approximating algorithm. The derivation of both the Open integration formula as well as the Closed integration formula, by its turn, is based on approximating polynomials which require the derivatives to be continuous up to a certain order. Only the Euler's integration method does not impose any requirement upon the continuity of the derivatives.

3. Integration using a Fourier Transform property.

Given the definition of the Fourier Transform of a time dependent function a(t) as :

$$A(w) = \int_{-\infty}^{+\infty} a(t)e^{-jwt}dt \qquad (6)$$

where w is the frequency domain and j is the imaginary unit number, we wish to calculate v(t) from A(w). One can demonstrate by integrating Eq. (6) by parts, after substituting a(t)=dv/dt that the following equation is true:

$$A(w) = jwV(w) = \int_{-\infty}^{+\infty} \frac{dv}{dt}e^{-jwt}dt \qquad (7)$$

In other words, to calculate v(t) from the Fourier transform A(w) of a(t), the following steps will produce the desired result:

a) Calculate the Fourier transform of a(t), generating A(w).
b) Calculate the velocity in the frequency domain V(w) using the first part of Eq. (7).
c) Calculate the inverse Fourier transform of V(w) to obtain v(t).

4. Integrating analytical functions.

In order to compare the performance of the different integration methods presented above, we studied the integration of simple analytical functions like sine, cosine, sum of sine and cosine as well as triangular shaped periodic functions. Since in machinery vibration analysis a important characteristic to be analyzed are the Fourier transform of the velocity signal into the frequency domain, we compared the velocity spectra, V(w) calculated by numerical integration with above formulas with that obtained by analytical integration of these functions.

The integration of acceleration signals to obtain velocities and then applying the Fourier transform to generate the velocity spectrum has a intrinsic problem which is the high gain in the lower frequency range. This is a unavoidable problem since as can be seen in Eq. (7), the velocity V(w) is divided by the frequency w. In frequency values approaching zero or the so called DC value, the velocity would theoretically tend to infinity. This high gain in the lower frequency range near the DC is also observed in the electronic analog integrators where gains of the order of 100 to 200 dB's are commonly found in typical commercially available instruments requiring therefore on board high pass filters.

This high gain in low frequencies is also present, of course, in velocity signals calculated from numerical integration schemes. In order to compare the error introduced by each numerical scheme we defined a error spectra calculated by the following formula:

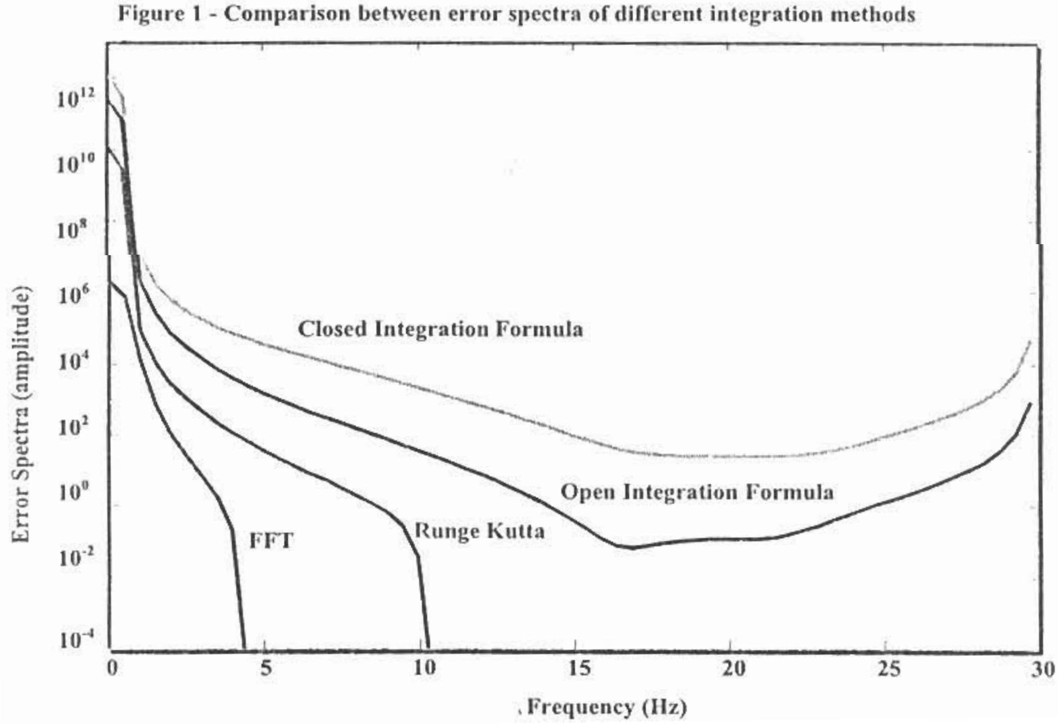
$$error(w) = \frac{V(w) - V_a(w)}{V_a(w)} \tag{8}$$

In Eq. (8), V(w) is the Fourier transform of the numerically integrated v(t) and $V_a(w)$ is the Fourier transform of the analytical function v(t). Since the most important effect of numerical integration errors show up at the low frequency range, particularly at the DC value, we also defined another performance index which we named the DC Gain, defined as follows:

$$Gain_{DC}(dB) = 20 * \log_{10}\left(\frac{V(0)}{V_a(0)}\right) \tag{9}$$

where V(0) and $V_a(0)$ are the magnitude of the velocity power spectrum density at frequency 0 Hz (DC value).

Figure 1 - Comparison between error spectra of different integration methods



All the algorithms were programmed using MATLAB software.

In Fig. (1), we can compare the error spectra as defined by Eq. (8) for the different integration methods used. The calculations were performed using the function a(t)=sine(2πft), with f=20Hz, the sampling frequency of 60Hz and a total number of points correspondent to 20 periods.

Figure 1. Comparison between errors spectra of different integration methods

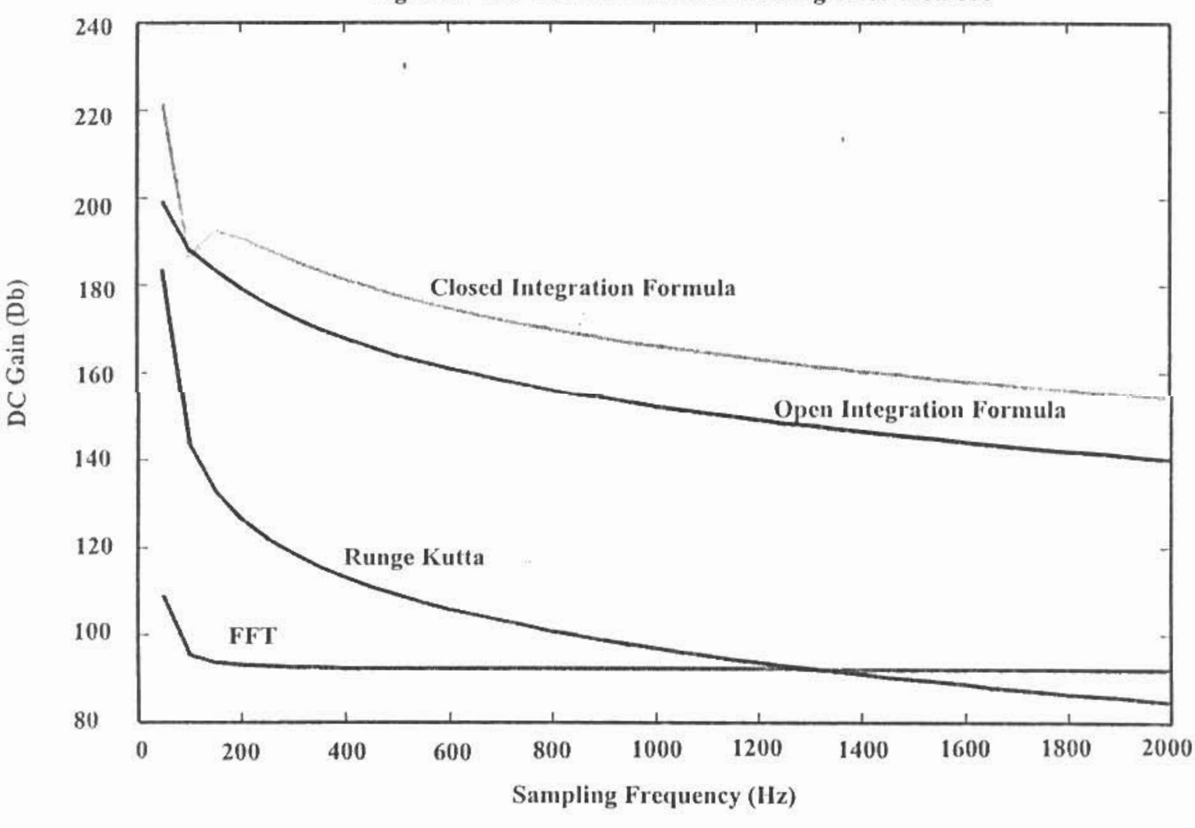
Analyzing Fig. (1), three important characteristics should be pointed out:

a) All integration methods introduce numerical noise. As expected, the most severe error shows up at the DC frequency which we denoted by DC gain. Since the velocity amplitudes in the frequency domain are divided by the frequency, for small frequency values even small errors are greatly amplified. The closed and the open integration formulas has inherently interpolation, truncation and rounding errors which, once amplified, produce the worst results (10^{12} and 10^{11}) at DC among the methods. The Runge-Kutta method has only truncation and rounding errors resulting in a smaller effect (10^{10}) at its DC value. The smallest DC gain error was produced by the (FFT) Fourier transform method with 10^{6}. This is because FFT method has only rounding errors when Nyquist criteria is applied (sampling frequency to be greater than two times the largest frequency component in the signal).

b) There is a region where the effect of low frequency error amplification is important. This region can be defined by a breaking frequency which can be defined as the frequency where the error amplitude drops to a certain specified error. If we define this error amplitude freely as being 10^{2}, the breaking frequencies of the methods would be 15, 10, 5 and 2.5 respectively. This result shows again that the FFT method is the best among the tested methods, for the same reasoning presented in item a above.

c) For methods with interpolation and truncation errors such as the closed and open integration formulas, these errors show up at every sampled point creating a high frequency component in the error spectrum as one can see in Fig. (1). This effect is negligible for Runge-Kutta and FFT methods.

Figure 2 - DC Gain for the different integration methods



Since the most important consequence of errors introduced by any given method shows up is in the DC value of the error spectra, we studied the influence of the sampling frequency in the DC gain as defined by the Eq. (9). The results are shown in Fig. (2) for the methods tested using the same signal used to produce the first figure. The most important conclusions drawn from this figure are:

a) All the numerical methods based on discretization and interpolation techniques (Closed and Open integration formulas and Runge-Kutta) has a monotonically decreasing behavior of the DC gain for increasing sampling frequency. This characteristic is expected since increasing the sampling frequency we are actually reducing the time interval between two adjacent points. These methods are strongly dependent on the time interval.

b) The method based on Fourier transform (FFT), as expected showed independence from the sampling frequency. The only dependence is on the Nyquist criteria which requires the sampling frequency to be larger than the double of the highest frequency component of the signal. In this case, the Nyqyist frequency is 120Hz. All arguments presented above can be seen in the Fig. (2). The DC gain drops sharply at the Nyquist frequency of 120Hz and is constant with changes in the sampling frequency.

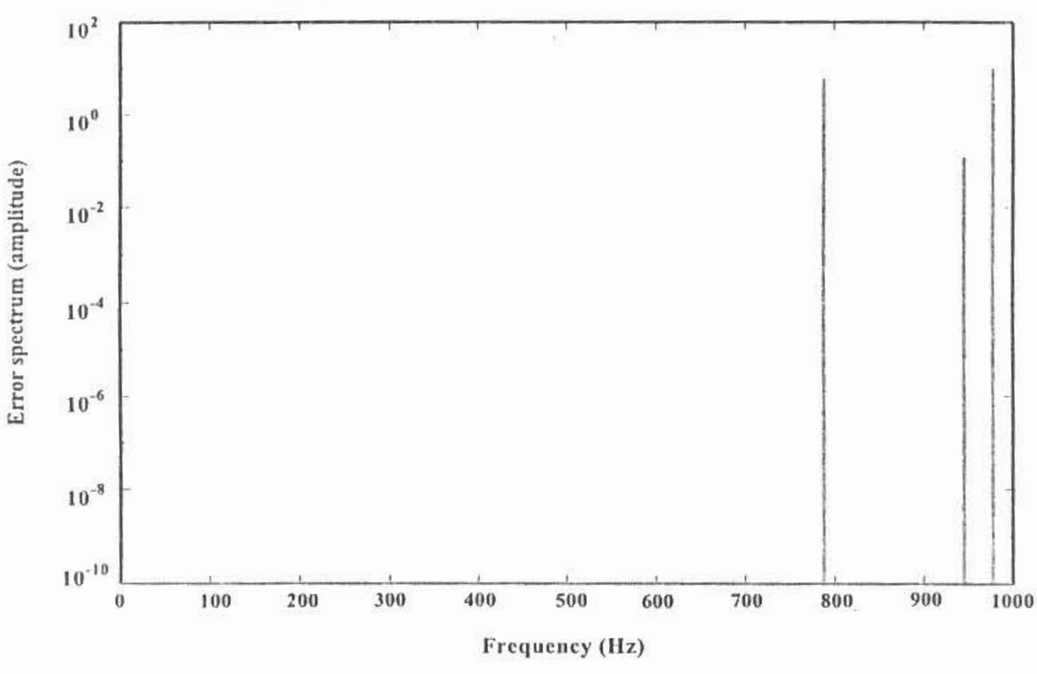
c) Runge-Kutta methods which in this case reduces to Simpson's rule or the trapezoidal rule can be as precise as the FFT method, provided the sampling frequency is large enough. In this particular case the sampling frequency needed is ten fold larger (120 and 1200 Hz) as can be seen in Fig. (2).

5. Integration of a actual vibration signal.

A set of simultaneous acceleration a(t) and velocity v(t) signals were acquired using analog electronic instruments on an experimental bench where rolling defective bearings were tested. The velocity signal was obtained by analog electronic integration. Both signals sampled at 2000 Hz were converted to digital files using a A/D card and the Labview software. The same methods were applied to these signals and the electronically acquired velocity signal spectrum was compared with the one obtained by numerical integration. One result in terms of error spectra as defined by Eq. (8) is shown in Fig. (3). It is the worst result obtained among all the numerical integration methods (the Closed Integration method).

Although no theoretical conclusions can be drawn from this figure, an interesting observation is that the errors in the frequency domain for both the electronic integrator and the numerical integration methods are the same since the error spectra (Eq. 8) are practically zero throughout the entire frequency domain. There are only three singular error values attributed to the Closed Integration Formula which are relatively small and caused probably by some local numerical idiosyncrasy. In other words, under the experimental condition where these signals were acquired, both Runge-Kutta methods as well as FFT method would be acceptable.

Figure 3 - Error spectra for a actual acceleration signal



6. Conclusions.

a)For the cases tested in this work, among the methods based on discretization such as the Closed and Open Integration formulas, Runge –Kutta methods and its derivatives like the Trapezoidal and Simpson's rules, the Trapezoidal rule proved to be the most efficient. Although all of them depend strongly on the sampling frequency.

b)For the cases studied in this work, among all the studied methods, the Fourier transform based method proved to be the most robust since it requires only the Nyquist criteria to be satisfied.

c)In view of the noise amplification at the lower frequency range, independent of the integration method chosen, the acquisition of the acceleration signal should be done as noise free as possible to reduce this influence in the vibration analysis. For low frequency phenomena this poses a special challenge to acquire a adequate signal for vibration analysis.

d)Comparing the numerical integration with the electronic-analogue one, the results are very similar, at least for the FFT and Runge-Kutta methods. Therefore, it is possible and enough to use numerical integration to obtain velocity signal from sampled acceleration signals, and consequently reduce cost of the measurement system.

e)Further calculations should be done to expand the validity of the conclusion presented in the present work.

7. References.

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