A Proposal for a Subcritical Reactivity Meter Based on Gandini and Salvatores' Point Kinetics Equations for Multiplying Subcritical Systems

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Abstract–**Multiplying Subcritical Systems were for a long time poorly studied and its theoretical description remains with plenty open questions. Great interest on such systems arose partly due to the improvement of hybrid concepts, such as the Accelerator-Driven Systems (ADS). Along with the need for new technologies to be developed, further study and understanding of subcritical systems are essential also in more practical situations, such as in the case of a PWR criticalization in their physical startup tests. Point kinetics equations are fundamental to continuously monitor the reactivity behavior to a possible variation of external sources intensity. In this case, quickly and accurately predicting power transients and reactivity becomes crucial. It is known that conventional Reactivity Meters cannot operate in subcritical levels nor describe the dynamics of multiplying systems in these conditions, by the very structure of the classical kinetic equations. Several theoretical models have been proposed to characterize the kinetics of such systems with special regard to the reactivity, as the one developed by Gandini and Salvatores among others. This work presents a discussion about the derivation of point kinetics equations for subcritical systems and the importance of considering the external source. From the point of view of the Gandini and Salvatores' point kinetics model and based on the experimental results provided by Lee and dos Santos, it was possible to develop an innovative approach. This article proposes an algorithm that describes the subcritical reactivity with external source, contributing to the advancement of studies in the field.**

I. INTRODUCTION

HYBRID reactors concepts, such the Accelerator- Driven Systems (ADS), are partly responsible for the increasing interest in studies about multiplying subcritical systems [1]. Those hybrid reactors would offer a range of important advantages over conventional ones, including greater security (and therefore greater public acceptance of nuclear energy), minor problems with respect to proliferation, significantly reduction of nuclear waste production and the possibility of transmutation of the existent nuclear waste [2, 3]. The ADS physics and its subcritical core have been well studied, however, many concepts are new and their understanding requires experimental validation.

Subcritical cores need to be further studied and understood also in practical situations, as in case of PWR criticalization. At the start up procedure, the agreement between performed calculations and physical tests, considering a predetermined tolerance, ensure the safety of the operation until the reactor is turned off for a posterior exchange of fuel. However, during the criticalization process, the reactivity is not determined directly. Since there are no methods to perform it, the reactivity measurements are done only on a relative basis [4], which might be considered a crucial problem in nuclear reactor physics.

A. The Subcritical Reactivity Meter

A reactivity meter is a device capable of measuring changes in neutron flux and to transform this signal into reactivity. It may be utilized by nuclear power plants operators in subcritical approaches and when changing temperature, moving control rods, or even when adding boron in moderator. This device is also commonly used in nuclear reactors at the initialization process, after refueling or during any interruption. In research facilities, a reactivity meter may be used in much more occasions [5].

For safety, properly monitor the subcriticality level is extremely important, often in order to maintaining it always below a certain threshold to avoid accidents. In 1999, in Tokai - Mura, Japan, a criticality accident took place after a dissolution operation of enriched uranium powder in nitric acid [6]. This event demonstrated that a reactivity monitoring device operating in subcritical regime should be essential in nuclear facilities. Furthermore, a recent growing demand is found both for the period reduction of tests and initialization processes to be performed. The tendency, for economic reasons, is that these extents of time are kept to a minimum while maintaining accuracy in the required measures and obtaining vital information from neutron characteristics with increasing detail and efficiency [7, 8].

Unfortunately, a device that provides the accurate reactivity of the system in the subcritical regime (especially in deep subcritical states) was not yet developed, by the very structure of the kinetic equations in which conventional reactivity meters are based [9].

B. The External Source

In subcritical domain, or even with the critical reactor operating at a very low power, the total number of neutrons in the core is very low and becomes comparable to the number of neutrons generated by the external source. In this case, the source dominates the behavior of the multiplying system and

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We would like to express our gratitude to IPEN/CNEN-SP, CAPES, ELETRONUCLEAR and FAPESP, institutions that make our research possible.

978-1-4799-9918-7/15/$31.00 ©2015 IEEE

influences the neutron counting and the determination of the reactivity [10].

When working in the subcritical domain, the core conditions may be considered similar to the equilibrium ones (the number of the neutrons in the core and the precursor's concentration constant in time). If one ignores the source term, the tendency of the reactivity meter is, then, to show a crescent result tending to zero, thus losing any connection to the real system. Needless to say, this behavior may lead to complications in a nuclear facility. Therefore, the source term must not be ignored in kinetic equations when in subcritical regime [11, 12].

C. *Point Kinetics Equations and the Importance Function*

Kinetic equations are closely related to importance functions. Based on transport equation and considering delayed neutrons, the point kinetics equations may be obtained utilizing the importance function as weight [13].

The concept of importance was first defined in the radiation transport field and then considered as the contribution of a neutron, through its progeny, to a response in a critical system. The concept was extended, with the interpretation of the adjoint function adopted in variational techniques [14].

Considering the classical point kinetics equations, the adjoint flux φ^* is identified as the weight function, or importance function. However, this is only true in critical regime. In this context, the determination of a set of point kinetics equations for subcritical systems with external source is especially challenging, since finding a suitable importance function for the description of the system is both complex and fundamental.

Several methods have been proposed to describe multiplying systems in the subcritical domain [15]. It is important to highlight that although this work relies on a particular theory, the other approaches have the same structure, differing only by the parameters utilized. Under this point of view, the choice of the model may be done conveniently.

The particular Gandini and Salvatores' model was chosen due to the fact that it considers the main characteristics of the subcritical multiplying system, as fission and external source. A highly important detail is that the importance function, in this case, is uniquely defined with respect to a given system response or a measurable amount, as counting rates of detectors [16, 17].

II. METHODS

A. *Gandini and Salvatores' point kinetics equations*

An interesting definition of subcriticality was proposed by Gandini and Salvatores, where an importance function n_0^* associated with the relative power level of the subcritical system is introduced. The model takes into account the presence of external source and fission source and the concept of generalized reactivity.

According to Gandini and Salvatores, the equations for the relative power between two subcritical states and the delayed neutron precursor's concentration are given by:

$$l_{eff}\frac{dP_N(t)}{dt} = \left(\rho_{gen} - \alpha\beta\right)P_N + \alpha\sum_{i=1}^{I}\lambda_i\xi_i + \zeta\left(1 - P_N\right) + \rho_{source} \qquad (1)$$

$$\frac{d\xi_j(t)}{dt} = \beta_j P_N(t) - \lambda_j\xi_j(t) \qquad (2)$$

Where:

l_{eff} represents the effective prompt neutron lifetime;

$P_{N(t)}$ represents the normalized power, or the relative power between two subcritical states of the reactor;

An important point to highlight is that "ρ_{gen}" is not the total reactivity, but the reactivity obtained after a perturbation, or even the generalized reactivity between two subcritical points;

α represents the relation between the prompt neutron spectrum and the delayed neutron spectrum;

β represents the effective delayed neutron fraction;

β_j represents the j'th delayed neutron fraction;

λ_j represents the j'th delayed neutron precursors decay constant;

ξ_j represents the j'th delayed neutron precursors concentration;

ζ represents the subcriticality index;

"ρ_{source}" represents the reactivity due to source variation. However, this work does not consider this term. The study was focused on fission reactors operating in subcritical regime driven by a static source, thus with no change in the source strength. In case of working with ADS system, for example, this term should be considered.

From equation (1) and (2), it is possible to notice that two unknowns are present: the relative power (P_N) and the subcriticality index (ζ). The first one may be obtained from the ratio of detector signals between two successive acquisitions. The obtainment of the subcriticality index (ζ) will be discussed below.

B. *The Obtainment of the Subcriticality Index ζ*

The subcriticality index (ζ) may be obtained through a recent experimental study conducted by Lee and dos Santos [18]. Their conclusion was that the index (ζ) corresponds to the reactivity obtained by classical point kinetics equations with external source.

Lee and dos Santos developed a method for determining with precision the reactivity levels of subcritical systems utilizing the Gandini and Salvatores' point kinetic equations. In their approach, the independence of the effective delayed neutron fraction and the prompt neutron generation time to the subcriticality level of the system were the only hypothesis made. The method described by the authors was based only on measured quantities such as counting rates of the employed detectors and the parameters arising from the least squares fitting of the Auto Power Spectral Density (APSD). An important aspect of their approach is that detector efficiencies, quantities required in other procedures, are unnecessary. Their technique permitted reactivity measurements in several subcritical configurations of the IPEN/MB-01 reactor, reaching -7000 pcm and achieving good final results.

The IPEN/MB-01 reactor has been considered international benchmark in several experiments of various configurations.

Thus, the IPEN/MB-01 may be considered of utmost importance to establish experimental parameters to validate kinetic models of subcritical systems [19-22].

As well known, the classical point kinetic equations with source are [23]:

$$\frac{dN(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} N(t) + \sum_{i=1}^{6} \lambda_i C_i(t) + S \tag{3}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} N(t) - \lambda_i C_i(t) \tag{4}$$

Where:

$N(t)$ is the neutron population at instant t;

$\rho(t)$ is the reactivity at instant t, which in a subcritical system correspond to ζ;

β is the effective delayed neutron fraction;

β_i is the i'th delayed neutron fraction;

λ_i is the i'th delayed neutron precursors decay constant;

$C_i(t)$ is the i'th precursors concentration at instant t;

Λ is the prompt neutron generation time;

S is the neutron source.

Therefore, considering Lee and dos Santos' experimental results, it is possible, from equations (3) and (4) to find an expression for the subcriticality index (ζ). Solving equation (4), substituting it into (3) and then solving for $\rho(t)$, the following is obtained:

$$\zeta = \rho(t) = \frac{\Lambda}{N(t)} \frac{dN(t)}{dt} + \beta - \frac{N_0}{N(t)} \sum_{i=1}^{I} \beta_i e^{-\lambda_i t} - \frac{1}{N(t)} \sum_{i=1}^{I} \lambda_i \beta_i e^{-\lambda_i t} \int_{t'}^{t} N(t'') e^{\lambda_i t''} dt'' - \frac{\Lambda}{N(t)} S \tag{5}$$

Similarities between equations (1) and (2) and the classic point kinetics equations (3) and (4) are only apparent. The relative power and generalized reactivity are distinct concepts that make a huge difference between models and should be used with careful.

III. A PROPOSAL FOR A SUBCRITICAL REACTIVITY METER BASED ON GANDINI AND SALVATORES' POINT KINETICS EQUATIONS FOR MULTIPLYING SUBCRITICAL SYSTEMS

In order to achieve an algorithm which can describe the subcritical reactivity meter, we may begin solving equation (2), substituting it into (1) and then solving for ρ_{gen}. The following equation is obtained:

$$\rho_{gen}(t) = \frac{\zeta}{P_N}(1 - P_N) + \alpha\beta - \frac{\ell_{eff}}{P_N} \frac{d}{dt} P_N(t) + \frac{\alpha}{P_N} \sum_{i=1}^{I} \lambda_i \xi_i(0) e^{-\lambda_i t} + \frac{\alpha}{P_N} \sum_{i=1}^{I} \lambda_i \beta_i e^{-\lambda_i t} \int_{t'}^{t} P_N(t'') e^{\lambda i t''} dt'' \tag{6}$$

A key point that should be clear is that equations (5) and (6) cannot be calculated simultaneously, but by steps. Thus, firstly the subcriticality index is calculated and then, after a perturbation in the system (as during the insertion of control rods), ρ_{gen} is computed. The interpretation of the generalized reactivity should also be highlighted; its value is cumulative and the final reactivity will be given by the sum of the previously calculated ρ_{gen}.

Considering a steady state reactor when t=0:

$$\xi_i(0) = \frac{\beta_i}{\lambda_i} P_N(0) \tag{7}$$

Substituting (7) into (6):

$$\rho_{gen}(t) = \frac{\zeta}{P_N}(1 - P_N) + \alpha\beta - \frac{\ell_{eff}}{P_N} \frac{d}{dt} P_N(t) + \alpha \frac{P_N(0)}{P_N} \sum_{i=1}^{I} \beta_i e^{-\lambda_i t} + \frac{\alpha}{P_N} \sum_{i=1}^{I} \lambda_i \beta_i e^{-\lambda_i t} \int_{t'}^{t} P_N(t'') e^{\lambda i t''} dt'' \tag{8}$$

The final algorithm for the reactivity meter may be obtained through equations (5) and (8), evaluating the derivatives and integrals in terms of small time intervals t_0, t_k, t_{k+1}, ..., t_{k+n}.

For equation (8) we may define:

$$A_k = \alpha\beta - \frac{\ell_{eff}}{P_{N_k}} \frac{d}{dt} P_N \Big|_{t_k} \tag{9}$$

$$B_k = \frac{\zeta}{P_{N_k}}(1 - P_{N_k}) \tag{10}$$

$$D_{i,k} = \lambda_i \xi_i(0) e^{-\lambda_i t_k} \tag{11}$$

$$F_{i,k} = \lambda_i \beta_i e^{-\lambda_i t_k} \int_{0}^{t_k} P_N(t'') e^{\lambda i t''} dt'' \tag{12}$$

The reactivity at the instant t_k is, then:

$$\rho_{gen_k} = A_k + B_k + \frac{\alpha}{P_{N_k}} \sum_{i=1}^{I} D_{i,k} + \frac{\alpha}{P_{N_k}} \sum_{i=1}^{I} F_{i,k} \tag{13}$$

And, at the immediately posterior instant, t_{k+1}, the reactivity is given by:

$$\rho_{gen_{k+1}} = A_{k+1} + B_{k+1} + \frac{\alpha}{P_{N_{k+1}}} \sum_{i=1}^{I} D_{i,k+1} + \frac{\alpha}{P_{N_{k+1}}} \sum_{i=1}^{I} F_{i,k+1} \tag{14}$$

Where:

$$A_{k+1} = \alpha\beta - \frac{\ell_{eff}}{P_{N_{k+1}}} \frac{d}{dt} P_N \Big|_{t_{k+1}} \tag{15}$$

$$B_{k+1} = \frac{\zeta}{P_{N_{k+1}}}(1 - P_{N_{k+1}}) \tag{16}$$

$$D_{i,k+1} = \lambda_i \xi_i(0) e^{-\lambda_i \Delta t} \tag{17}$$

$$F_{i,k+1} = F_{i,k} e^{-\lambda_i \Delta t} + \lambda_i \beta_i e^{-\lambda_i t_{k+1}} \int_0^{t_{k+1}} P_N(t'') e^{\lambda i t''} dt'' \tag{18}$$

In (17) and (18), $\Delta t=(t_{k+1}-t_k)$ represents the time interval utilized by the reactivity meter algorithm to perform the generalized reactivity calculus. It is important to notice that an algorithm for the obtainment of the subcriticality index (ζ) may be find analogously starting from (5).

Finally, equations (5), (14), (15), (16), (17) and (18) constitute a step algorithm, where the reactivity at the instant t_{k+1} is obtained utilizing the known parameters calculated at the immediately previous instant t_k.

It should be emphasized that this model is valid until a certain level of subcriticality. The behavior of deeply subcritical systems remains completely unknown.

IV. DISCUSSION

Many theories have been proposed to describe subcritical systems but most involve very complicated parameters to be applied in practical situations. Gandini and Salvatores' theory differs from others by being based on simple parameters, thus is the most convenient choice to be made for the development of a reactivity meter and its application.

The experimental work conducted by Lee and dos Santos showed that Gandini and Salvatores' theory is able to successfully describe subcritical systems and demonstrated the obtainment of the subcriticality index, predicted by the model. An important implication of their experimental work was the demonstration that the analysis could be based on simple measures, such as detector counting rates, achieving results of good quality.

Based on both works, the theory proposed by Gandini and Salvatores and the experimental results achieved by Lee and dos Santos, this paper presents an algorithm of a reactivity meter able to calculate the generalized and final reactivity for a subcritical system with external source. The developed algorithm is an important application of new concepts and may support further studies in subcritical systems.

V. CONCLUSIONS

Subcriticality is subject of great interest not only from a theoretical point of view, but also from a practical one, since there are still many important questions unanswered. An accurate subcritical reactivity meter should be fundamental in any nuclear facility for safety reasons. In this context, the algorithm proposed is innovative, principally due to the fact that it considers a subcritical multiplying system with external source.

From the point of view of the Gandini and Salvatores' point kinetics equations and based on the experimental results provided by Lee and dos Santos, it was possible to conceive an algorithm that aims to describe the subcritical system in a relatively simple manner, as all the analysis is based on counting rates of detectors.

Everything indicates that the proposed approach may yield great results, though more studies in this field are fundamental. In order to achieve a better understanding of the studied system kinetics, more computer simulations and also the production of more experimental data are crucial.

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