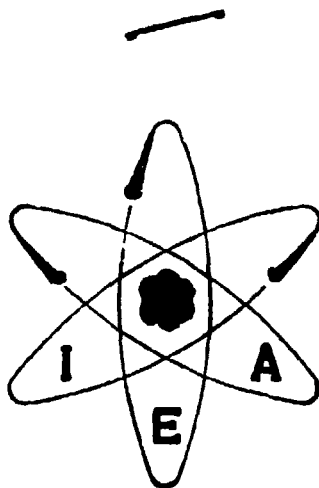


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OF PHOTONS IN AN ANTHROPOMORPHIC MODEL OF THE HUMAN BODY**

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# MONTE CARLO ESTIMATION OF DOSE FROM INTERNAL SOURCES OF PHOTONS IN AN ANTHROPOMORPHIC MODEL OF THE HUMAN BODY

Walter S. Snyder\*

## ABSTRACT

To estimate the dose from a source of photons within the body, attempts are made to model the body and its principal organs approximately taking into account their sizes, shapes, compositions and densities, and Monte Carlo Method is used. Using this technique one attempts to compute representative "histories" of photons originating in the source. At each point of interaction one determines the type of interaction, the energy loss in the tissue and the new direction of the photon using the well established laws of physics.

Estimation of dose from a source of photons within the body is not easy because of the complexities of the geometrical structures and the inhomogeneities of the body. Also it will generally be true that a calculation using only the first interactions of the photons will not be sufficiently accurate since the buildup factor may be as much as 2000 at 20 mean free paths in water or in tissue. This paper presents one approach; one attempts to model the body and its principal organs approximately, taking into account their sizes, shapes, compositions and densities, and one uses a method of calculation suitable for such a model, the so called Monte Carlo method. Using this technique one attempts to compute representative "histories" of photons originating in the source. At each point of interaction one determines the type of interaction, the energy loss in the tissue and the new direction of the photon using the well established laws of physics. These laws are not deterministic and one can only predict a distribution of interactions of energy losses and of directions and velocities for the photon. Thus at each alternative of the photons "history" one must play a "game of chance" using random numbers which decide what path a photon should follow. It is because of this probabilistic character that it is termed the Monte Carlo Method.

The phantom is shown in outline in Fig. 1A, and in Fig. 1B various "organs" are sketched. Each such "organ" of the phantom is defined by rather simple mathematical expressions. For example, the stomach is an ellipsoid specified in an appropriate coordinate system by

$$\left(\frac{x-8}{4}\right)^2 + \left(\frac{y+4}{3}\right)^2 + \left(\frac{z-35}{8}\right)^2 \leq 1 \quad (1)$$

The ribs have a more complicated set of inequalities

$$\begin{aligned} \left(\frac{x}{17}\right)^2 + \left(\frac{y}{9,8}\right)^2 &\leq 1 \\ \left(\frac{x}{16,5}\right)^2 + \left(\frac{y}{9,3}\right)^2 &\geq 1 \\ 35,1 &\leq z \leq 67,3 \end{aligned} \quad (2)$$

$$\sin \Pi \left(\frac{z-35,1}{1,4}\right) > 0$$

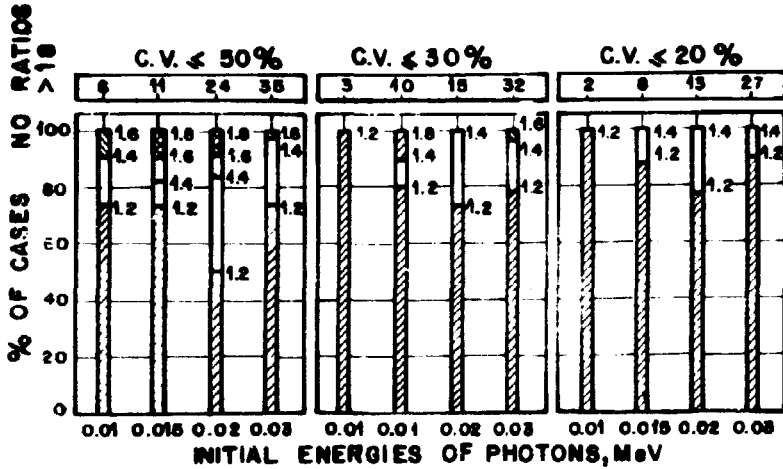


Fig. 1(a) - Distribution of reciprocity ratios: tissue organ and tissue target

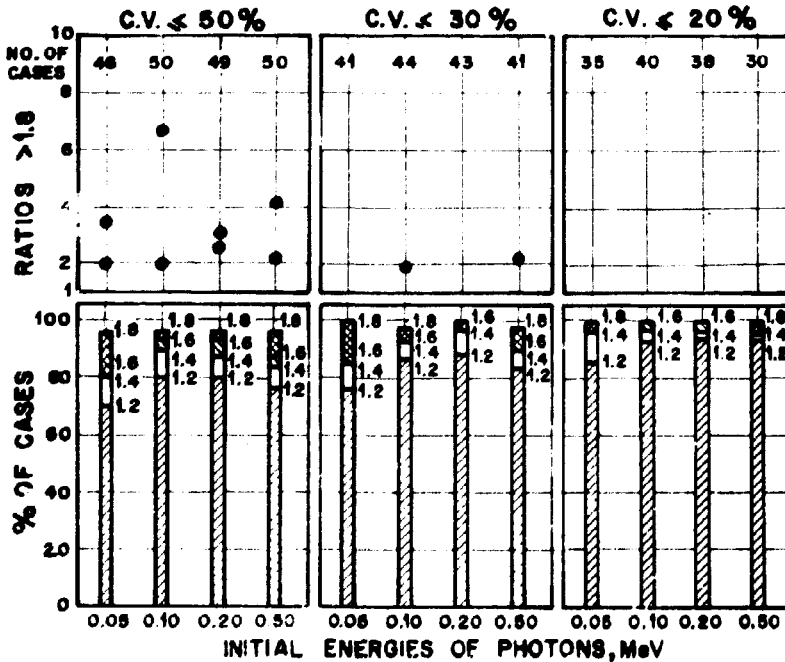


Fig. 1(b) - Distribution of reciprocity ratios: tissue organ and tissue target

All five inequalities must be satisfied for the point (x,y,z) to be in the ribs. Of course, these are only examples and for the complete description of the phantom one must consult Ref. 1 of the bibliography. The point is that these equations are of simple type which one can check quickly on a modern digital computer. The phantom is described in these mathematical terms and until recently it had no physical existence. Within the last year such a phantom has been built at Oak Ridge National Laboratories in order to check measurements of dose against the calculated values. These measurements have agreed as well as might have been expected, taking into account that the Monte Carlo Method is statistical so that always one must be satisfied with an estimate of dose. Of course, the difficulties of measurement are also considerable but on the whole, taking both sources of error into account, the two methods gave comparable values for the organs tested.

The phantom contains three types of tissue, lung tissue (density  $\sim 0.3 \text{ g/cm}^3$ ), bone plus marrow (density  $\sim 1.5 \text{ g/cm}^3$ ) and soft tissue (density  $\sim 1 \text{ g/cm}^3$ ). The list of specified organs and their masses is contained in Table 1. The composition of the three tissues is shown in brief in Table 2. This composition is based on autopsy data from 150 adults who were judged to have been grossly normal prior to death, supplemented by literature values for the more abundant elements. The composition was specified by Dr. Isabelle Tipton who was responsible for the data on trace elements obtained as a result of these studies

Of course, the phantom was not always this complicated. At first it was only a homogeneous cylinder of tissue, then legs and a head were added but the arms are considered as part of the body. At first it was subdivided into regular geometrical portions and only later were the organs defined and then the composition was determined for the different tissues. This process has not ended and almost every study described below has resulted in some changes in the phantom. Clearly, as the needs of dosimetry become greater the phantom must change to meet these needs. There may be some who question this elaboration and will wonder if the computed doses approximate those received by individuals. Certainly, it would be foolhardy to expect more than approximate agreement with the dose received by a 60 kg man, or an 80 kg man since our phantom has a mass of 70 kg. Thus the doses are only approximations to the doses received by real people and the phantom offers us the chance to study how such doses vary with the mass of the body or of the various organs.

Calculations have been carried out for a source uniformly distributed in 16 different organs and for twelve monoenergetic photons with energies ranging from 0.01 Mev to 4 Mev. The results have been reported in Snyder et al (1969). The data are given in the form of an absorbed fraction of energy  $\psi$  where

$$\psi = \frac{\text{energy absorbed by target organ}}{\text{energy emitted by source}}$$

The absorbed fractions of energy are not all equally reliable. This is because the Monte Carlo Method is statistical and consequently dose to a small organ will interact within the organ. Thus the dose to the gonads, or to the thyroid, is generally rather inaccurately determined. For this reason the code also calculates a standard deviation for each estimate. In many cases this amounts to 50 percent or more of the value being estimated and we have not published such values, preferring to leave a blank in the table. Experience has shown that even when the standard deviation is 30 percent or so of the estimate, the estimate may be inaccurate by a

factor of 2 or more. There have been a number of studies carried out in the attempt to obtain values which would be useful in filling these gaps in the table.

One of these is concerned with the validity of the reciprocity theorem. This theorem predicts that in any two regions of a homogeneous and infinite medium the dose rates are equal in the two regions if the same activity as a source is uniformly distributed in the other region, that is, if A and B are the two regions then the theorem asserts that

$$\frac{D(B \leftarrow A)}{D(B \rightarrow A)} = 1 \quad (3)$$

if the activities in the source organs are the same and are uniformly distributed and  $D(B \leftarrow A)$  represents the dose rate in target organ B from source in A.

This theorem is frequently used by radiologists in spite of the fact that the body is neither homogeneous nor is it infinite as the hypotheses of the theorem demand. Whenever two computations have been completed at the same source energy but for different source organs, one can check whether the doses in rads per photon are approximately equal or not. Since the estimates are statistical there is always some margin of error. The above fractions, taking as numerator the larger of the two doses, have been studied for those organs where the coefficient of variation ( $= 100 \sigma/\text{mean}$ ) does not exceed 50% and the results are shown as a bar graph in Fig. 1a, 1b and 1c. It is clear that in most cases the two results agree within a factor of 2. In fact, in about 70% of the cases the ratio is within a factor of 1.3. Moreover, if we restrict attention to only those organs for which the coefficient of variation does not exceed 30% the results are even better and the exceptional organs for which the fraction exceeds 2 are greatly diminished. Finally, if we restrict attention to those organs where the coefficient of variation is less than 20% these exceptional cases disappear and the fraction is always less than 2. These results offer considerable evidence, not that the reciprocity theorem is exact, but that for organs composed of soft tissue the theorem holds within 20 – 30%.

In the above cases we have used only the data on the organs composed of soft tissue. If we now examine the evidence where one of the organs is the skeleton (= bone + marrow) we find that the situation is somewhat different. The similar results are displayed in the upper portion of Fig 2a and 2b. The quantity plotted is  $\ln(D_{\text{skel} \rightarrow T}, D_T \rightarrow \text{skel})$  where T denotes a target organ composed of soft tissue. This time the results do not improve as we restrict ourselves to results which are statistically better and we have to conclude that the difference is real and is not the result the statistical character of the estimates. If one applies a correction factor to the ratio, namely  $\mu_{\text{ab}}^{\text{skel}} / \mu_{\text{ab}}^T$ , where  $\mu_{\text{ab}}$  denotes the mass energy absorption coefficient, the results are better. In Fig 2a and 2b these results are displayed and one sees that the variation is by about a factor of 2 – 3. These results are discussed further in Snyder et al (1972).

Another result of considerable interest is the extent to which the absorbed fraction is proportional to the mass. Assuming that the source and target organs are distinct and are separated by a reasonable distance we have tested this by computing the dose to 3 bladders plus contents one of 502 g, a second of 382 g and a third of 254 g. The dose should be identical if the absorbed fraction is truly proportional to the mass and the resulting dose estimates are plotted

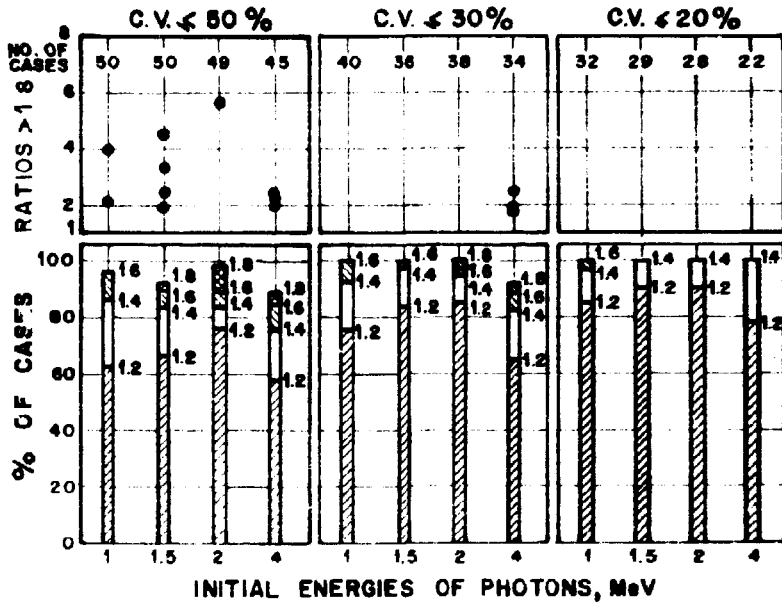


Fig. 1(c) - Distribution of reciprocity ratios: tissue organ and tissue target



in Fig 3 for the source located in the ovaries and in kidneys. It is seen that the dose is practically identical. Only at the lowest energy of 10 keV it appears there might be a spread of values but here the statistics of the estimates are rather poor. This data is taken from Snyder, W.S. et al (1972).

When the source organ and the target organ are identical there is no simple rule. One can calculate that if the organ is spherical in shape and if multiple scattering can be neglected the absorbed fraction should vary inversely with the cube root of the mass. The quantity  $AF/(\mu_{ab} M^{1/3})$  is plotted in Fig 4 and the fraction is approximately constant for energies above 100 keV. Although the organs plotted are in several cases not approximately spherical, the proportionality seems to hold remarkably well although the constant factor certainly varies with shape. However, at energies below 100 keV it is clear that the above quantity is not approximately constant and we should expect some departure because the assumption that multiple scattering can be neglected is no longer valid. Unfortunately, there is no good rule which will predict the behaviour of the absorbed fraction for these energies. The data on the variation of the absorbed fraction with mass is discussed further in Snyder, W.S. et al (1972).

By use of the reciprocity theorem we can estimate the absorbed fraction for some of the source-target organs for which the direct Monte Carlo estimate is unreliable. But this will not suffice to fill all the gaps in the tables given in MIRD Pamphlet No 5, that is, in Snyder, W.S. et al (1969). For example, if both the organs are small, neither estimate may be valid. The use of the build-up factor enables one to compute the specific absorbed fraction, that is the absorbed fraction per gram, and this estimate is found to be remarkably similar to the results obtained by the direct Monte Carlo calculation. The specific absorbed fraction is given by

$$\phi = \frac{\mu_{ab}}{|S||T|} \int_S dx \int_T dy \frac{e^{-\mu|x-y|}}{4\pi|x-y|^2} B(\mu|x-y|) \quad (4)$$

In which S and T are the source and target organs with masses |S| and |T| respectively,  $\mu$  is the mass attenuation coefficient,  $\mu_{ab}$  the mass-energy absorption coefficient,  $B(\mu|x-y|)$  is the build-up factor for the distance  $|x-y|$  and it is assumed that B is known for a medium of near unit density. In the actual integration, which is over three dimensions for organ S and also three dimensions for organ T, the build-up factor computed for water as tabulated by Berger (1968) was used. Typical results are shown in Fig 5 and 6 which are for photon energies of 0.1 MeV and 0.5 - 4 MeV respectively. More tests of the validity of this procedure are reported in Snyder and Ford (1972) and the reader is referred to these publications for further information. The examples shown in Figs 5 and 6 are remarkable in that the value of  $\phi$  as estimated by equation 4 (which implies that both organs are part of an infinite homogeneous medium) and the values obtained by direct Monte Carlo calculation agree within a factor of 2 for those organs where the coefficient of variation is less than 50%, and in most cases the difference of the ratio lies between 0.7 and 1.3. There is no reason to suspect that the accuracy of the method depends on the size of the organs and thus this alternative method, while not producing estimates of high accuracy, should produce estimates which depart from the accurate value by a factor of 2 at worst and generally will be within 20-30% of the correct value.

The use of an anthropomorphic phantom and of the Monte Carlo Method of calculation

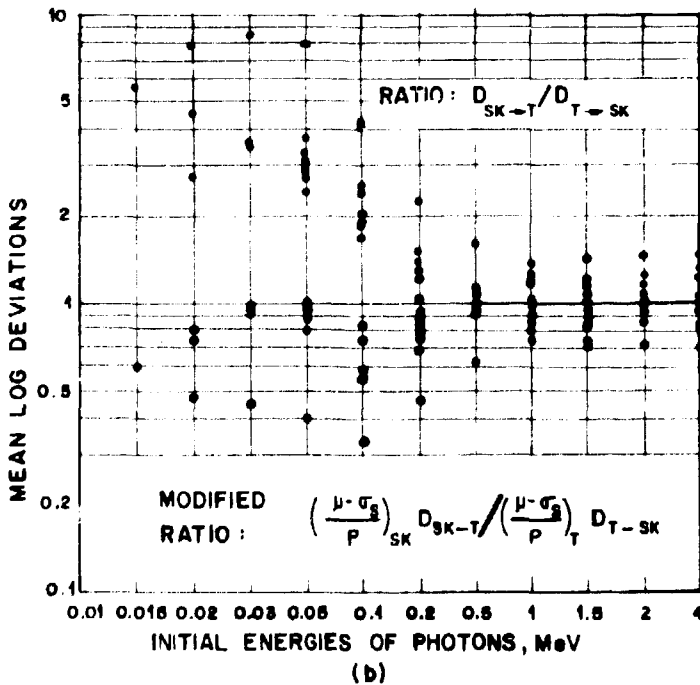
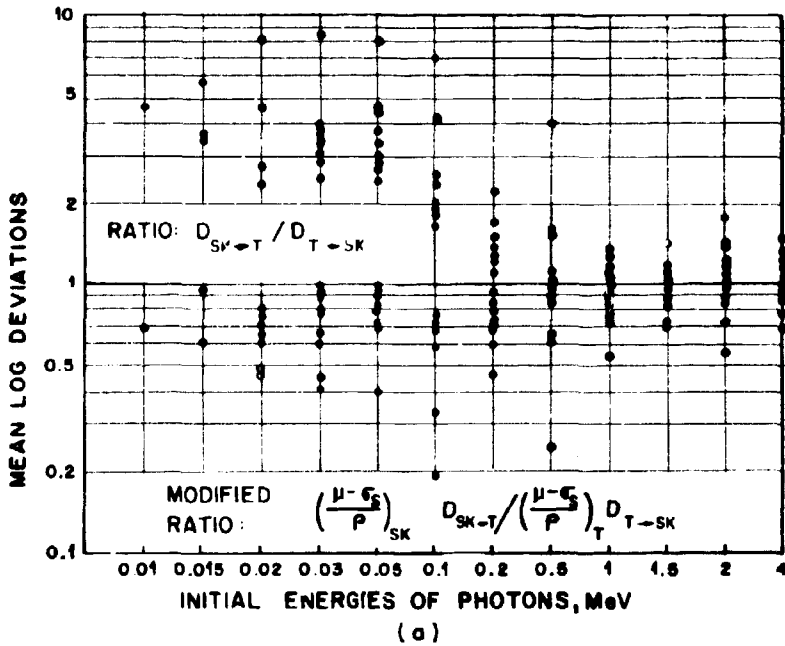


Fig. 2 - Reciprocity ratios for skeleton to tissue (a) Coefficient of variation,  $\leq 50\%$ . (b) Coefficient of variation,  $\leq 20\%$ .

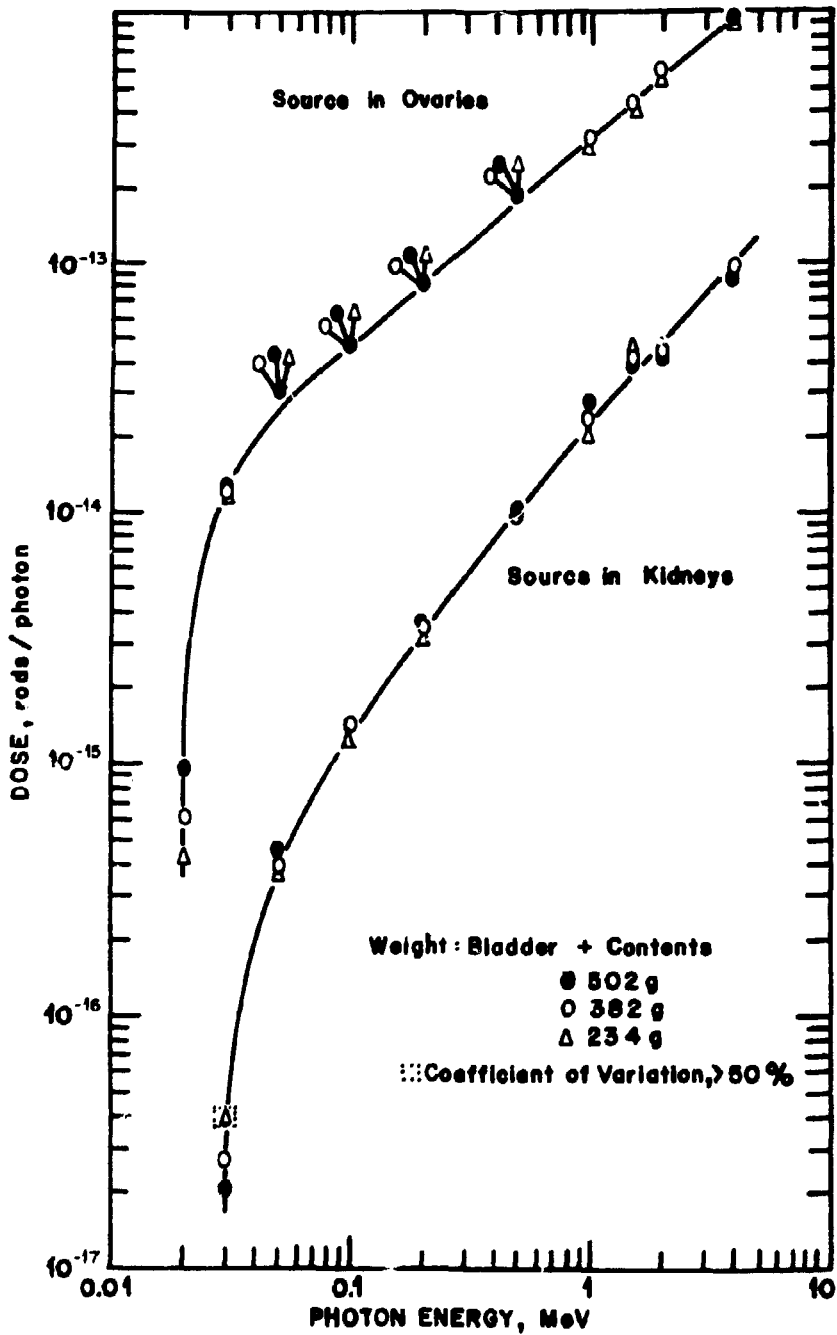


Fig.3 - Dose for three bladder sizes and two source organs

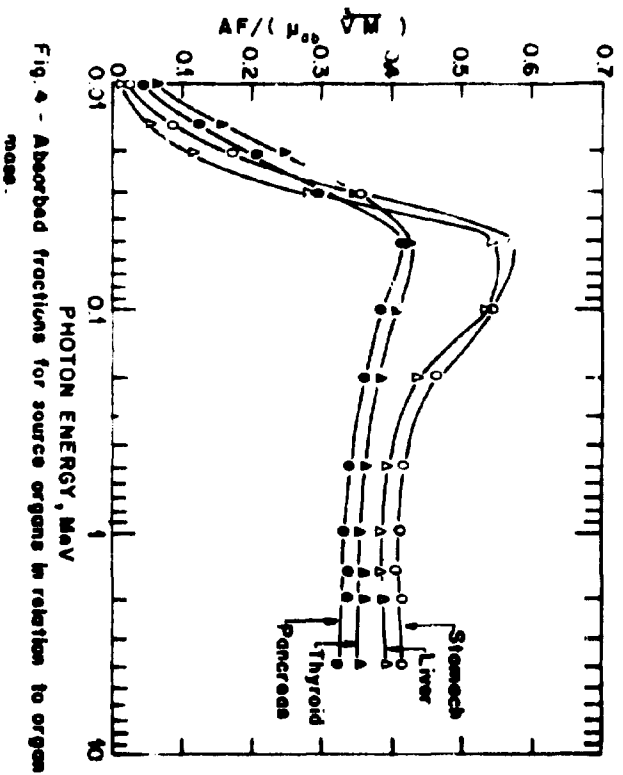


Fig. 4 - Absorbed fractions for source organs in relation to organ mass.

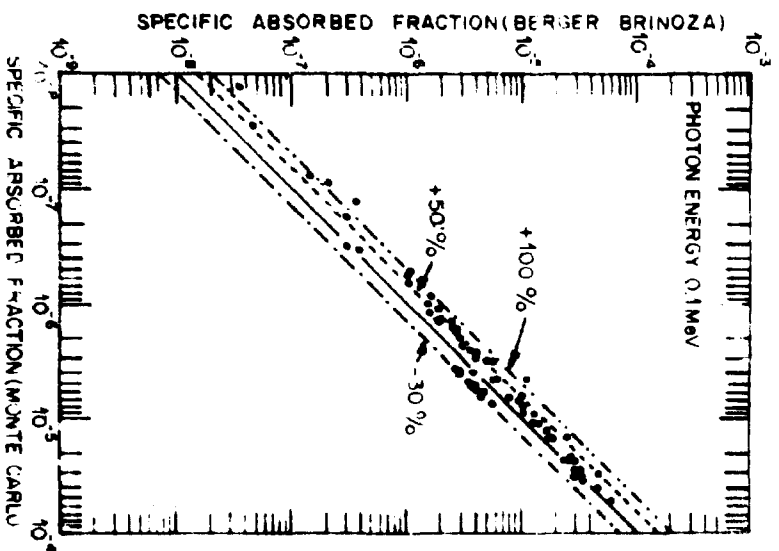


Fig 5 - Specific absorbed fraction in an anthropomorphic phantom and in an infinite medium.

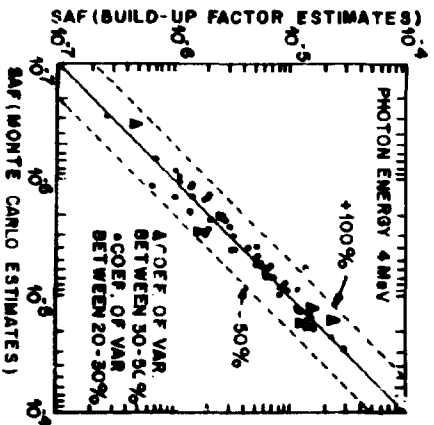
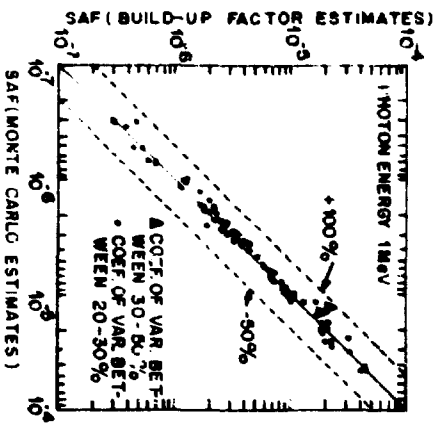
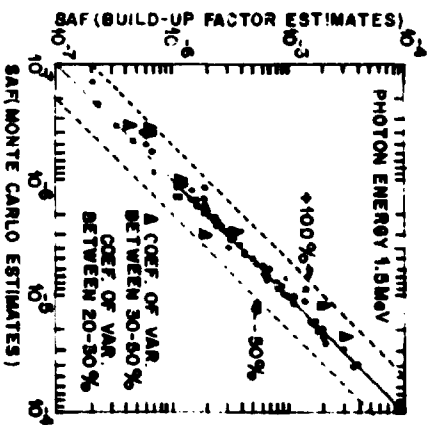
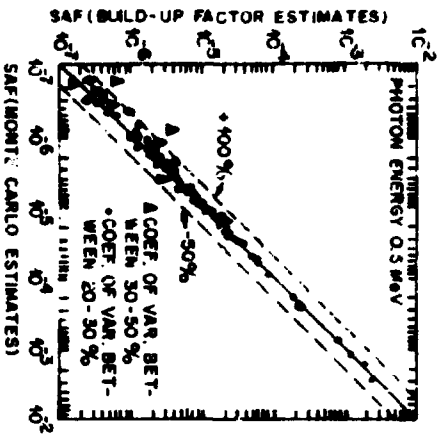


Fig 6. Monte Carlo estimates of specific absorbed fractions compared with build-up factor estimates

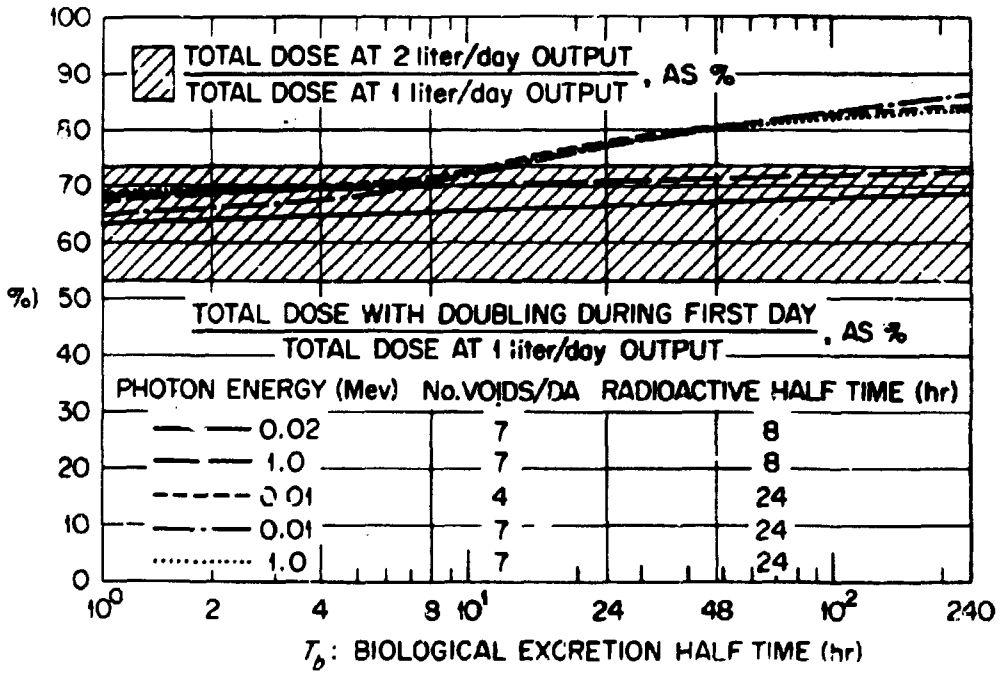


Fig 7 Effect of Doubling Urinary Output

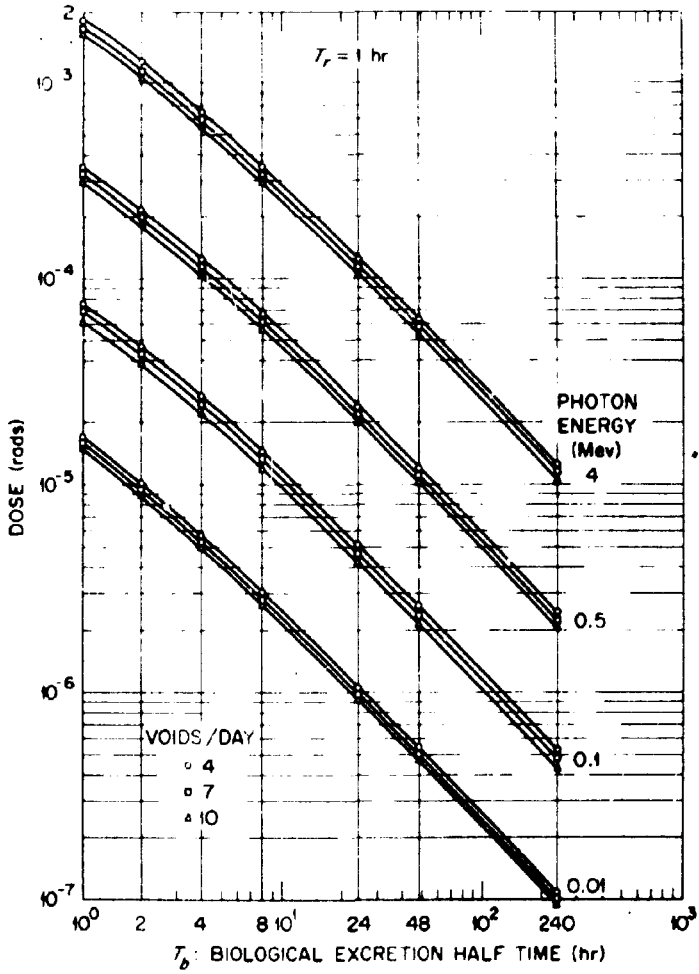
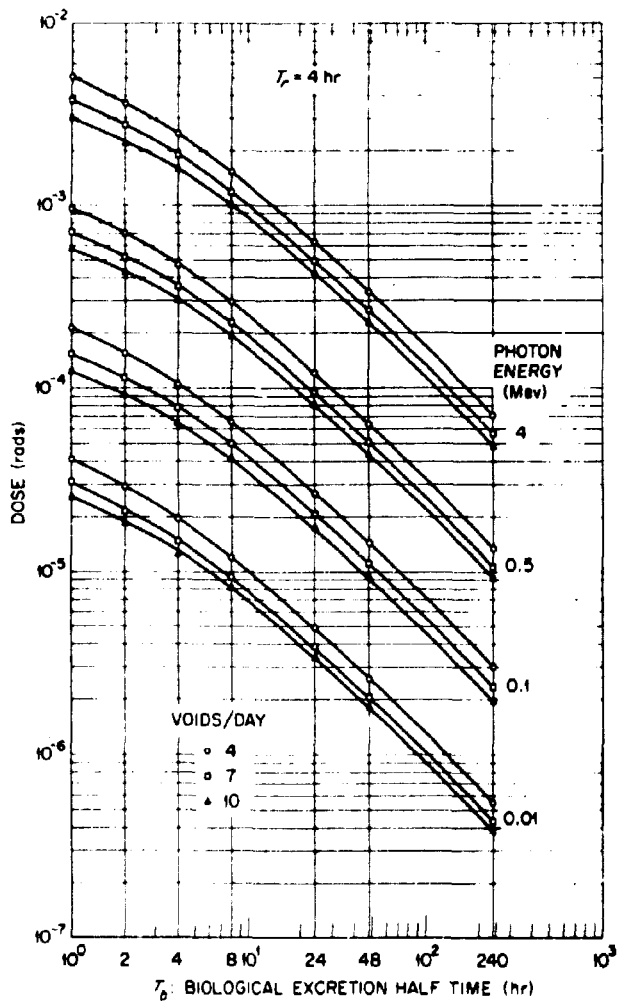


Fig. 8 - Variation of Total Dose for 4,7 and 10 VOIDS/DAY  
(Urine Volume = 1400 ml)



**Fig 8 b · Variation of Total Dose for 4,7 and 10 VOIDS/DAY.  
(Urine Volume = 1400 ml)**



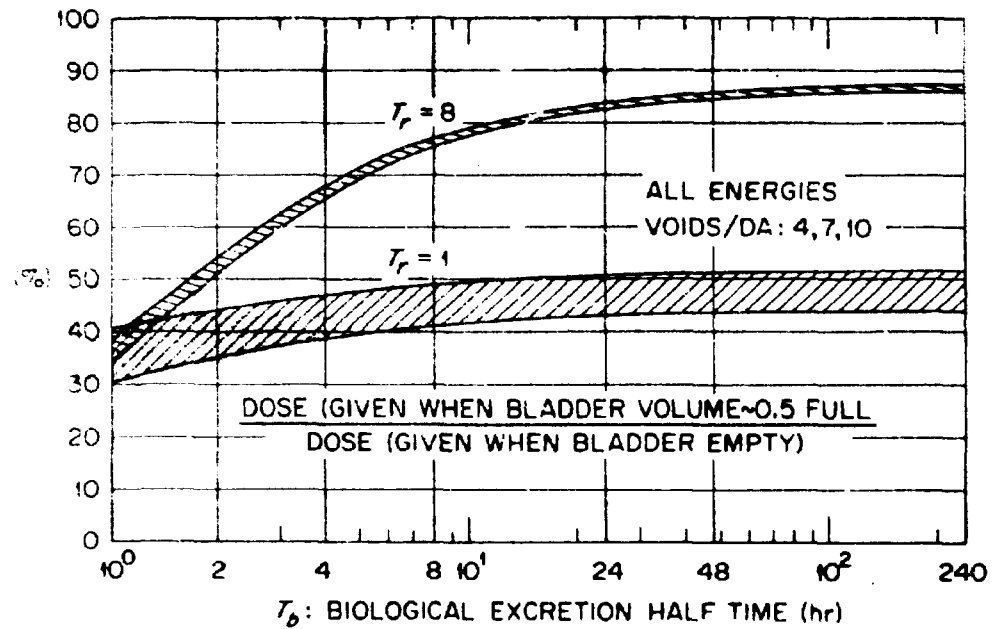


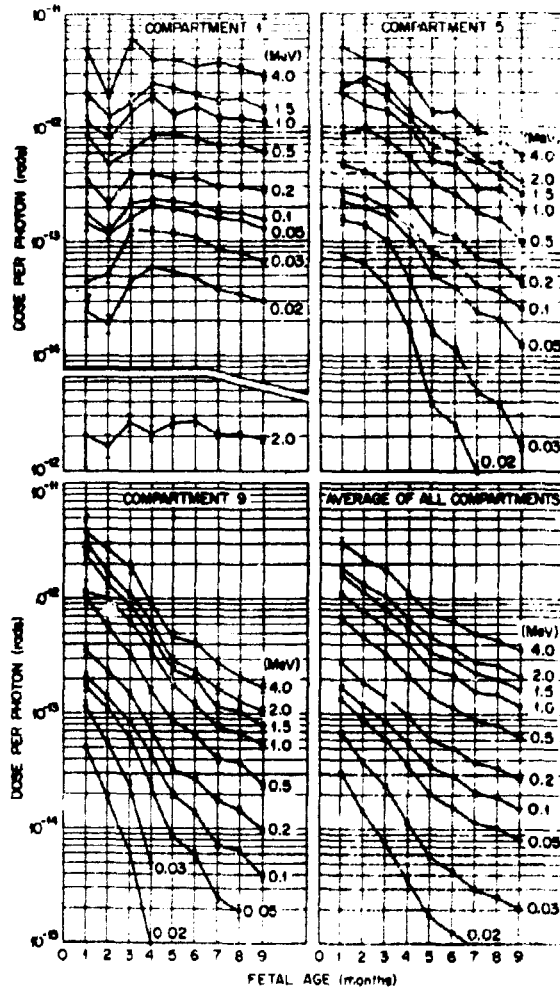
Fig 9 - Effect on Dose of Choice of Time of Administration.  
(Urine Output 1400 ml/day)

also allows one to explore many special questions of dosimetry. For example, the bladder dose rate from a photon emitter in the urine, varies greatly during the course of a filling as might be expected on the basis of the inverse square law alone. This variation of dose rate with the volume of urine has been studied by Snyder and Ford (1973). They have varied the photon energy, the biological half time for the radionuclide to enter the urine, the radioactive half-time, the daily output of urine and the schedule of voidings in all, over 4500 results were tabulated. In Fig. 7 the ratio of total dose at 2 liters per day output of urine to the total dose at 1 liter per day output is indicated. Individual points are not shown but in every case the ratio lay in the shaded area, indicating that one can avoid 25-50% of the dose to the bladder by doubling the output of urine. If this increased formation of urine leads to a more frequent schedule of voidings the dose is further decreased as is indicated in Figs 8a and 8b although the reduction is only a small factor. Finally, if the clinician can administer the radio nuclide when the bladder is approximately half-full there is a saving of about 25-50% for radionuclides of short half-life, that is less than 8 hours. This result is illustrated in Fig. 9. The interested reader is referred to the original publication for further details on the method and the results. A study of the beta portion of the dose to the bladder has been undertaken and similar results appear to hold. When this study is completed a complete report on the dosimeter of the bladder will be published.

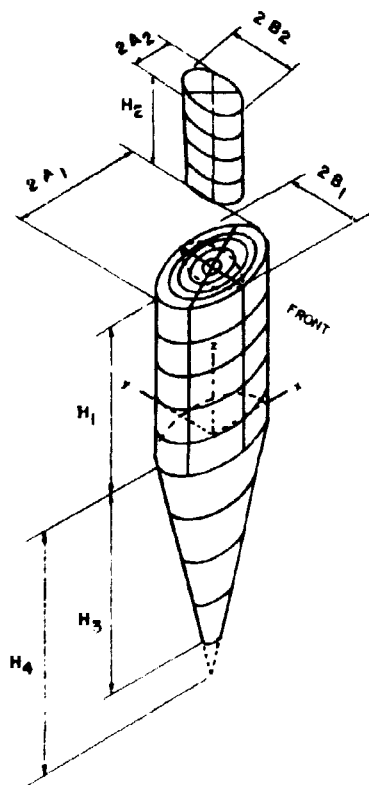
Another special study concerns dose to a fetus from a photon emitter present in the bladder. Many radionuclides are excreted to a considerable extent in urine and thus the bladder constitutes a source of photons which lie close to the fetus, in fact it touches the uterus. This question has been studied by Cloutier, Smith and Snyder (1973) and Fig. 10 indicates the course of the dose to different portions of the fetus. The fetus has been subdivided into 12 portions in order to get some idea of the variation of dose within it. Subdivisions 1 and 5 lie close to the bladder and consequently get the lightest dose. As the pregnancy progresses the portions of the fetus tend to move away from the source and this is the primary reason for the general decline of dose. The reader should refer to the published paper for further details on either the method or on the results.

Finally, the variation of photon dose with body size, that is, with age, has been explored in a preliminary way. To avoid the great labor of redesigning the organs of the phantom, the adult phantom has been shrunk by transformations which operate independently on the head region, the trunk and the leg region of the phantom. Thus, although the resulting "scaled phantoms" cannot claim to be entirely accurate, the major body portions have been adjusted to representative masses for various ages. The phantoms defined in this way represent individuals of ages of 0 (newborn), 1, 5, 10 and 15 years respectively. The masses and dimensions of these phantoms are specified in Fig. 11. Some of the results obtained in this way are plotted in Figs 12-15 which indicate dramatic increases in the value of the specific absorbed fractions of photon energies as age (body size) decreases. This increase is greatest for the lower energies but it is always substantial, that is, one to two orders of magnitude. Since dose per photon is directly proportional to the value of the specific absorbed fraction this indicates similar increases in dose per unit of administered activity. Of course, clinicians do adjust the dose for the age of the person, but these calculations offer an objective bit of evidence on the extent of the adjustment necessary to the achieve about the same absorbed dose. In practice of health physics these will frequently be factors (smaller consumption of air, water, foods, etc.) which may largely offset these increases in dose, but it is important to be able to judge objectively whether or not the reduction of intake balances the increase in dose per unit intake.

Of course, these are only some examples of the many studies made possible by use of the



**Fig. 10 - Average Dose per Bladder Filling (rads/photon) of Certain Compartments and Average of All Compartments for Photons of Energies Shown**



PHANTOM DIMENSIONS AND DOSE REGIONS

Age (yr)	Weight (kg)	H <sub>1</sub> (cm)	H <sub>2</sub> (cm)	H <sub>3</sub> (cm)	H <sub>4</sub> (cm)	A <sub>1</sub> (cm)	B <sub>1</sub> (cm)	A <sub>2</sub> (cm)	B <sub>2</sub> (cm)
0	3.473	23	13	15	20	5.5	5	4.5	5
1	10.171	33	18	27	36	8	7	6.5	7
5	19.654	45	20	46	57.5	11	7.5	6.5	2.5
10	31.902	54	22	64	80	14	8	6.5	8
15	54.041	65	23	78	97.5	18	9	7	9
20	70.037	70	24	80	100	20	10	7	10

Fig.11- Masses and dimensions of phantoms

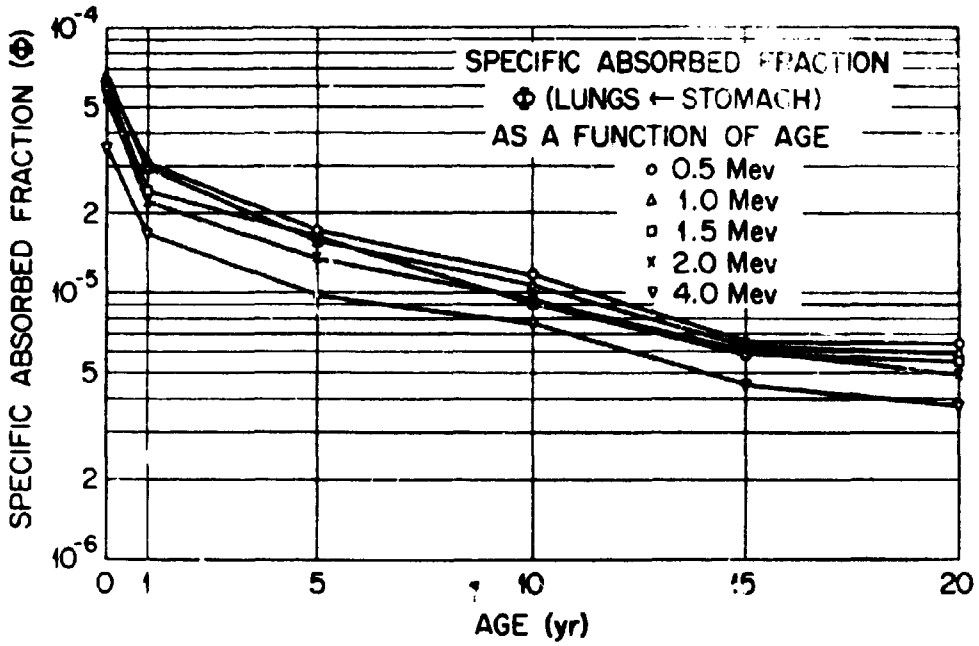


Fig. 12 - Age Variation of the Specific Absorbed Fraction for Photons.

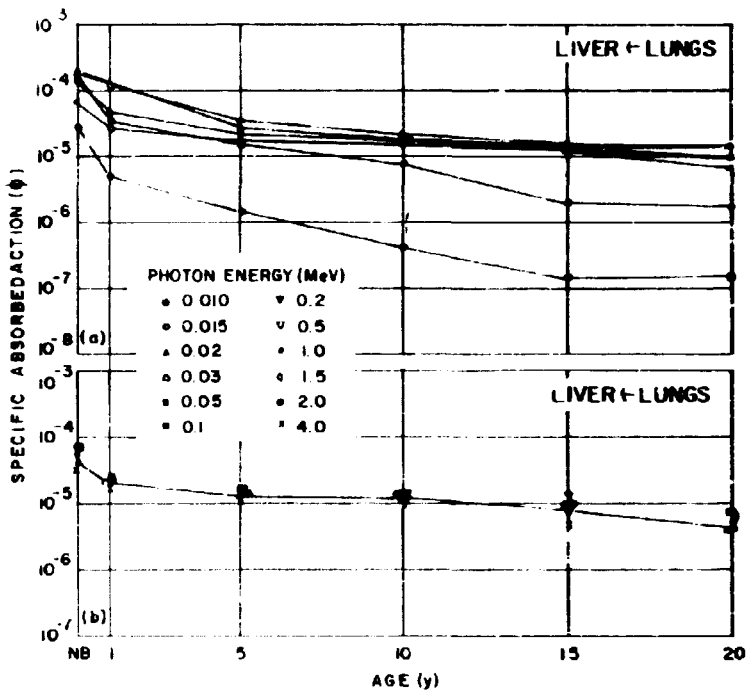


Fig.13-Specific Absorbed Fraction as a Function of Age and Photon Energy

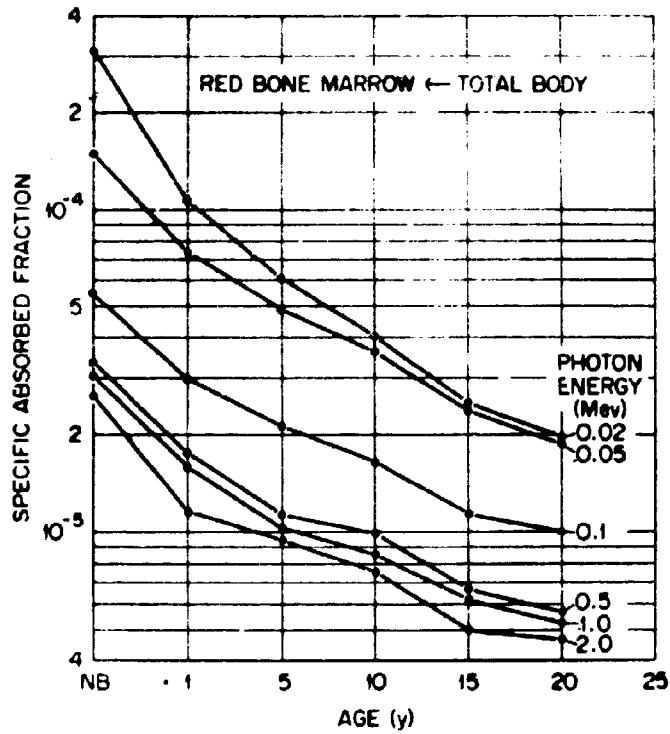


Fig 14 Specific Absorbed Fraction of Red Bone Marrow as a Function of Age and Photon Energy

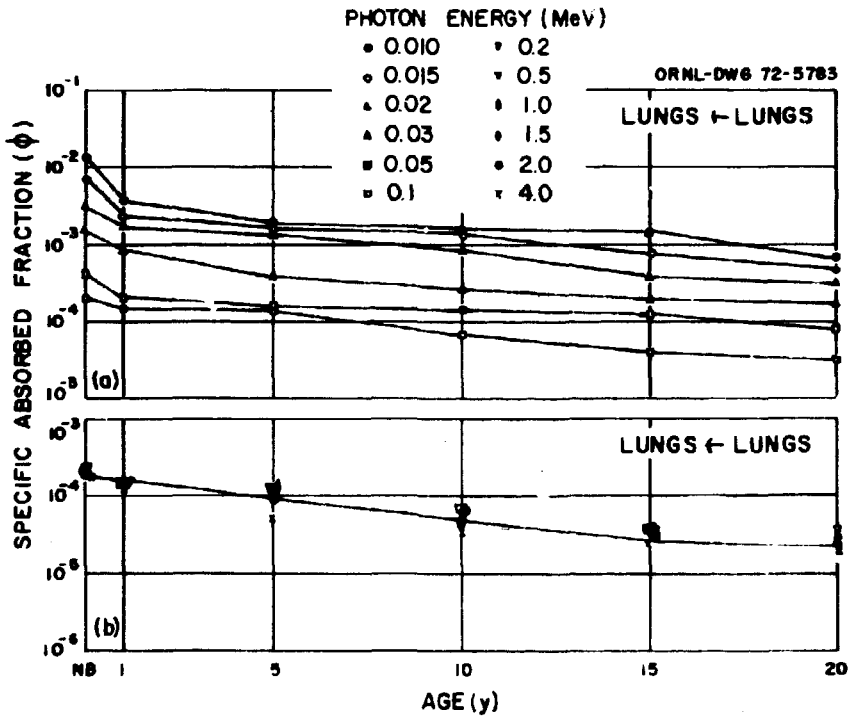


Fig. 15 - Specific Absorbed Fraction of Lungs as Function of Age and Photon Energy



phantom and of the Monte Carlo technique. A physical model of the phantom has been constructed at Oak Ridge National Laboratory and dosimetric studies to test the validity of the calculations are underway. Measurements of dose also have their sources of error (energy dependence, perturbation of the radiation field, etc.) but the preliminary estimates agree about as well as can be expected, taking both the statistical errors of the estimates and the errors of the measurements into account. Doubtless the method of calculation and the refinement of the phantom will continue and in the future there may be many more realistic studies of dose within the body. Already a number of improvements are underway at the Instituto de Energia Atômica at São Paulo. Goro Hiromoto is designing an improved model of the kidneys to include separate areas for the cortex and medulla which frequently have rather different retention of radionuclides. Roosevelt Rosa is making a better dosimetric model for bone and for bone + marrow in the phantom and this should improve the estimation of dose in these tissues particularly at energies below 100 keV. Suely Machado is attempting to design a phantom which will be truly representative of a 5 year old child and Vera Segreti is designing a fetus in various stages of development in relation to the displacement of the Gastro-intestinal tract. Finally, Suderhaque F. de Deus is studying the dosimetry of the gastro-intestinal tract in all its aspects but especially with reference to the dose the mictosing cells in the crypts receive from beta radiation emitted in the contents of the tract. These studies are typical of the kind of question one may hope to study by this method of estimation and so discover what features of the exposure situation are truly important in influencing the dose received.

## RESUMO

Na estrutura da dose devido a uma fonte de fótons dentro de um corpo humano, foi usado um modelo aproximado do corpo e seus principais órgãos, levando em conta seus tamanhos, formas, composições e densidades e, o Método de Monte Carlo. Usando esta técnica tenta-se calcular as "histórias" representativas de fótons que se originam na fonte. Em cada ponto de interação, determina-se o tipo de interação, a perda de energia nos tecidos e a nova direção do fóton usando as conhecidas leis da física.

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