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NEUTRON TRANSPORT IN TWO DISSIMILAR MEDIA WITH ANISOTROPIC SCATTERING

A. R. Burkart*, Y. Ishiguro and C. E. Siewert*

ABSTRACT

The elementary solutions of the one speed neutron transport equation with linearly anisotropic scattering are used in conjunction with Chandrasekhar's invariance principles to solve in a concise manner the Milne problem for two adjoining half-spaces and the critical reactor problem for a reflected slab

I – INTRODUCTION

The elementary solutions of Case (1960) were used by Kuszell (1961) to study neutron diffusion for problems defined by the presence of two dissimilar media. That work, however, was limited to isotropic scattering and was left in a somewhat cumbersome final form Later work by Mendelson and Summerfield (1964) added to the general area of multi region problems; however, it is to the basic work of McCormick (1969) and McCormick and Doyas (1969) that we must look for the most significant contribution to two-media problems with the effects of anisotropic scattering included. Today in the field of neutron-transport theory many researchers consider the fundamental paper by Case (1960) to be the cornerstone of the theory of "exact" solutions Later Pahor and Zweifel (1969) in an elegant paper demonstrated how the work of Chandrasekhar (1950) and Case (1960) could be coupled and utilized at the same time to obtain in a profitable and concise manner certain results for a variety of single medium problems

In a recent note, Siewert and Burkart (1975) demonstrated how the principles of invariance, as developed by Chandrasekhar (1950), could be used effectively to analyze the critical reactor problem for a reflected slab with isotropic scattering. In this work, we wish to show explicitly the complications that arise when the same critical problem and the Milne problem for two adjoining half-spaces are solved for the case of linearly anisotropic scattering.

II – THE MILNE PROBLEM FOR TWO HALF-SPACES

We consider the one-speed neutron transport equation for region 1, $x > 0$, and region 2, $x < 0$, written in the familiar manner:

$$\mu \frac{\partial}{\partial x} \Psi_\alpha(x, \mu) + \Psi_\alpha(x, \mu) = \frac{1}{2} c_\alpha \int_{-1}^{1} \Psi_\alpha(x, \mu') (1 + b_\alpha \mu\mu') \, d\mu' \qquad (1)$$

Here $\Psi_\alpha(x, \mu)$ denotes the neutron angular density in region α, as a function of position (in optical units) x and the direction cosine of the propagating neutrons, μ. In addition, c_α denotes the mean number of secondary neutrons per collision in region α, and b_α is the coefficient of anisotropic scattering.

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For the considered Milne problem, we seek a diverging (as $x \longrightarrow \infty$) solution of Equation (1) such that

(i)
$$\lim_{x\to\infty} \Psi_1(x,\mu)e^{x/\nu_0} < \infty, \tag{2a}$$

(ii)
$$\lim_{x\to-\infty} \Psi_2(x,\mu) = 0 \tag{2b}$$

and

(iii)
$$\Psi_1(0,\mu) = \Psi_2(0,\mu), \qquad \mu\in(-1,1). \tag{2c}$$

Here ν_0 denotes the discrete eigenvalue in region 1 — we use, with only slight modification, the notation of Case and Zweifel (1967) so that many of the basic quantities need not be redefined here.

Relying on the basic work of McCormick and Kuščer (1965), we can immediately write solutions to Equation (1) that satisfy the boundary conditions listed as Equations (2a) and (2b):

$$\Psi_1(x,\mu) = A(\nu_0)\phi_1(\nu_0,\mu)e^{-x/\nu_0} + \phi_1(-\nu_0,\mu)e^{x/\nu_0} + \int_0^1 A(\nu)\phi_1(\nu,\mu)e^{-x/\nu}d\nu \tag{3a}$$

and

$$\Psi_2(x,\mu) = B(-\eta_0)\phi_2(-\eta_0,\mu)e^{x/\eta_0} + \int_0^1 B(-\eta)\phi_2(-\eta,\mu)e^{x/\eta}d\eta. \tag{3b}$$

Here

$$\phi_\alpha(\xi_\alpha,\mu) = \frac{c_\alpha\xi_\alpha}{2} R_\alpha(\xi_\alpha,\mu)\frac{1}{\xi_\alpha-\mu}, \tag{4}$$

where $R_\alpha(x,y) = 1 + \ell_\alpha xy$, $\ell_\alpha = b_\alpha(1-c_\alpha)$, $\xi_1 = \nu_0$, $\xi_2 = \eta_0$, and $\xi_\alpha\notin[-1,1]$ is the positive zero of

$$\Lambda_\alpha(z) = 1 - c_\alpha zR_\alpha(z,z)\tanh^{-1}\frac{1}{z} + c_\alpha\ell_\alpha z^2. \tag{5}$$

Also

$$\phi_\alpha(\xi,\mu) = \frac{c_\alpha\xi}{2} R_\alpha(\xi,\mu)\frac{P}{\xi-\mu} + \lambda_\alpha(\xi)\delta(\xi-\mu), \qquad \xi\in(-1,1), \tag{6}$$

with

$$\lambda_\alpha(\xi) = 1 - c_\alpha\xi R_\alpha(\xi,\xi)\tanh^{-1}\xi + c_\alpha\ell_\alpha\xi^2. \tag{7}$$

Since the solutions given by Equations (3) inherently satisfy Equations (2a) and (2b), we need simply to constrain them to obey Equation (2c), which we choose to write as

$$\Psi_1(0,\mu) = \Psi_2(0,\mu) \text{ and } \Psi_1(0,-\mu) = \Psi_2(0,-\mu), \qquad \mu\in(0,1). \tag{8}$$

At this point we can use the S function of Chandrasekhar (1950) to write

$$\Psi_2(0,\mu) = \frac{1}{2\mu}\int_0^1 S_2(\mu',\mu)\Psi_2(0,-\mu')d\mu', \qquad \mu\in(0,1), \tag{9}$$

where

$$S_2(\mu',\mu) = \frac{c_2\,\mu\mu'}{\mu+\mu'}\,[1 - \hat{c}_2(\mu+\mu') - \hat{c}_2\mu\mu']H_2(\mu')H_2(\mu) \tag{10}$$

Here $H_2(\mu)$ is Chandrasekhar's H function for region 2 and, in general,

$$\hat{c}_\alpha = \frac{c_\alpha \ell_\alpha a_{\alpha,1}}{2 - c_\alpha a_{\alpha,0}},\quad \hat{q}_\alpha = \frac{2(1-c_\alpha)}{2 - c_\alpha a_{\alpha,0}},\quad \text{and}\quad a_{\alpha,\beta} = \int_0^1 H_\alpha(\mu)\mu^\beta d\mu \tag{11}$$

If we now enter Equation (8) into Equation (9), we can obtain

$$\Psi_1(0,\mu) = \frac{1}{2\mu}\int_0^1 S_2(\mu',\mu)\Psi_1(0,-\mu')d\mu',\qquad \mu\in(0,1) \tag{12}$$

We consider that Equation (12) is the basic equation now to be satisfied, since if $A(\nu_0)$ and $A(\nu)$ are established by Equation (12), then $B(-\eta_0)$ and $B(-\eta)$ can be obtained immediately from Equation (8) by using *the half-range orthogonality relations* of McCormick and Kuščer (1965).

On substituting Equation (3a) into Equation (12), we find that we can evaluate the integral over μ' to obtain

$$\frac{A(\nu_0)}{H_2(\nu_0)}[\phi_1(\nu_0,\mu) - W(\nu_0)] + \int_0^1 \frac{A(\nu)}{H_2(\nu)}[\phi_1(\nu,\mu) - W(\nu)]\,d\nu$$

$$= -\frac{1}{H_2(-\nu_0)}[\phi_1(-\nu_0,\mu) - W(-\nu_0)],\quad \mu\in(0,1), \tag{13}$$

where

$$W(\xi) = \frac{1}{2}\,\frac{c_1\xi}{R_2(\xi,\xi)}\,[\xi(\ell_1 - \ell_2)(\hat{q}_2H_2(\xi) - 1) - \hat{c}_2R_1(\xi,\xi)]. \tag{14}$$

Equation (13) clearly is a singular integral equation that can be regularized by using the half-range orthogonality relations for one medium (McCormick and Kuščer, 1965) Thus, if we multiply Equation (13) by $[\phi_1(\nu_0,\mu) + \frac{c_1\nu_0}{2}\hat{c}_1]\mu H_1(\mu)$ and integrate over μ, we find

$$\frac{A(\nu_0)}{H_2(\nu_0)}[N_1(\nu_0)H_1(\nu_0) - \nu_0\hat{q}_1W(\nu_0)] - \nu_0\hat{q}_1\bar{A} = -\frac{1}{H_2(-\nu_0)}[J(-\nu_0,\nu_0) - \nu_0\hat{q}_1W(-\nu_0)], \tag{15}$$

where

$$\bar{A} = \int_0^1 \frac{A(\nu)}{H_2(\nu)}W(\nu)d\nu, \tag{16}$$

$$N_1(\nu_0) = \frac{c_1\nu_0^2}{2}\,R_1(\nu_0,\nu_0)\Big[\frac{c_1R_1(\nu_0,\nu_0)}{\nu_0(\nu_0^2-1)} - \frac{(1-c_1)R_1(3\nu_0,\nu_0)}{\nu_0R_1(\nu_0,\nu_0)}\Big], \tag{17}$$

and

$$J(-\nu_0,\xi) = \frac{c_1\nu_0\xi}{2(\nu_0+\xi)H_1(\nu_0)}[1 - \ell_1\nu_0\xi + \hat{c}_1(\nu_0+\xi)]. \tag{18}$$

In a similar manner, we can multiply Equation (13) by $[\phi_1(\nu', \mu) + \frac{c_1 \nu'}{2} \hat{c}_1]\mu H_1(\mu)$, $\nu' \in (0,1)$, and integrate to obtain

$$-\frac{A(\nu_o)}{H_2(\nu_o)} \nu' \hat{q}_1 W(\nu_o) + \frac{A(\nu')}{H_2(\nu')} N_1(\nu') H_1(\nu') - \nu' \hat{q}_1 \bar{A} = -\frac{1}{H_2(-\nu_o)} [J(-\nu_o, \nu') - \nu' \hat{q}_1 W(-\nu_o)] \quad (19)$$

Here

$$N_1(\nu) = \nu [\{\lambda_1(\nu)\}^2 + \{\frac{c_1 \nu \pi}{2} R_1(\nu, \nu)\}^2] \quad (20)$$

If now we rearrange Equation (19) and multiply by $W(\nu)$ we can integrate to find

$$\bar{A} = -\frac{c_1 K_1 R_1(\nu_o, \nu_o)}{4 H_1(\nu_o) H_2(-\nu_o)} + \frac{A(\nu_o) H_1(\nu_o) N_1(\nu_o) K_2}{\nu_o H_2(\nu_o)}, \quad (21)$$

where the two constants K_1 and K_2 are given by

$$K_1 = \int_0^1 \frac{\nu W(\nu)(\nu_o - \nu)}{N_1(\nu) H_1(\nu)(\nu_o + \nu)} d\nu \quad \text{and} \quad K_2 = \int_0^1 \frac{\nu W(\nu)}{N_1(\nu) H_1(\nu)} d\nu \quad (22)$$

Equation (21) can be used in Equation (15) to find $A(\nu_o)$, and subsequently $A(\nu)$ can be found from Equation (19)

$$A(\nu_o) = \frac{H_2(\nu_o)}{H_2(-\nu_o)} \frac{[c_1 \nu_o \{1 - \ell_1 \nu_o + 2\nu_o c_1 + \hat{q}_1 K_1 R_1(\nu_o, \nu_o)\} - 4\nu_o H_1(\nu_o) \hat{q}_1 W(-\nu_o)]}{[4 N_1(\nu_o) H_1(\nu_o) H_1(\nu_o)(1 - \hat{q}_1 K_2) - 4\nu_o H_1(\nu_o) \hat{q}_1 W(\nu_o)]} \quad (23)$$

and

$$A(\nu) = \frac{\nu H_2(\nu)}{N_1(\nu) H_1(\nu)} \left[\frac{A(\nu_o) H_1(\nu_o) N_1(\nu_o)}{\nu_o H_2(\nu_o)} - \frac{c_1(\nu_o - \nu) R_1(\nu_o, \nu_o)}{4(\nu_o + \nu) H_2(-\nu_o) H_1(\nu_o)} \right] \quad (24)$$

Equations (23) and (24) are explicit expressions for the expansion coefficients $A(\nu_o)$ and $A(\nu)$. Though our final results were obtained differently and are different in appearance from those of McCormick (1969) and McCormick and Doyas (1969), they are similar in that there appear extra terms, in our case, W terms, in regard to either the isotropic scattering case, $\ell_1 = \ell_2 = 0$, or the single medium result, $c_2 = 0$

We note that the neutron density in region 1 is given by

$$\rho_1(x) = \int_{-1}^{1} \Psi_1(x, \mu) d\mu = A(\nu_o) e^{x/\nu_o} + e^{x/\nu_o} + \int_0^1 A(\nu) e^{x/\nu} d\nu. \quad (25)$$

Also, an asymptotic solution can be written as

$$\rho_{1a}(x) = A(\nu_o) e^{x/\nu_o} + e^{x/\nu_o} \quad (26)$$

Thus if we write

$$z_o = -\frac{\nu_o}{2}\ \ell n[-A(\nu_o)]. \tag{27}$$

then Equation (26) becomes

$$\rho_{1_a}(x) = e^{x/\nu_o} - e^{(x+2z_o)/\nu_o} \tag{28}$$

It is therefore clear that z_o is the extrapolated endpoint for the considered Milne problem.

III – THE CRITICAL PROBLEM FOR A REFLECTED SLAB

We consider the transport equations for the core, $-a \leqslant x \leqslant a$, and the reflector, $|x| > a$, written as Equation (1), where $\alpha = 1$ implies the core and $\alpha = 2$ implies the reflector. Clearly we take $c_1 > 1$ and $c_2 < 1$. We thus seek solutions of Equation (1) such that $\Psi_\alpha(-x, -\mu) = \Psi_\alpha(x, \mu)$, $\Psi_1(a, \mu) = \Psi_2(a, \mu)$, $\mu \in (-1,1)$, and $\Psi_2(\infty, \mu) = 0$. We consider that c_1 and c_2 are given and thus seek the critical half-thickness a.

For the core, we can write the desired solution as

$$\Psi_1(x,\mu) = \mathrm{\AA}(\nu_o)\,[\phi_1(\nu_o,\mu)e^{-x/\nu_o} + \phi_1(-\nu_o,\mu)e^{x/\nu_o}] + \int_0^1 \mathrm{\AA}(\nu)\,[\phi_1(\nu,\mu)e^{-x/\nu} + \phi_1(-\nu,\mu)e^{x/\nu}]\,d\nu, \tag{29}$$

which clearly satisfies the symmetry condition. For the reflector, we need only consider $x > a$, and thus we write

$$\Psi_2(x,\mu) = \hat{B}(\eta_o)\phi_2(\eta_o,\mu)e^{-x/\eta_o} + \int_0^1 \hat{B}(\eta)\phi_2(\eta,\mu)e^{-x/\eta}d\eta, \quad x > a. \tag{30}$$

As in a previous work for isotropic scattering (Siewert and Burkart, 1975), we would now like to use the continuity condition at $x = a$ in the reflection equation

$$\Psi_2(a,-\mu) = \frac{1}{2\mu}\int_0^1 S_2(\mu',\mu)\Psi_2(a,\mu')d\mu', \quad \mu \in (0,1), \tag{31}$$

to obtain, what we consider to be, our basic boundary condition:

$$\Psi_1(a,-\mu) = \frac{1}{2\mu}\int_0^1 S_2(\mu',\mu)\Psi_1(a,\mu')d\mu', \quad \mu \in (0,1). \tag{32}$$

If we now substitute Equation (29) into Equation (32), we can evaluate the integral over μ' to obtain

$$\frac{\mathrm{\AA}(\nu_o)}{H_2(\nu_o)}\, e^{a/\nu_o}[\phi_1(\nu_o,\mu) - W(\nu_o)] + \int_0^1 \frac{\mathrm{\AA}(\nu)}{H_2(\nu)}\, e^{a/\nu}[\phi_1(\nu,\mu) - W(\nu)]d\nu$$

$$= -\frac{\mathrm{\AA}(\nu_o)}{H_2(-\nu_o)}\, e^{a/\nu_o}[\phi_1(-\nu_o,\mu) - W(-\nu_o)] - \int_0^1 \mathrm{\AA}(\nu)e^{a/\nu}[H_2(\nu)K(\nu)\phi_1(-\nu,\mu) - G(\nu)]d\nu, \quad \mu \in (0,1); \tag{33}$$

where

$$K(\nu) = \frac{c_2(c_1 - 1)R_2(\nu,\nu) - c_1(c_2 - 1)R_1(\nu,\nu)}{c_1 R_1(\nu,\nu)} \tag{34}$$

and

$$G(\nu) = \frac{c_1\nu}{2R_2(\nu,\nu)}[H_2(\nu)K(\nu)\{\hat{c}_2 R_1(\nu,\nu) - \nu(\ell_1 - \ell_2)\} + \nu(\ell_1 - \ell_2)\hat{q}_2] \tag{35}$$

Equation (33) is a singular integral equation that can be regularized by using the half-range orthogonality relations of McCormick and Kuščer (1965). Thus we multiply Equation (33) by $\mu H_1(\mu)\,[\phi_1(\nu_0,\mu) + \frac{c_1\nu_0}{2}\hat{c}_1]$ and integrate to obtain

$$\frac{e^{2a/\nu_0}}{H_2(\nu_0)}[N_1(\nu_0)H_1(\nu_0) - \nu_0\hat{q}_1 W(\nu_0)] - \nu_0\hat{q}_1\bar{D} = -\frac{1}{H_2(-\nu_0)}[J(-\nu_0,\nu_0) - \nu_0\hat{q}_1 W(-\nu_0)]$$

$$-\int_0^1 D(\nu)e^{-2a/\nu}[H_2(\nu)K(\nu)J(-\nu,\nu_0) - \nu_0\hat{q}_1 G(\nu)]\,d\nu, \tag{36}$$

where we have introduced

$$D(\nu) = \frac{\hat{A}(\nu)e^{a/\nu}e^{a/\nu_0}}{\hat{A}(\nu_0)} \quad \text{and} \quad \bar{D} = \int_0^1 \frac{D(\nu)}{H_2(\nu)}W(\nu)d\nu. \tag{37}$$

In a similar manner, we can multiply Equation (33) by $\mu H_1(\mu)\,[\phi_1(\nu',\mu) + \frac{c_1\nu'}{2}\hat{c}_1]$, $\nu' \in (0,1)$, and integrate to get

$$-\frac{e^{2a/\nu_0}}{H_2(\nu_0)}\nu'\hat{q}_1 W(\nu_0) + \frac{D(\nu')}{H_2(\nu')}N_1(\nu')H_1(\nu') - \nu'\hat{q}_1\bar{D} = -\frac{1}{H_2(-\nu_0)}[J(-\nu_0,\nu') - \nu'\hat{q}_1 W(-\nu_0)]$$

$$-\int_0^1 D(\nu)e^{2a/\nu}[H_2(\nu)K(\nu)J(-\nu,\nu') - \nu'\hat{q}_1 G(\nu)]\,d\nu, \quad \nu' \in (0,1), \tag{38}$$

where

$$J(-\nu,\xi) = \frac{c_1\nu\xi}{2(\nu+\xi)H_1(\nu)}[1 - \ell_1\nu\xi + \hat{c}_1(\nu+\xi)], \quad \xi = \nu_0 \quad \text{or} \quad \nu' \in (0,1). \tag{39}$$

Equations (36) and (38) are the two regular integral equations that we must solve simultaneously to obtain the critical half-thickness a. Though it is perhaps unreasonable to expect to be able to find analytical solutions to Equations (36) and (38), to construct a numerical solution certainly poses no problem.

Before listing our final results, obtained by solving Equations (36) and (38) iteratively, we note that there are two immediately available approximations we can introduce. The simplest is to set $D(\nu) = 0$ and ignore completely Equation (38). This approximation leads to the critical condition, from Equation (36),

$$a_{0B} = \frac{1}{2}\pi|\nu_0| - z_{0B}, \tag{40}$$

where

$$z_{oB} = \frac{\nu_o}{2} \text{ Log } \{ \frac{H_2(-\nu_o)}{H_2(\nu_o)} [4N_1(\nu_o)H_1(\nu_o)H_1(\nu_o) - 4\nu_o H_1(\nu_o)\hat{q}_1 W(\nu_o)]$$

$$\div [c_1\nu_o \{ 1 - \ell_1\nu_o^2 + 2\nu_o\hat{c}_1 \} - 4\nu_o H_1(\nu_o)\hat{q}_1 W(-\nu_o)] \} \tag{41}$$

On the other hand, we might approximate Equations (36) and (38) in such a way that we can utilize the results for the Milne problem developed in Section II We observe that if we neglect the integral terms on the right-hand sides of Equations (36) and (38), then the two approximated equations will have the solutions

$$D_M(\nu) = A(\nu)$$

and

$$e^{2a_{oM}/\nu_o} = A(\nu_o), \tag{42}$$

where $A(\nu_o)$ and $A(\nu)$ are the Milne solutions given by Equations (23) and (24). From Equation (42), we get the critical condition

$$a_{oM} = \frac{1}{2}\pi|\nu_o| - z_{oM}, \tag{43}$$

where

$$z_{oM} = \frac{\nu_o}{2} \text{ Log } \{ \frac{H_2(-\nu_o)}{H_2(\nu_o)} [4N_1(\nu_o)H_1(\nu_o)H_1(\nu_o)(1 - \hat{q}_1 K_2) - 4\nu_o H_1(\nu_o)\hat{q}_1 W(\nu_o)]$$

$$\div [c_1\nu_o \{ 1 - \ell_1\nu_o^2 + 2\nu_o\hat{c}_1 + \hat{q}_1 K_1 R_1(\nu_o,\nu_o) \} - 4\nu_o H_1(\nu_o)\hat{q}_1 W(-\nu_o)] \}. \tag{44}$$

For the case of isotropic scattering, $\ell_1 = \ell_2 = 0$, or for the case of a bare reactor, $c_2 = 0$, the two approximations given by Equations (40) and (43) are identical, but in general they are different. In Table I, we list some typical values of z_{oB} and z_{oM}, and in Table II, we list a_{oB}, a_{oM} and our "exact" results obtained by solving Equations (36) and (38) iteratively. We note that our "exact" results are identical to those reported by Carol and Aronson (1973). Also z_{oM} and a_{oM} agree with the asymptotic results of Doyas and McCormick (1968). Numerical results in addition to those given here can be found in the thesis of Burkart (1975).

Table I

Extrapolated Endpoints

c_1	b_1	c_2	b_2	z_{oB}	z_{oM}
1.01	0.0	0.9	0.0	1.818175	1.818175
1.01	1.0	0.9	0.0	2.697328	2.697147
1.01	0.0	0.9	1.0	1.533918	1.537910
1.01	1.0	0.9	1.0	2.285898	2.291668
1.06	0.0	0.9	0.0	1.564092	1.564092
1.06	0.0	0.9	1.0	1.366538	1.370686
1.20	0.0	0.4	0.0	0.6992548	0.6992548
1.20	1.0	0.4	0.0	1.091650	1.090817
1.20	0.0	0.4	1.0	0.660828	0.6631535
1.20	1.0	0.4	1.0	1.037642	1.040783
1.50	0.0	0.9	0.0	0.7914220	0.7914220
1.50	1.0	0.9	0.0	1.190381	1.182627
1.50	0.0	0.9	1.0	0.7479395	0.7516228
1.50	1.0	0.9	1.0	1.142157	1.140871
1.60	0.0	0.4	0.0	0.5097969	0.5097969
1.60	1.0	0.4	0.0	0.8605117	0.8578493
1.60	0.0	0.4	1.0	0.4900796	0.4920241
1.60	1.0	0.4	1.0	0.8343955	0.8360061

Table II

Critical Sizes

c_1	b_1	c_2	b_2	a_{oB}	a_{oM}	a
1.01	0.0	0.9	0.0	7.214751	7.214751	7.214751
1.01	1.0	0.9	0.0	8.393351	8.393532	8.393532
1.01	0.0	0.9	1.0	7.499008	7.495017	7.495017
1.01	1.0	0.9	1.0	8.804780	8.799010	8.799010
1.06	0.0	0.9	0.0	2.052395	2.052395	2.052360
1.06	0.0	0.9	1.0	2.249949	2.245801	2.245784
1.20	0.0	0.4	0.0	1.182975	1.182975	1.182419
1.20	1.0	0.4	0.0	1.328226	1.329059	1.328237
1.20	0.0	0.4	1.0	1.221402	1.219077	1.218626
1.20	1.0	0.4	1.0	1.382234	1.379093	1.378425
1.50	0.0	0.9	0.0	0.2910616	0.2910616	0.2825876
1.50	1.0	0.9	0.0	0.3002338	0.3079674	0.2889746
1.50	0.0	0.9	1.0	0.3345442	0.3308609	0.3248086
1.50	1.0	0.9	1.0	0.3484569	0.3497437	0.3347018
1.60	0.0	0.4	0.0	0.4509333	0.4509333	0.4468885
1.60	1.0	0.4	0.0	0.4922023	0.4948646	0.4817374
1.60	0.0	0.4	1.0	0.4706506	0.4687061	0.4653684
1.60	1.0	0.4	1.0	0.5183184	0.5167079	0.5051470

RESUMÉ

Les solutions elementaires de l equation de transport de neutrons monoenergetiques avec deviation esotropique lineaire sont utilisees conjointement aux principes d invariance de Chaudrasekhar afin de resoudre de maniere concise le probleme de Milne pour deux especes adjoints et le probleme du reacteur critique type parois reflechissante

RESUMO

As soluções elementares da equação de transporte de neutrons monoenergeticos com espalhamento linearmente anisotropico são usadas em conjunto com os principios de invariância de Chandrasekhar para resolver de maneira concisa o problema de Milne para dois semi espaços adjuntos e o problema do reator critico tipo parede refletida

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