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A new procedure to analyze angular correlation experimental data

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Abstract. In this work an approach is proposed to the analysis of directional angular correlation data in which the function $W(\theta)$ used to fit the experimental data already includes the relations between the A_{kk} parameters and the mixing ratios (δ), so that the angular correlation function is fitted explicitly in terms of the mixing ratio of the transition of interest; the normalization factor that arises from the fit can also be used, under certain situations, to determine the intensity of one of the transitions. In order to verify the usability and consistence of this method, some well-determined cascades from ^{152}Eu and ^{60}Co standard sources, as well as from the β^- decay of ^{193}Os, were analyzed, and the results compared to the literature values, allowing for a discussion of the performance of this approach.

1 Introduction

Directional $\gamma\,\gamma$ angular correlation experiments have been widely used to measure nuclear multipolar mixing ratios ($\delta_{L,L+1}$) for more than 40 years [1], and has the ability to measure not only the magnitude but also the signal of the mixing ratio.

The basic concept behind these measurements is that, in a sample where nuclear spins are randomly aligned, once you choose nuclei that emmitted a photon γ_1 in a direction $\hat{x}_1$, if there is a second transition γ_2 emmitted subsequently by the same nucleus, the direction $\hat{x}_2$ of this emmission is related to the nature and multipolarity of both transitions.

Strictly speaking, if a nuclear state decays by the successive emmission of two gamma transitions γ_1 and γ_2, which are in turn detected by two finite detectors which make an angle θ between their axes, the count rate will follow the angular correlation function [1], where the series was truncated to the order of $k = 4$ due to experimental sensitivity limitations:

$$W(\theta) = \alpha\,[1 + A_{22}Q_{22}P_2(\cos\theta) + A_{44}Q_{44}P_4(\cos\theta)] \qquad (1)$$

where A_{kk} are the angular correlation coefficients related to the properties of the transitions, P_k are the Legendre Polynomials of the k^{th} order, Q_{kk} are coefficientes related to finite solid angle corrections (see [2,3]) and α is a normalization constant.

The A_{kk} coefficients can be split in two separate coefficients, each related to one of the transitions involved:

$$A_{kk} = A_k(\gamma_1)\cdot A_k(\gamma_2) \qquad (2)$$

and these coefficients can be expressed as:

$$A_k(\gamma_1) = [1+\delta^2(\gamma_1)]^{-1}\times[F_k(L_1L_1I_iI) + \\ + (-1)^{L_1+L_i'}2\delta(\gamma_1)F_k(L_1L_1'I_iI) + \delta^2(\gamma_1)F_k(L_1'L_1'I_iI)] \qquad (3)$$

and

$$A_k(\gamma_2) = [1+\delta^2(\gamma_2)]^{-1}\times[F_k(L_2L_2I_fI) + \\ + 2\delta(\gamma_2)F_k(L_2L_2'I_fI) + \delta^2(\gamma_2)F_k(L_2'L_2'I_fI)] \qquad (4)$$

where the F_k are the Frauenfelder coefficients (tabulated in [4]), L is the angular momentum carried away by the transition, $L' = L+1$, and δ is the *multipolar mixing ratio*, obtained by the ratio of the reduced matrix elements of the multipolar electromagnectical transitions of the order L e L'.

2 Fitting the angular correlation function

The usual way to fit the experimental points to the angular correlation function $W(\theta)$ is to fit directly the three parameters that appear in equation (1) (α, A_{22} and A_{44}) without taking into account the constraints between the latter two and the multipolar mixing ratio (δ), which can be seen in equations (3) and (4); in this fit all the data can be normalized in respect to a certain angle, thus eliminating the normalization constant α – introducing, as a side effect, some correlation between the data – and then finding the value of δ which minimizes the χ^2.

In this work, a different approach will be taken, fitting the angular correlation function directly as a function of the parameters α and δ. The main difference between the two procedures can be understood by noticing that, although equations 3 and 4 impose a constraint between the values of A_{22} and A_{44}, only the second procedure effectively guarantees that this constraint will be respected.

Formally, this procedure can be described as follows; the equation to be fitted is:

$$W_{teo} = \left(\frac{S\cdot Y_{12}}{4}\right)\cdot\epsilon\cdot R\cdot X \qquad (5)$$

where S is the number of disintegrations in the sample in a time interval Δt, Y_{12} is the probability that the pair of transitions $\gamma_1\gamma_2$ will occur per disintegration, and ϵ is the efficiency matrix (a diagonal $N\times N$ matrix, where N is the number of independent detector pairs); ϵ, R and X are defined as follows. If $J_0^k(E_l)$ is the absolute gamma ray efficiency of the detector k to the energy E_l, then the efficiency matrix is

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given by:

$$\epsilon = \begin{pmatrix} J_0^a(E_1) \cdot J_0^b(E_2) \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & J_0^c(E_1) \cdot J_0^d(E_2) \end{pmatrix}. \quad (6)$$

Moreover, if Q_{kk}^{ij} is the k^{th} order solid angle correction factor (as defined in [2,3]) for the detector pair ij, then

$$R = \begin{pmatrix} 1 \; P_2(cos\theta_{ij}) \cdot Q_{22}^{ij} \; P_4(cos\theta_{ij}) \cdot Q_{44}^{ij} \\ \vdots \qquad\qquad \ddots \qquad\qquad \vdots \\ 1 \; P_2(cos\theta_{ij}) \cdot Q_{22}^{ij} \; P_4(cos\theta_{ij}) \cdot Q_{44}^{ij} \end{pmatrix} \quad (7)$$

and, finally,

$$X = \begin{pmatrix} 1 \\ A_{22}(\delta_1, \delta_2) \\ A_{44}(\delta_1, \delta_2) \end{pmatrix}. \quad (8)$$

The parameter to be minimized is, then,

$$\chi^2 = (W_{exp} - W_{teo})^t \cdot M^{-1} \cdot (W_{exp} - W_{teo}) \quad (9)$$

where M is the covariance matrix:

$$M = M_{exp} + M_{\epsilon}. \quad (10)$$

3 Experimental procedure and data analysis

In order to test the proposed method, the values of the multipole mixing ratio for several transitions from standard sources of ^{152}Eu and ^{60}Co, as well as transitions from the β^- decay of ^{193}Os, were measured and compared to the compiled values [5,6].

These experimental measurements were performed using the planar multidetector array system assembled at the Laboratrio do Acelerador Linear (LAL), in the Physics Institute of the São Paulo University [7]. For this experiment, four HPGe detectors, with volumes ranging from 50 to 120 cm^3, were used in coincidence mode, placed around the sample. The coincidence electronics is a regular fast-slow system that checks for a coincidence between two or more detectors within 200 ns and, for each valid event, stores both time and energy information for each detector involved. Both the detector setup and the electronics are shown in figure 1.

The ^{193}Os samples were produced by irradiating 5 mg of 99% enriched ^{192}Os for 5 minutes in the IEA-R1 reactor, under a neutron flux of about 10^{12} cm$^{-2} \cdot$ s^{-1}, resulting in a total of $\sim 10^8$ total events; for the standard sources of ^{60}Co and ^{152}Eu the total number of events was approximately 10^5 and 10^6, respectively.

The data analysis was performed subtracting accidental coincidences, through the use of a time gate, and fitting bidimensional gaussian peaks compensated for Compton remains of other transitions.

The data fitting procedure was performed using a covariant fitting procedure developed in the MatLab platform, and

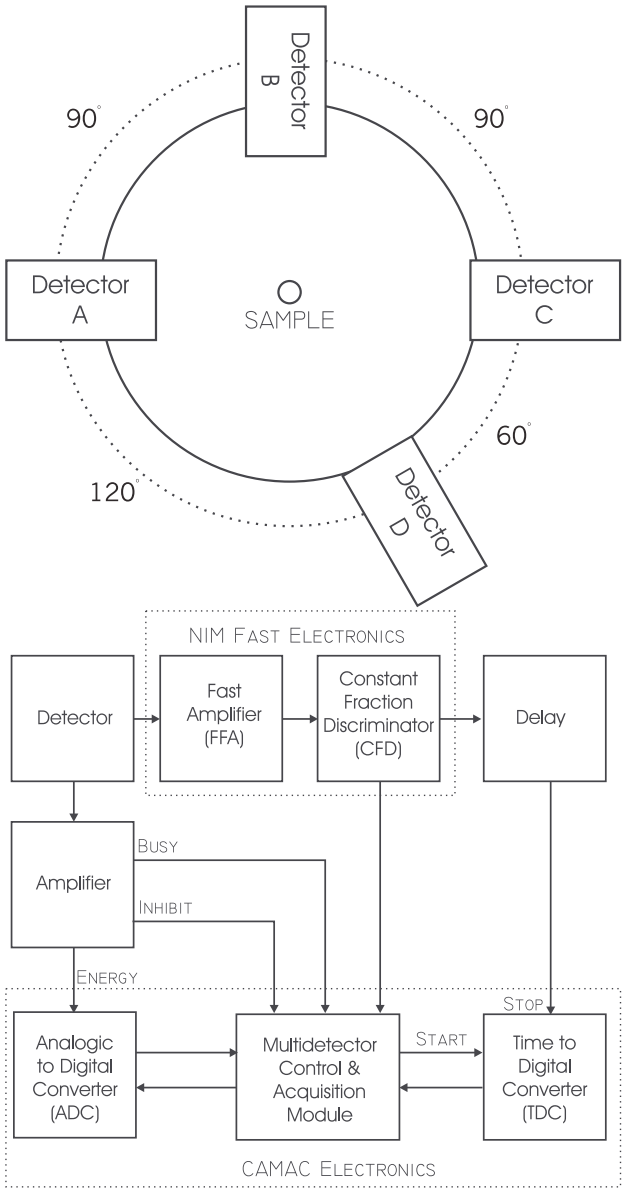

Fig. 1. Top: Schematics of the planar detector setup used in the present measurements; Bottom: Electronical setup used in the present experiment.

special care was taken as to fit only one of the mixing ratios on a cascade, using the tabulated value (or the result of a previous fit using other cascade) for the other, in order to avoid the problems described in [8]. Also, both "sides" of a cascade (i.e., $\gamma_1 \times \gamma_2$ and $\gamma_2 \times \gamma_1$) were fitted together.

3.1 Angular correlation plots

One additional problem in this type of analysis is the matter of the graphical representation of the results. As the Q_{kk} coefficients affect non-linearly the angular correlation function $W(\theta)$ (eq. (1)) and only make sense for real, finite detectors, the fitted angular correlation function can only be directly compared to the experimental results at the angles where real

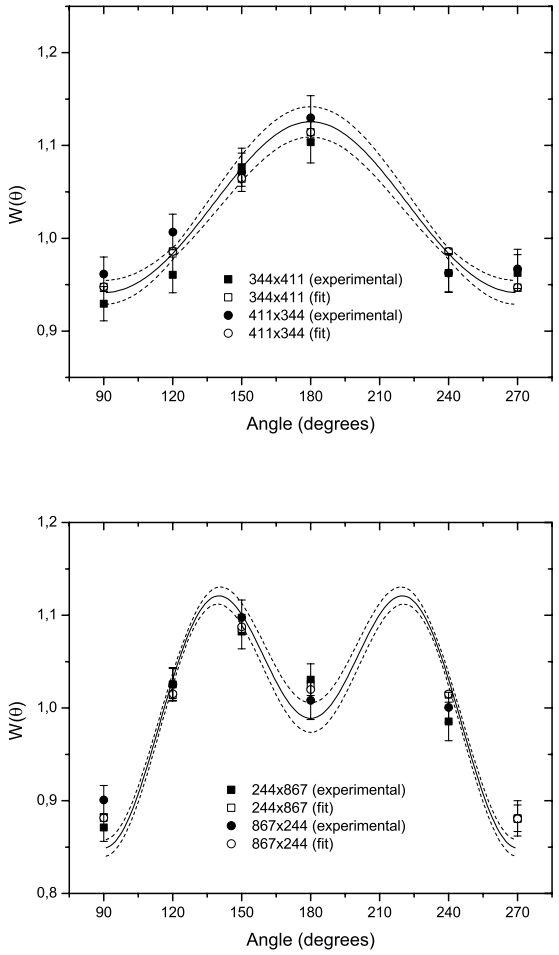

Fig. 2. Experimental results for the 344 keV × 411 keV (top) and 867 keV × 244 keV (bottom) cascades from ^{152}Eu.

Table 1. Experimental values for some transitions with well-determined mixing ratios (δ) compared to the compiled values from [6] (for ^{193}Os) and [5].

Nuclide	Transition (keV)	δ (this work)	δ (refs. [5,6])
^{60}Co	1332.5	+0.001(15)	0
^{152}Eu	1213.0	−0.007(17)	0.00(2)
	867.4	−5.3(4)	−6.5(3)
	411.1	+0.032(22)	0
	778.9	+0.026(10)	+0.002(6)
^{193}Os	460.5	−0.634(17)	−0.64(3)
	107.1	+0.171(13)	+0.164(8)
	321.6	+0.236(17)	+0.234(10)
	361.8	−0.314(27)	−0.33(3)
	251.6	−0.132(7)	−0.079(20)

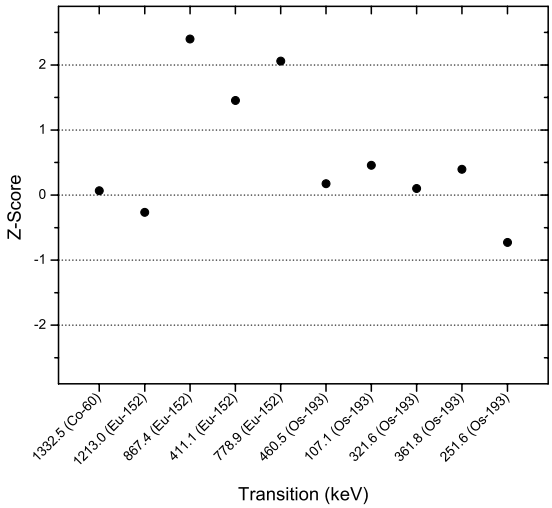

Fig. 3. Z-Score comparison of the multipolar mixing ratio values found in this work to the tabulated values [5,6] for each of the transitions shown in table 1.

detectors exist, so that the correction can be applied to the calculated values; as a consequence, it's not possible to draw a continuous curve for $W(\theta)$ that can be compared to the experimental results. A suggestion to overcome this is shown in figure 2, where the experimental data is plotted together with the fitted results (corrected for the real detectors pairs) and the continuous $W(\theta)$ curve which was **not** corrected; together with that, the graph shows the confidence bands calculated as the band covered within 1σ intervals for each of the fit parameters (α and δ).

4 Results and discussion

Overall, the multipolar mixing ratios for 10 transitions were evaluated: the 1332.5 keV transition from the decay of ^{60}Co, the 1213.0 and 867.4 keV transitions from the electron capture decay of ^{152}Eu, the 411.1 and 778.9 keV transitions from the β^- decay of ^{152}Eu and the 460.5, 107.1, 321.6, 361.8 and 251.6 keV transitions from the β^- decay of ^{193}Os (the ^{193}Os decay data are part of a larger work, which can be found in [9,10]). The results of the multipolar mixing ratio (δ) for these transitions are compared to the reference values (found in [5] for the standard sources and in [6] for the ^{193}Os decay) in table 1; the Z-Score for the comparison of the present values and the reference ones are shown in figure 3. This comparison shows that the results found in the present work are basically compatible to the reference values, with the exception of the 887.4 and 778.9 keV transitions from the ^{152}Eu source; in the first case, our results agree to the fact that the 867.4 keV transition is a strongly-mixed E2+M1 transition and also agrees on the sign of the mixing ratio, but the resulting Z-Score is more than two with our results indicating a smaller M1 contribution than the reference value; in the second case, the 778.9 keV transition is expected to be a E1 transition, but our results show a distinct M2 contribution.

Conclusions

The methodology proposed in this work was applied to the determination of the multipolar mixing ratios for 10 transitions from both ^{60}Co and ^{152}Eu standard sources and from the β^- decay of ^{193}Os. The results obtained using the proposed

methodology were mostly compatible with the ones found in the literature, with only two of the values differing significantly (Z-Score > 2) from the compiled ones.

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