A METHOD FOR CALCULATING THE CURRENT DISTRIBUTION AND MAGNETIC FIELD AROUND A LIGHTNING STROKED STRUCTURE

Carlos A. F. Sartori, M. Sc José Roberto Cardoso, Ph. D.

Dept. of Electrical Engineering - Escola Politécnica da Universidade de São Paulo
05508-900 Cid. Universitária -S.Paulo-Brasil

***Abstract* - One of the most important problem related to Electromagnetic Compatibility analysis is the evaluation of the electromagnetic environment around a structure struck by a direct lightning. In this paper is introduced a discretization technique in the space and time of the phenomenon based on transmission line and wave propagation model to calculate the current distribution in a structure and on the infinitesimal time-varying dipole and image method to calculate the magnetic field around it. This work was proposed due to the necessity of developing an electromagnetic field calculation methodology in the interior of a lightning protection system during a lightning stroke, suitable for the evaluation of energy and interference level associated with them.**

I. INTRODUCTION

The evaluation of electromagnetic environment is necessary in a great number of EMC engineering applications.

Beyond the classified areas, where the insurance aspects related to explosion hazards are very important, the evaluation of the effects resulting from lightning has been assuming a special concern in our modern society due to the use of high susceptibility equipments, like electronic ones, in many social activities in a wide variety of functions.

This evaluation can be necessary, for instance, to determine the best lay-out of a control room , which safety aspects are relevant to the performance of complex industrial plants. It can be done through the comparison of the resulted field with the susceptibility of electronic circuits.

The objective of the present work is to carry out a discretization technique in the space and time that meets the need of calculating the current distribution in a lightning protection system and the flux density or magnetic field inside the protected volume.

II. DESCRIPTION OF THE METHOD

To satisfy the necessities mentioned before, it was developed a three-dimensional element or cell, where the central node corresponds to a junction of transmission lines, forming a impedance discontinuity in each line. Fig. 1 shows this element.

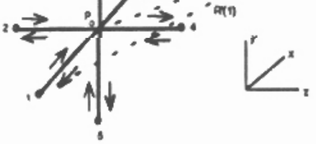


Fig. 1: Proposed Three-dimensional Element

The response of the cell to incident voltage pulses and the current distribution were determined by transmission line and wave propagation theory.

Then, using the dipole and images theory was calculated the contribution of each reflected and incident current of the element to the magnetic field.

The total magnetic field as a function of the time, in any point of the space, can then be determined by superposition, after subdividing the protection system conductors into a finite number of elementary units, represented by the proposed three-dimensional element or cell.

This model differs from others, presented up to this moment in the scientific literature, due to their intrinsic aspects. In this model it is not necessary to solve any equation system and thanks to the direct methodology there is not problems related to the convergence of the solution.

Another characteristic, to be mentioned about this method, is the determination of magnetic field considering the incident and reflected current, not the current resulted from the sum of them. It is important because the validation time should be follow during the calculation of the magnetic field.

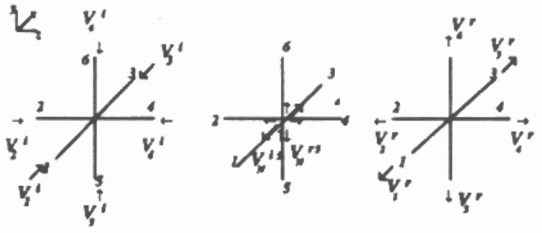
It was assumed in the present model some simplifications that could be reduced in a second round, without damage to the general philosophy adopted up to this moment.

Among these simplifications are the calculation of transmission line parameters through traditional formulas, not taking in account the non-linear ionization phenomena, and the simulation of the lightning stroke by a unidirectional current source.

Some numerical results are shown in order to evaluate the physical and mathematical consistence of the proposed methodology.

III. IMPULSE RESPONSE OF THE CELL

The impulse response of the cell at instants k, k+1and k+2 is illustrated by figure 2.



(K) (K+1) (K+2)

Figure 2: Impulse response of the cell at instants k, k+1and k+2

If at time $t=(k-2)\Delta l/2$, voltage impulses ${}_{k-2}V^{i}_{n}$ on lines 1 to n are incident on any series node of the cell, then, the reflected voltages on these lines at time $t = k\,\Delta l/2c$ will be represented by equation 01.

$$
{}_{k}\left[V^{r}_{n}(z,x,y)\right]=\begin{bmatrix}
(\sigma_h-1) & \sigma_h & \sigma_h & \sigma_h & \sigma_v & \sigma_v \\
\sigma_h & (\sigma_h-1) & \sigma_h & \sigma_h & \sigma_v & \sigma_v \\
\sigma_h & \sigma_h & (\sigma_h-1) & \sigma_h & \sigma_v & \sigma_v \\
\sigma_h & \sigma_h & \sigma_h & (\sigma_h-1) & \sigma_v & \sigma_v \\
\sigma_h & \sigma_h & \sigma_h & \sigma_h & (\sigma_v-1) & \sigma_v \\
\sigma_h & \sigma_h & \sigma_h & \sigma_h & \sigma_v & (\sigma_v-1)
\end{bmatrix}{}_{K-2}\left[V^{i}_{n}(z,x,y)\right]
$$

The parameter Δl is the length between two series nodes of the cell and σ_h and σ_v are the refraction coefficients of lines 1-4 and 5-6 respectively.

Equation (02) shows us how reflected impulses, from neighboring cells, becomes incident impulses on the cell (z,x,y) :

$$\begin{bmatrix} {}_K V_1^i(z,x,y) \\ {}_K V_2^i(z,x,y) \\ {}_K V_3^i(z,x,y) \\ {}_K V_4^i(z,x,y) \\ {}_K V_5^i(z,x,y) \\ {}_K V_6^i(z,x,y) \end{bmatrix} = \left[C(I,J)\right] \begin{bmatrix} {}_K V_1^r(z,x+\Delta l,y) \\ {}_K V_2^r(z+\Delta l,x,y) \\ {}_K V_3^r(z,x-\Delta l,y) \\ {}_K V_4^r(z-\Delta l,x,y) \\ {}_K V_5^r(z,x,y+\Delta l) \\ {}_K V_6^r(z,x,y-\Delta l) \end{bmatrix} \tag{02}$$

The matrix $c(i,j)$ is :

$$\left[C(I,J)\right] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The boundary condition for series nodes of cells, connected to earth, can be represented by equation (03):

$${}_K V_6^r(z,x,y-\Delta l) = {}_K V_5^i(z,x,y) = \frac{R_{at} - Z_v}{R_{at} + Z_v} {}_K V_5^r(z,x,y) \tag{03}$$

In the equation (03), R_{at} is the earth resistance and Z_v is the characteristic impedance of the vertical lines. The reflected and incident current along line "n", at instant "k", will be respectively:

$${}_K\left[I_n^r(z,x,y)\right] = \left[\frac{1}{Z_n}\right] SR^D \, {}_K\left[V_n^r(z,x,y)\right] \tag{04}$$

$${}_K\left[I_n^i(z,x,y)\right] = \left[\frac{1}{Z_n}\right]\left[C(I,J)\right] SR^D \, {}_{K-1}\left[V_n^r(z,x,y,\Delta l)\right] \tag{05}$$

Z_n is the characteristic impedance of line "n" and the value of SR^D depends on the position and direction of the wave propagation. The following table shows these values, when z,x,y are positive (0) or negative (1) coordinates of the cell.

TABLE I

VALUE OF SR^D AS A FUNCTION OF THE COORDINATES OF CELLS

Z	X	Y	I/n	1	2	3	4	5	6
0	0	0	1	-1	-1	1	1	-1	1
0	0	1	2	-1	-1	1	1	1	-1
0	1	0	3	1	-1	-1	1	-1	1
0	1	1	4	1	-1	-1	1	1	-1
1	0	0	5	-1	1	1	-1	-1	1
1	0	1	6	-1	1	1	-1	1	-1
1	1	0	7	1	1	-1	-1	-1	1
1	1	1	8	1	1	-1	-1	1	-1

IV. MAGNETIC FIELD CALCULATION

The field generated by a elemental dipole of current propagating along y axis with speed v is given by [02], where the geometrical factors are shown in the fig. 3 :

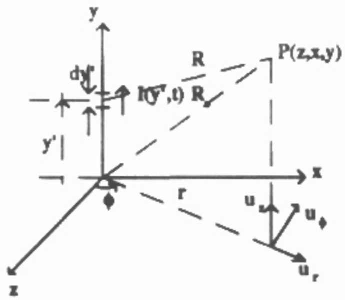


Fig. 3: Definition of geometrical factors used in the field computation.

$$dB_{\phi}(r,\phi,y,t)=\frac{\mu_0 dy'}{4\pi}\left(\frac{r}{R^3}i(y',t-\frac{R}{c})+\frac{r}{cR^2}\frac{\partial i(y',t-\frac{R}{c})}{\partial t}\right) \tag{06}$$

In order to obtain the magnetic field, each current will be considered as a step function :

$$i(y',t)=I_0 u(t-\frac{y'}{v}),\text{ where } u(\xi)=\begin{matrix}0,\xi\langle 0\\ 1,\xi\geq 0\end{matrix} \tag{07}$$

Then, the magnetic field from the reflected and incident currents along lines 5 or 6, can be found by using the image method and integrating, properly, (8) plus (9).

$$\frac{\mu_0 I_0 r}{4\pi R^3}dy',t)\,{}^{R}\!/_{c}+{}^{y'}\!/_{v} \tag{08}$$

$$\frac{\mu_0 I_0 r}{4\pi c[(y-y')^2+r^2]}\frac{1}{[\frac{1}{v}-\frac{(y-y')}{c\sqrt{(y-y')^2+r^2}}]}dy',t)\,{}^{R}\!/_{c}+{}^{y'}\!/_{v} \tag{09}$$

The same though can be used to obtain the magnetic field due to the currents along lines 1,3 and 2,4. The total field will be, then, obtained by superposition.

V. RESULTS

With the model described, many simulations were done to verify the validity of this methodology and its sensibility with parameters.

The following example, represented by Fig. 4, shows some results that were considered during these process :

In this example, the characteristic impedance of vertical and horizontal lines are assumed as 323,97Ω and 424,92Ω, the length between two series nodes (Δl) as 6m, the earth resistance (Rat) as 5Ω and the current source is represented by the function :

$$\mathrm{I}=5\,\mathrm{t}\,(\mathrm{kA})\text{ for }\mathrm{t}\leq 2\,\mu s\text{ and }\mathrm{I}=10\,[\,0.5\,\mathrm{t}-0.51\,(\,\mathrm{t}-2\,)\,]\,(\mathrm{kA})\text{ for }\mathrm{t}\rangle\,2\mu s$$

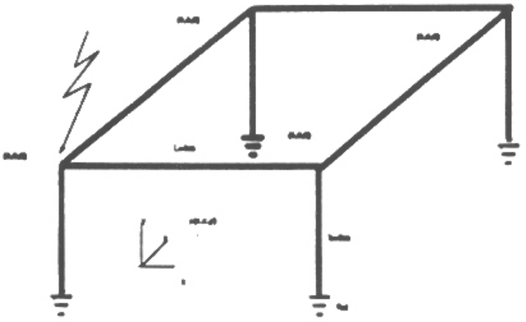


Fig. 4 : Sketch of a three-dimensional structure struck by a lightning at point (0,0,0) .

The current distribution in the cell (000) and the magnetic flux density as a function of time (μs) at point (0,3,-1.25) m, due the current distribution in the structure, are represented in the fig. 5 and 6.

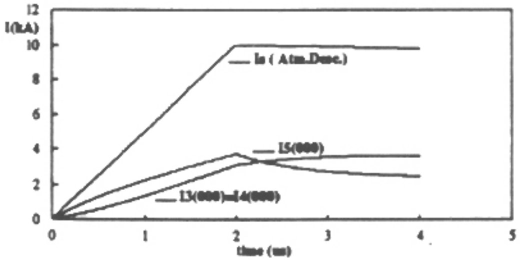


Fig. 5 : Current distribution in the cell (000)
(Example 1)

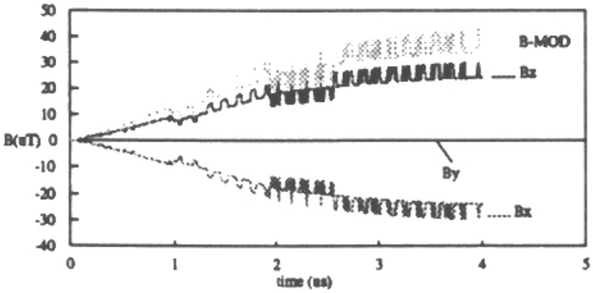


Fig. 6: Magnetic flux density B (μT) and components as a function of time (μs) at point (0,3,-1.5)m, due the current distribution in the structure (Example 1).

The magnetic field distribution along the diagonal of the axis z and x, for the configuration of Fig. 4, at time t = 2μs and for coordinate y = -1.5m is represented in Fig. 7.

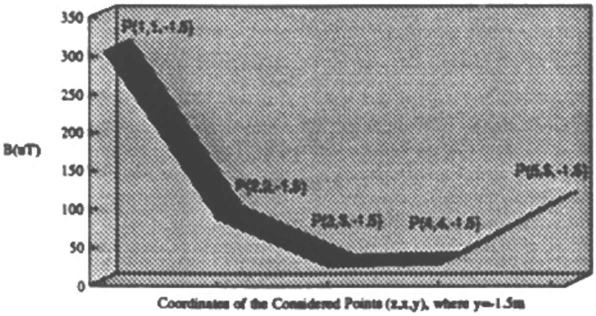


Fig. 7: Magnetic flux density B (μT) along the diagonal of axis z and x, t = 2μs.

A qualitative validation was done by comparison with theoretical and practical works of others authors.

The amplitude level and the wave shapes of the currents and magnetic field and the field profile was analyzed and by the engineering point of view a good agreement was obtained.

VI. CONCLUSION

The evaluation of electromagnetic environment allow us to identify potential EMI problems, anticipating the best solution before the installation of equipaments.

The proposed methodology is a suitable numerical method to be used when the evaluation of electromagnetic environment is necessary. It is a fast and direct method, when computational aspects are considered.

At the present, authors are implementing the electric field calculation.

This method will be improved, in the future, when the nonlinear, the ground network and the lightning channel effects will be considered.

VI. REFERENCES

[01] Johns, P. B.; Beurle, R. L. Numerical solution of 2-dimensional scattering problems using a transmission line matrix. Proc. IEE, v.118, n.9, p.1203-8, Sept. 1971.

[02] Rubinstein M.; Uman, M. A. Transient electric and magnetic fields associated with establishing a finite electrostatic dipole, revised. IEEE Transactions on Electromagnetic Compatibility, v.33, n.4, p.312-20, Nov. 1991.

[03] Mattos, M. A. Calculation of the lightning induced voltage on overhead transmission line. In: 1991 International Symposium on EMC, Cherry Hill - N.J, 1991. Proceedings. p.96-98.

[04] Kuramoto, S.; Sato, M.; Ohta, M. Surge current and voltage distribution in a reinforced concrete building caused by direct lightning stroke. In: 1991 International Symposium on EMC, Cherry Hill - N.J, 1991. Proceedings. p.84-89.

[05] Geri, A.; Veca, A. A complete lightning protection system simulation in the EMI analysis. In: 1991 International Symposium on EMC, Cherry Hill - N.J, 1991. Proceedings. p.90-95.

[06] Sowa,A. Surge current distribution in building during a direct lightning stroke. In: 1991 International Symposium on EMC, Cherry Hill - N.J, 1991. Proceedings. p.103-105.

[07] Cardoso, J.R. Curso de linhas de transmissão - Transitórios e Regime Permanente - Ondas eletromagnéticas. Universidade de São Paulo., in press.

[08] Sartori, Carlos A. F. Avaliação do ambiente eletromagnético em estruturas atingidas por descargas atmosféricas. Qualification for Master Thesis. Universidade de São Paulo. Oct. 1993.

[09] Sartori, Carlos A. F.; Cardoso, J. R. Evaluation of electromagnetic environment around a structure during a lightning stroke. In: 1994 International Symposium on EMC, Roma - Italy, Sept. 1994. Proceedings. p. 746-749.