

**A POWER CALIBRATION METHOD USING THE
XENON POISONING**

**CALIBRAÇÃO DA POTÊNCIA PELO ENVENENAMENTO
POR XENON**

MARCELLO DAMY DE SOUZA SANTOS

PAULO SARAIVA DE TOLEDO

INSTITUTO DE ENERGIA ATÔMICA

Publicação I E A — N.º 5

— 1958 —

INSTITUTO DE ENERGIA ATÔMICA

Caixa Postal 11049 (Pinheiros)

CIDADE UNIVERSITÁRIA "ARMANDO DE SALLES OLIVEIRA"

SÃO PAULO — BRASIL

CONSELHO NACIONAL DE PESQUISAS

Presidente — Prof. Dr. João Christovão Cardoso
Vice-Presidente — Prof. Dr. Athos da Silveira Ramos

UNIVERSIDADE DE SÃO PAULO

Reitor — Prof. Dr. Gabriel Sylvestre Teixeira de Carvalho
Vice-Reitor — Prof. Dr. Francisco João Humberto Maffei

INSTITUTO DE ENERGIA ATÔMICA DIRETOR

Prof. Dr. Marcello Damy de Souza Santos

CONSELHO TÉCNICO-CIENTÍFICO

Representantes do Conselho Nacional de Pesquisas

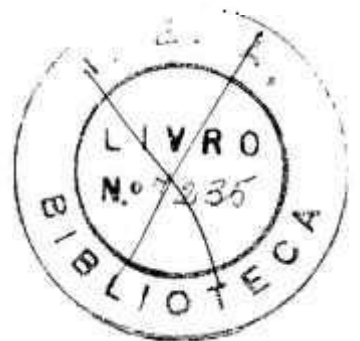
Prof. Dr. Luiz Cintra do Prado
Prof. Dr. Paulus Aulus Pompéia

Representantes da Universidade de São Paulo

Prof. Dr. Francisco João Humberto Maffei
Prof. Dr. José Moura Gonçalves

CONSELHO DE PESQUISAS

Prof. Dr. Marcello Damy de Souza Santos
— Chefe da Divisão de Física Nuclear
Prof. Eng. Paulo Saraiva de Toledo
— Chefe da Divisão de Física de Reatores
Prof. Dr. Fausto Walter Lima
— Chefe da Divisão de Radioquímica
Prof. Dr. Rômulo Ribeiro Pieroni
— Chefe da Divisão de Radiobiologia



A Power Calibration Method Using the Xenon Poisoning

By Marcello Damy de Souza Santos and Paulo Saraiva de Toledo*

In this paper a method is given of reactor power measurement applicable to reactors in which the thermal neutron flux is of the order of 10^{12} neutrons per square centimeter per second. The method was developed in order to have an independent method of power measurement of the Instituto de Energia Atômica swimming pool reactor in the region of 1 Mw.

Essentially, the method consists in measuring the reactivity variation due to xenon poisoning after the pool temperature has reached an equilibrium value. The power is easily calculated using an approximate quadratic expression for the time variation of xenon poisoning.

Results of measurements performed at the Instituto de Energia Atômica with the reactor at a nominal 1 Mw power are given and it is shown that the method is quite reliable provided the bulk pool temperature is constant or slowly varying.

XENON CONCENTRATION AS A FUNCTION OF TIME OF OPERATION

The differential equation governing the growth of xenon concentration in the core of a thermal reactor is:¹

$$\frac{dX(\mathbf{r},t)}{dt} = \lambda_1 I(\mathbf{r},t) + \gamma_2 \Sigma_f \phi(\mathbf{r}) - \lambda_2 X(\mathbf{r},t) - \sigma_2 \phi(\mathbf{r}) X(\mathbf{r},t). \quad (1)$$

The notation is the one of Ref. 1 with one single modification: the spacial and temporal dependence of the xenon concentration $X(\mathbf{r},t)$ is written explicitly.

The flux $\phi(\mathbf{r})$ is supposed to be constant in time. This assumption is quite good since the xenon poisoning is compensated in our reactor by the withdrawal of a single regulating rod and the time needed to take the necessary data is of the order of three hours. During this time and with the reactor operating at about 1 Mw with a thermal flux of the order of 10^{12} neutrons/cm² sec, the distortion of the thermal flux due to the withdrawal of the regulating rod is quite small. Strictly speaking, this distortion must exist since the distributed effect of the xenon poisoning is compensated by the quasi-local withdrawal of the regulating rod.

Integrating Eq. (1) with the condition that at $t=0$

we have $X(0)=I(0)=0$ and X_0 being the equilibrium value of xenon poisoning, we get after rearranging the corresponding expression of Ref. 1:

$$X(\mathbf{r},t) = X_0(\mathbf{r}) \left\{ 1 + \frac{\gamma_1}{\gamma_1 - \gamma_2} \left[\frac{\lambda_1}{\lambda_2 - \lambda_1 - \sigma_2 \phi(\mathbf{r})} \times \left(e^{-(\lambda_2 + \sigma_2 \phi(\mathbf{r}))t} - e^{-\lambda_1 t} \right) - \frac{\gamma_2}{\gamma_1} e^{-(\lambda_2 + \sigma_2 \phi(\mathbf{r}))t} - e^{-\lambda_1 t} \right] \right\}. \quad (2)$$

with

$$\lambda_1 = \frac{\gamma_1 + \gamma_2}{\lambda_2 + \sigma_2 \phi(\mathbf{r})}, \quad \Sigma_f \phi(\mathbf{r}). \quad (3)$$

Now, if we consider that

$$\lambda_2 = 2.1 \times 10^{-5} \text{ sec}^{-1}; \quad \sigma_2 = 2.88 \times 10^{-18} \text{ cm}^2 \\ \lambda_1 = 2.9 \times 10^{-5} \text{ sec}^{-1}; \quad (4)$$

we can see that for $\phi(\mathbf{r}) = 10^{12}$ n/cm² sec:

$$\lambda_2 = [\lambda_2 + \sigma_2 \phi(\mathbf{r})] = 4.98 \times 10^{-5} \text{ sec}^{-1}.$$

So, if $t \ll 10^5$ sec (30 hr), we have:

$$\lambda_2 t \ll 1, \\ \lambda_1 t \ll 1.$$

Then, for times of the order of 3 to 5 hr we will have a good approximate expression for $X(\mathbf{r},t)$ if we develop the exponentials in Eq. (2) in a power series and retain only the terms up to the second power. Doing this we get:

$$X(\mathbf{r},t) = X_0(\mathbf{r}) \frac{1}{\gamma_1 + \gamma_2} \left[\gamma_2 \lambda_1^2 t^2 + \frac{1}{2} \lambda_2 \sigma_2 \lambda_1^2 t^2 - \lambda_2 \gamma_2 \sigma_2 \lambda_1 t^2 \right] \quad (5)$$

The definition of poisoning $P_0(\mathbf{r},t)$ is:

$$P_0(\mathbf{r},t) = X(\mathbf{r},t) \sigma_2 / \Sigma_u, \quad (6)$$

in which Σ_u is the macroscopic absorption cross section of the fuel in the fuel elements.

Let us define a mean poisoning $P_0(t)$ as:

$$P_0(t) = \frac{1}{V} \int_V P_0(\mathbf{r},t) dV \quad (7)$$

where V is the core volume. Taking into account that this mean poisoning is related to the variation of reactivity $\Delta\rho(t)$ through the following equation:

$$P_0(t) = \Delta\rho(t) [\Sigma_m + \Sigma_u] / \Sigma_u \quad (8)$$

with Σ_m the macroscopic absorption cross section of

* Instituto de Energia Atômica, São Paulo (on leave of absence from São Paulo University).

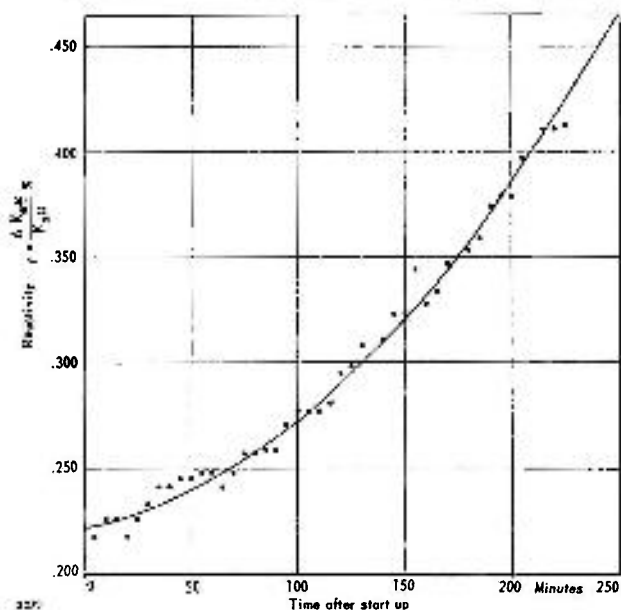


Figure 1

the moderator and structural materials of the core, we get:

$$[\Sigma_m + \Sigma_u] \Delta\rho(t) = \frac{\sigma_2}{V} \int_V X_0(\mathbf{r}) \frac{1}{\gamma_1 + \gamma_2} [\gamma_2 \bar{\lambda}_2 t + \frac{1}{2} (\gamma_1 \lambda_1 - \gamma_2 \bar{\lambda}_2) \bar{\lambda}_2 t^2] dV. \quad (9)$$

Remembering the expression for $X_0(\mathbf{r})$ and the definition of $\bar{\lambda}_2$ we have:

$$[\Sigma_m + \Sigma_u] \Delta\rho(t) = \frac{\sigma_2 \gamma_2 \Sigma_f t}{V} \cdot t \cdot \int_V \phi(\mathbf{r}) dV + \frac{\sigma_2 \Sigma_f t^2}{2V} \cdot \int_V (\lambda_1 \gamma_1 - \bar{\lambda}_2 \gamma_2) \phi(\mathbf{r}) dV. \quad (10)$$

Since $\bar{\lambda}_2 = \lambda_2 + \sigma_2 \phi(\mathbf{r})$, it follows that the coefficient of t^2 depends in a complicated way on the space variation of the thermal flux. This space variation is small in the swimming pool reactor due to the effect of the water reflector, and it will be a good approximation to consider $\bar{\lambda}_2$ as space independent. This approximation for the coefficient of the t^2 term is very good indeed, since $\bar{\lambda}_2 \gamma_2$ contributes less than 5% of the total value of the coefficient, even for a thermal flux as high as 10^{13} n/cm² sec. Considering then a space independent value for $\bar{\lambda}_2$ and calling it $\bar{\lambda}_2^{(0)}$ we obtain from: ¹⁰

$$[\Sigma_m + \Sigma_u] \Delta\rho(t) = [\sigma_2 \gamma_2 \Sigma_f \bar{\phi}] t + \left[\frac{\sigma_2}{2} (\lambda_1 \gamma_1 - \bar{\lambda}_2^{(0)} \gamma_2) \Sigma_f \bar{\phi} \right] t^2 \quad (11)$$

with $\bar{\phi}$, a mean thermal flux, defined as:

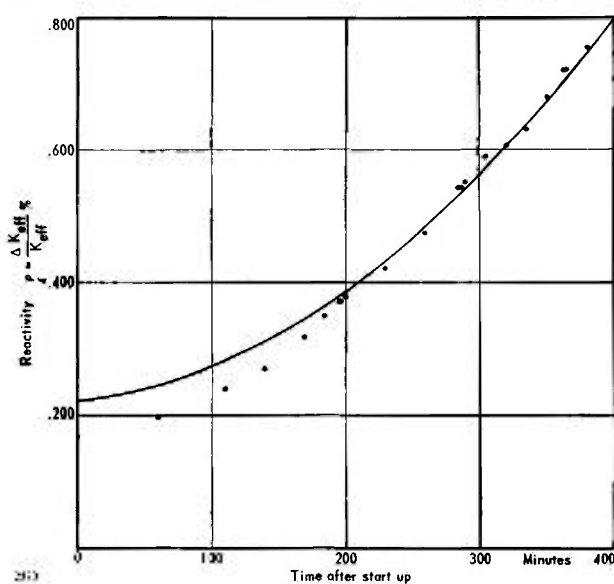


Figure 2

$$\bar{\phi} = \frac{1}{V} \int_V \phi(\mathbf{r}) dV. \quad (12)$$

Finally, since the power P is given by:

$$P = \frac{1}{3.1 \times 10^{10}} \Sigma_f \bar{\phi} V_f \quad (13)$$

(with V_f the fuel volume) it follows that:

$$\Delta\rho(t) = \left[\frac{3.1 \times 10^{10} \sigma_2 \gamma_2}{V_f (\Sigma_m + \Sigma_u)} P \right] t + \left[\frac{3.1 \times 10^{10} \sigma_2}{2 V_f (\Sigma_m + \Sigma_u)} (\lambda_1 \gamma_1 - \bar{\lambda}_2^{(0)} \gamma_2) P \right] t^2. \quad (14)$$

Writing for the time dependence of $\Delta\rho(t)$:

$$\Delta\rho(t) = \alpha t + \beta t^2 \quad (15)$$

and determining experimentally the values of α and β we can calculate P through any of those values. The coefficient of t^2 could be used but we would have in the determination of P through it an error due to the approximation we made when we considered $\bar{\lambda}_2$ as constant. Besides, to calculate this constant we would need at least the mean value of the thermal flux, which is unknown also.

All these drawbacks do not exist if we use the value of α to calculate P , since now all constants are known and the only approximations made are the ones related to the use of Eq. (8); those are inherent to this method and cannot be avoided.

The expression to be used to calculate P will then be:

$$P = \frac{1}{3.1 \times 10^{10}} \cdot \frac{V_f (\Sigma_m + \Sigma_u)}{\sigma_2 \gamma_2} \cdot \alpha. \quad (16)$$

Table 1

Experiment	Nominal power	β_0	α	β	$(\beta/\alpha)_{\text{exp}}$	$(\beta/\alpha)_{\text{app}}$ (calculated)	P calculated Eq. 17
IEA-(A exp)	1 Mw	0.22×10^{-2}	2.099×10^{-4}	3.026×10^{-9}	2.40×10^{-4}	2.58×10^{-9}	1 Mw

Since $V_t N_u = M_u = \text{mass of fuel in the core (in grams)}$, we have, with σ_u the absorption cross section of the fuel:

$$P = \frac{1}{3.1 \times 10^{10}} \cdot \frac{M_u}{\gamma_2} \left(1 + \frac{\sum_m}{\sum_u} \right) \left(\frac{\sigma_u}{\sigma_2} \right) \cdot a \quad (\text{watts}). \quad (17)$$

It is interesting to note that even for $\phi = 10^{13} \text{ n/cm}^2 \text{ sec}$ the experimentally determined ratio (β/a) must agree with the theoretical value computed taking for $\lambda_2^{(0)}$ the value λ_2 . Indeed the correct value for (β/a) is:

$$(\beta/a) = \frac{\lambda_1 \gamma_2 - \lambda_2^{(0)} \gamma_2}{2\gamma_2}$$

and this should be compared with the approximate value taking $\lambda_2^{(0)} \cong \lambda_2$:

$$(\beta/a)_{\text{app}} = \frac{\lambda_1 \gamma_1 - \lambda_2 \gamma_2}{2\gamma_2}$$

Introducing the numerical values for the constants and taking $\phi = 10^{13} \text{ n/cm}^2 \text{ sec}$ we get:

$$\frac{(\beta/a) - (\beta/a)_{\text{app}}}{(\beta/a)} = 0.03,$$

that is, there is an error of 3%, certainly inferior to the experimental errors, in the measurements of reactivity variation.

RESULTS OF EXPERIMENTS

During the preliminary steps of power reactor calibration, with the power measured by a temperature differential method,² the variation of the regulating rod position as a function of time was determined after the pool temperature was almost stabilized. The experimental points of $\rho(t)$ versus t for an experiment performed, at a nominal power of 1 Mw, are given in Fig. 1.

Fitting the points by a curve of the form:

$$\rho(t) = \rho_0 + \alpha t + \beta t^2 \quad (18)$$

was done by a least squares method. The values obtained for the coefficients, the power calculated by use of Eq. (17), the experimental and theoretical approximate ratio (β/a) are given in Table 1 for our A experiment.

In a second experiment (B) at the same nominal power of 1 Mw, the experimental points obtained are given in Fig. 2. It can be seen that after 200 minutes, the pool temperature being then almost stabilized, the experimental points are very well fitted by a curve with the same values of α and β . As a matter of fact, the value of ρ_0 for the curve of Fig. 2 is the same as the one for the curve of Fig. 1; but this is merely a coincidence, since the only thing one could expect is that

the two curves be parallel, especially since the temperatures are different in the two experiments.

It is important to note that if one wishes to fit the experimental points by a curve using the method of least squares, the number of experimental points must be quite large; otherwise the computed values of the coefficients will be in error, due to the greater weight given by the least square method to the points corresponding to high values of t . In the analysis of our two experiments, we took as final values of the coefficients the mean between the values obtained putting the time origin for the application of the least square method at $t=0$, $t=T/2$ and $t=T$, where T is the total time in which the measurements were taken. The fitting of the experimental points by a curve of the form given by Eq. (11) is better than the one using a form without ρ_0 owing to fluctuations that occur in the position of the control rod due to small statistical fluctuations in the power.

Taking these precautions and especially considering only the measurements taken after the pool temperature has attained its equilibrium value, the method seems quite reliable.

One final remark must be made: the experimental points to be used in the determination of the coefficients α and β , are the ones obtained after the pool temperature has stabilized; but the real time origin must always be taken at the time in which the steady operation of the reactor has been initiated, in order that the requirements for the applicability of Eq. (2) be fulfilled. With a reactor already poisoned by xenon Eq. (2) is not applicable.

It seems, then, that the method is quite suitable for power measurements and that there are good grounds to expect it to give reasonably reproducible results. Anyway, the final decision is dependent on further experiments and its precision can only be estimated theoretically after a careful examination of the precision of the constants used. Those experiments will be done as soon as possible at the Instituto de Energia Atômica, depending on the schedule for operation at 1 Mw of the IEA Reactor.

ACKNOWLEDGEMENTS

We take the opportunity to thank Miss Ewa Wanda Cybulska for doing almost all the numerical calculations and to Professor Ivan Cunha Nascimento, Dr. Americo Frascino and Dr. Azor Camargo Penteadó for getting the data for experiments A and B.

REFERENCES

1. Glasstone and Edlund, *The Elements of Nuclear Reactor Theory*, D. Van Nostrand Co., New York (1955).
2. M. D. de Souza Santos et al., *Preliminary Results of 5 Mw Operation with the Brazilian Swimming Pool Reactor*, P/2279, Vol. 10, these Proceedings.

ERRATA

- 1) pag 544, second column, 11th line :
 instead of : $\phi(\mathbf{r}) = 10^{12} \text{ n/cm}^2 \text{ sec}$:
 read : $\phi(\mathbf{r}) = 10^{13} \text{ n/cm}^2 \text{ sec}$:
- 2) pag 545, first column, 19th line :
 instead of : from (10)
 read : from (10)
- 3) pag 545 : Table I should be substituted by the following

TABLE I

Experiment	<i>Nominal</i>						
	power	ρ_0	α (sec ⁻¹)	β (sec ⁻²)	$(\beta/\alpha)_{\text{exp}}$ sec ⁻¹	$(\beta/\alpha)_{\text{app}}$ calculated (sec ⁻¹)	P eq (17)
IEA-(A exp)	1 MW	0.22×10^{-2}	3.498×10^{-8}	8.405×10^{-12}	2.40×10^{-4}	2.60×10^{-4}	1 MW

- 4) pag 546 :
 The first four lines of first column should be substituted by the following :
 "Since $V_f N_u = M_u \frac{N_u}{A_u}$ where M_u is the mass of fuel in the core, in grams, N_u is the Avogadro's number and A_u is the atomic mass of the fuel, we have, with σ_u the absorption cross section of the fuel :

$$P = \frac{1}{3.1 \times 10^{10}} \cdot \frac{M_u}{\gamma_2} \cdot \frac{N_u}{A_u} \cdot \left(1 + \frac{\sum_{\text{th}}}{\sum_{\text{a}}} \right) \left(\frac{\sigma_u}{\sigma_2} \right) x. \quad (\text{watts}) \quad (17)$$

- 5) pag 546, first column, 10th line :
 instead of : $(\beta/\alpha) = \frac{\lambda_1 \gamma_2 - \bar{\lambda}_2^{(0)} \gamma_2}{2 \gamma_2}$
 read : $(\beta/\alpha) = \frac{\lambda_1 \gamma_1 - \bar{\lambda}_2^{(0)} \gamma_2}{2 \gamma_2}$
- 6) pag 546, first column, 16th and 17th lines :
 instead of :

$$\frac{(\beta/\alpha) - (\beta/\alpha)_{\text{app}}}{(\beta/\alpha)} = 0.03$$

that is, there is an error of 3%,

read :

$$\frac{(\beta/\alpha) - (\beta/\alpha)_{\text{app}}}{(\beta/\alpha)} \approx 0.06$$

that is, there is an error of the order of 6%,