Unloaded Q-Factor Measurements in Klystron Cavities

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Abstract—**This paper presents some experimental results of unloaded Q-factor, Q_0, measurements in klystron cavities using the critical points method. The circuit model and impedance locus involving this method are outlined in order to present the experimental procedure. The values calculated for a typical reentrant S-band klystrons cavity, using a vector network analyzer, were 3394 for the unloaded Q_0, 1.2715 for the coupling coefficient and 2.85818GHz for the load resonant frequency.**

Keywords – Q-Factor measurements, impedance measurements, network analyzer, Klystron cavities.

I. Introduction

The knowledge of the lumped circuit parameters of cavity klystrons is very relevant in klystron modeling because it is fundamental to describe the power balancing between the RF field in the cavity and the electron beam. Besides *R/Q* parameter, the three basic parameters of the equivalent cavity klystron are: the unloaded resonant frequency f_0, the unload Q-factor Q_0, and the coupling factor k. The three parameters are the subject of this paper. The unloaded Q-factor establishes an upper limit on the cavity performance, while the coupling coefficient k, describes how the cavity interacts with microwave circuits interfaced with it, and the loaded resonant frequency f_L, represents the shift of frequency from the natural resonance frequency f_0 when the cavity in coupled to the external circuit.

A great number of papers have been published on the characterization of the cavity resonator Q-factors, Ginzton [1]-[2]. Among these, the critical-points method, that uses one-port or reflection technique, developed by Sun and Chao [3], is a good option for fast and accurate measurement of the unloaded Q. In that method only four frequencies are needed from which the unloaded Q-factor is evaluated using the measured one-port input reflection coefficient Chua and Syahkal [4] extended that work to directly extract all the elements of an equivalent circuit used to describe a high-Q cavity and its coupling element to the external circuit. It can be found a very comprehensive textbook in [5].

In this work, the critical points method is used to measure of unloaded Q-factor of an S-band reentrant klystron-type cavity resonator, including the coupling elements.

This paper is organized as follows. Section II presents the circuit model of the cavity problem. The description of the experiment setup and results are shown and discussed in section III. Finally, the conclusion is presented in Section IV.

II. Circuit Model overview

It is well known that the input impedance of a high-Q cavity near the resonance can be represented by an equivalent lumped circuit as shown in Fig. 1. So, the input impedance at port b can be represented as follows

$$Z_b(\omega)=R_e+j\omega L_e+\frac{n^2R_0}{1+jQ_0\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}, \quad (1)$$

where $R_0'=n^2R_0$ is the cavity losses transformed by the coupling structure with n turns ratio, $Q_0=R_0/\omega_0L=\omega_0C/G_0$ is the unloaded Q-factor, and $\omega_0=1/\sqrt{LC}$ is the angular natural frequency. The loss and the reactance of coupling structure parameters are represented as R_e and L_e.

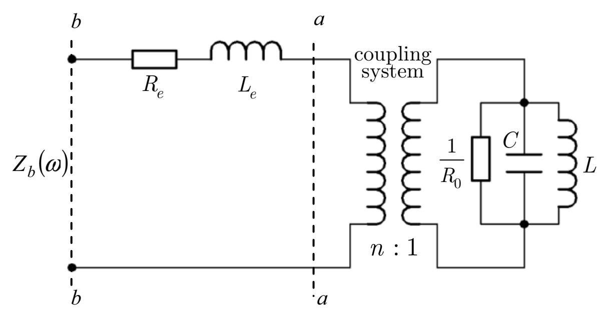

Fig. 1. Equivalent circuit of a high-Q cavity resonator including coupling losses and transformer ratio. In this work n =1.

In the vicinity ω_k of the natural resonant angular frequency ω_0, the denominator of (1) can be written

$$\frac{\omega_k}{\omega_0}-\frac{\omega_0}{\omega_k}=\frac{2(\omega_k-\omega_0)}{\omega_0}\frac{\left(\frac{\omega_k-\omega_0}{2\omega_0}+1\right)}{\left(1+\frac{\omega_k-\omega_0}{\omega_0}\right)}=2\delta_k\frac{\left(1+\frac{\delta_k}{2}\right)}{(1+\delta_k)}=2\delta_kD_k, \quad (2)$$

or

$$\frac{\omega_k}{\omega_0}-\frac{\omega_0}{\omega_k}\approx 2\delta_k \text{ if } \delta_k \ll 1, \quad (3)$$

where

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$$\delta_k = (\omega_k - \omega_0)/\omega_0 , \tag{4}$$

is the frequency-tuning parameter, and

$$D_k = (1+\delta_k/2)/(1+\delta_k) \tag{5}$$

is the deviation factor of linearity. Thus, the input impedance (1) can be written by

$$Z_b(\omega_k) = \left[R_e + \frac{R_0'}{1+(2Q_0\delta_k D_k)^2}\right] + j\left[\omega_0(1+\delta_k)L_e - \frac{2Q_0\delta_k D_k R_0'}{1+(2Q_0\delta_k D_k)^2}\right], \tag{6}$$

or

$$Z_b(\omega_k) = \left[R_e + \frac{R_0'}{1+(2Q_0\delta_k)^2}\right] + j\left[\omega_0(1+\delta_k)L_e - \frac{2Q_0\delta_k R_0'}{1+(2Q_0\delta_k)^2}\right] \tag{7}$$

for $\delta_k \ll 1$. At resonance frequency, from (1), the input impedance writes $Z_b(\omega_k) = (R_e + R_0') + j\omega_0 L_e$, and at the detuned crossover point there are two angular frequencies ω_3 and ω_4 on Fig. 2 where, from (6), $Z_b(\omega_3) = Z_b(\omega_4)$, i. e.,

$$\frac{1}{1+(2Q_0\delta_3 D_3)^2} = \frac{1}{1+(2Q_0\delta_4 D_4)^2}\text{, and} \tag{8}$$

$$\omega_0\delta_3 L_e - \frac{2Q_0\delta_3 D_3 R_0'}{1+(2Q_0\delta_3 D_3)^2} = \omega_0\delta_4 L_e - \frac{2Q_0\delta_4 D_4 R_0'}{1+(2Q_0\delta_4 D_4)^2} . \tag{9}$$

Solving (8) and (9) simultaneously and excluding the trivial solution, i.e., $\omega_3 = \omega_4$, from (8), one gets

$$\delta_3 D_3 = -\delta_4 D_4 , \tag{10}$$

and, from (9), one can write

$$\frac{t}{a} = \frac{1}{1+(2Q_0\delta_3 D_3)^2} , \tag{11}$$

where

$$t = \frac{\omega_0 L_e}{2Q_0 R_0'} \text{ and } a = \frac{2D_3}{1+D_3/D_4} . \tag{12}$$

From Fig. 2, it can be seen the nearly circular locus is deformed by a factor t/a called the factor of circle deformation. From nearly circular impedance locus, it can be got two points corresponding to two extreme values, maximum and minimum, of reactance within this circle. The corresponding frequencies ω_1 and ω_2 are both shown in Fig. 2 and should be in vicinity of the resonant frequency for high Q_0. Thus (6) and (8) can be used to find these extreme frequency-tuning parameters, δ_1 and δ_2. Using the commonly procedure to determine the extreme values of a function, one has to take the first-order partial derivatives with respect to δ_k in the second term of (6) (i.e., reactance of the input impedance) and then finding points

$$\frac{\partial \operatorname{Im}[Z_b(\omega_k)]}{\partial \delta_k} = 2Q_0 R_0'\left[t + \frac{2(2Q_0\delta_k)^2}{\left[1+(2Q_0\delta_k)^2\right]^2} - \frac{1}{1+(2Q_0\delta_k)^2}\right] = 0 . \tag{13}$$

The points ω_k, solutions of (13), are called of critical points. Defining $2Q_0\delta_k = x$, (13) can be written as

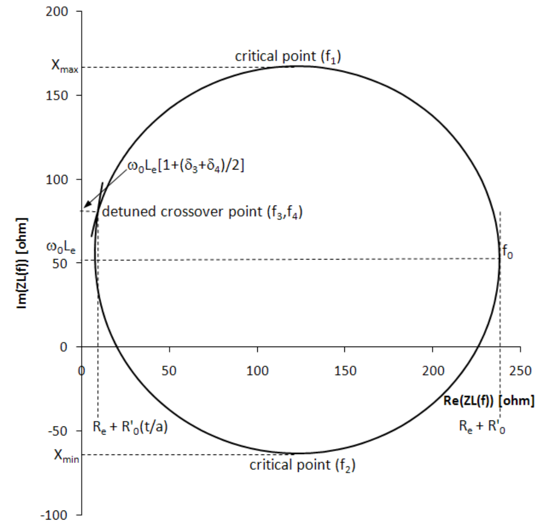

Fig. 2. Input impedance locus of a coupled resonator including the loss and the reactance in the vicinity of resonant frequency.

$$t + \frac{2(2Q_0\delta_k)^2}{\left[1+(2Q_0\delta_k)^2\right]^2} - \frac{1}{1+(2Q_0\delta_k)^2} = t - \frac{1-x^2}{(1+x^2)^2} = 0 \tag{14}$$

or

$$\frac{1-x^2}{(1+x^2)^2} = t . \tag{15}$$

Using (12) and defining b as $(\delta_3 D_3/\delta_k)^2$, (15) writes

$$\frac{1-x^2}{(1+x^2)^2} = \frac{a}{1+bx^2} . \tag{16}$$

Solving (16), it can be seen that the only reasonable solution is

$$x = \left[\frac{(b-2a-1)+\sqrt{(b-2a-1)^2 - 4(b+a)(a-1)}}{2(b+a)}\right]^{1/2} . \tag{17}$$

Knowing a, b, and x values, the unloaded Q-factor can be calculated as follows

$$Q_0 = \frac{|x|}{2|\delta_k|} . \tag{18}$$

The angular frequency ω_0 can be evaluated by $\omega_0 = (\omega_1 + \omega_2)/2$ and the bandwidth of the critical points is $2Q_0|\delta_k| = |\omega_1 - \omega_2|$. So the Q_0 (18) can also be written as

$$Q_0 = \frac{\omega_1 + \omega_2}{2|\omega_1 - \omega_2|}|x| , \tag{19}$$

and, for $|x| \approx 1$, one gets

$$Q_0 \approx \frac{f_1 + f_2}{2|f_1 - f_2|}. \tag{20}$$

It must be observed that b can also be written as

$$b = \left(\frac{\delta_4 D_4 - \delta_3 D_3}{\delta_4 - \delta_1} \right)^2. \tag{21}$$

At resonant frequency the input impedance (1) writes

$$Z_b(\omega_0) = (R_e + R_0') + j\omega_0 L_e, \tag{22}$$

and at the detuned crossover points, the input impedance writes

$$Z_b(\omega_3) = Z_b(\omega_4) = \left[R_e + R_0' \left(\frac{t}{a} \right) \right] + jX_3, \tag{23}$$

and

$$X_3 = \omega_0 L_e \left[1 + \frac{(\delta_3 + \delta_4)}{2} \right]. \tag{24}$$

in the case of an inductive coupling. From (22) and (23), R_0, R_e and L_e can be found, resulting in

$$R_0 = R_0' = \frac{\mathrm{Re}\left\{ Z_b(\omega_0) - 0.5\left[Z_b(\omega_3) + Z_b(\omega_4) \right] \right\}}{1 - t/a}, \tag{25}$$

$$R_e = \frac{\mathrm{Re}\left\{ -(t/a) Z_b(\omega_0) + 0.5\left[Z_b(\omega_3) + Z_b(\omega_4) \right] \right\}}{1 - t/a}, \text{ and} \tag{26}$$

$$L_e = \frac{X_3 + X_4}{\omega_0 (2 + \delta_3 + \delta_4)}. \tag{27}$$

In order to compute the coupling coefficient k, and the load resonant frequency f_L, the admittance to the left-hand side of plane *a-a* can be written, considering that the circuit is coupled to a R_s=50Ω input impedance network analyzer, as

$$Y_a(\omega) = G_a + jB_a(\omega), \tag{28}$$

where

$$G_a(\omega) = \frac{R_s}{(R_s + R_e)^2 + X_e^2}, \tag{29}$$

and

$$B_a(\omega) = \frac{-X_e}{(R_s + R_e)^2 + X_e^2}. \tag{30}$$

The total admittance in *a-a* plane is given by

$$Y_T(\omega) = Y_a(\omega) + Y_0(\omega), \tag{31}$$

or

$$Y_T(\omega) = (G_a + G_0) + j\left[G_0 Q_0 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) + B_a \right]. \tag{32}$$

The f_L is the frequency at which the imaginary part of (31) is zero, i.e., $\mathrm{Im}[Y_T(\omega_L)] = 0$. Under this assumption, one obtains

$$f_L = f_0 \left[1 - \frac{B_a(\omega_0)}{2 G_0 Q_0} \right]. \tag{33}$$

The effect of the conductance G_e, in parallel to G_0, is to lower the overall Q, producing a new value Q_L, which is expressed as

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_e}, \tag{34}$$

where the external Q-factor, Q_e, is defined, in analogy with Q_0, as

$$Q_e = \omega_0 C / G_e \tag{35}$$

On the other hand, the coupling coefficient k, defined as the ratio of power dissipated in the external circuit to the power dissipated in the resonator, is written as

$$k = \frac{G_e}{G_0} = \frac{Q_0}{Q_e}. \tag{36}$$

By eliminating Q_e from (34) with the use of (36), one obtains the relationship between the unloaded and the loaded Q as follows

$$Q_L = \frac{Q_0}{1 + k}. \tag{37}$$

III. Experiment and Results

The critical-points method was used to measure the Q_0-factor of a typical S-band klystron cavity resonator. The dimensions of the cavity are shown in Fig. 3. The resonant mode within the cavity is TM_{010} and it was excited using a loop assembled from SMA coaxial connector placed at center of the cavity sidewall.

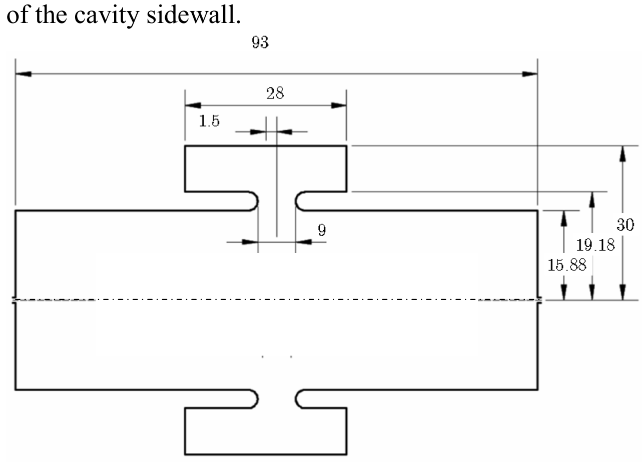

Fig. 3. Geometry and dimensions of the S-Band reentrant cavity used to unloaded Q-factor measurements. The dimensions are in mm.

Two copper foils of 0.1 mm thickness each were placed between the cavity body and the openings of the cavity, to act as electromagnetic chokes, in order to stop the electromagnetic leakage from the cavity. The measured frequencies at the critical and the detuned crossover points and their corresponding impedance are summarized in Table I. During the determination of the impedances presented in

Table I, it was found that a large number of points were concentrated around the detuned region when the measurement was taken with 1601 frequency points equally spaced over a span of 40 MHz on an Agilent N5230C PNA-L Network Analyzer.

TABLE I. RESULTS OF THE CRITICAL-POINTS METHOD APPLIED TO MEASURE THE UNLOADED *Q*-FACTOR CAVITY. IT WAS USED THE REENTRANT CAVITY SHOWN IN FIG. 3.

Quantity	f (GHz)	$Z(f)$ (Ω)
Resonance frequency f_0	2.85833	72.1 - j18.2
Critical point f_1	2.85769	40.9 + j55.4
Critical point f_2	2.85853	49.2 - j28.6
Detuned point f_3	2.84527	1.27 + j13.8
Detuned point f_4	2.87025	1.28+ j13.8

In order to locate accurately the critical points from the measured data, a second measurement was taken around the resonant frequency with the same number of points over a narrower span of 2.7 MHz from 2.85693 to 2.85956GHz. The Q_0, R_0, R_e and L_e values calculated using (19), (25), (26) and (27), respectively, are shown in Table II.

TABLE II. EXTRACTED PARAMETERS OF CAVITY AND COUPLING CIRCUITS OBTAINED USING THE CRITICAL POINTS METHOD SHOWN IN TABLE I.

Quantity	***Values***
Q_0	3394
Q_L	1482
k	1.2715
f_L	2.85818 GHz
R_e	1.185 Ω
n^2R_0	70.9 Ω
L_e	0.7691 nH

Fig. 4 shows the critical points and the detuned crossover point on the Smith Chart. It can be seen the loop overcouples with the cavity. From Table II one can also see that the cavity is overcoupled (k>1) with the external circuit. Also there was a lower of Q-factor due to the coupling and a frequency shift.

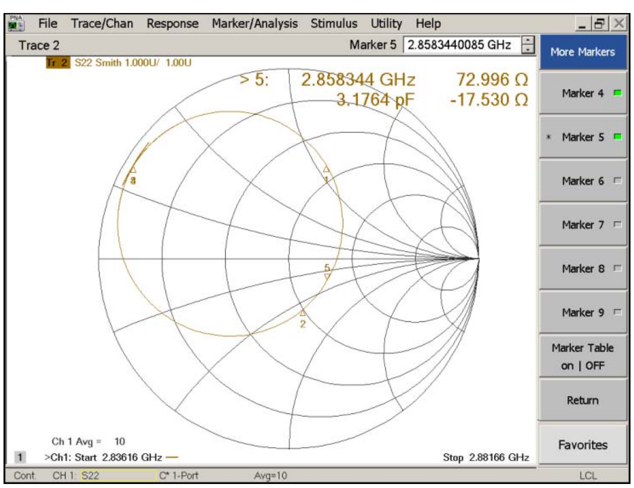

Fig. 4. Measured input reflection coefficient of the S-Band klystron cavity. It can be seen that the cavity is overcoupled.

IV. CONCLUSION

In this work the results of klystron cavity unloaded Q-factor measurements, using the critical points method, were presented. Besides of determination of Q_0, all cavity parameters and coupling circuits were determined as well. This kind of cavity characterization is very relevant in klystron amplifier design and modeling.

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