

# SENSITIVITY AND UNCERTAINTY EVALUATION APPLIED TO THE FAILURE PROCESS OF NUCLEAR FUEL

**Daniel S. Gomes<sup>1</sup>**

<sup>1</sup> Instituto de Pesquisas Energéticas e Nucleares (IPEN / CNEN - SP)  
Av. Professor Lineu Prestes 2242  
05508-000 São Paulo, SP  
dsgomes@ipen.br

## ABSTRACT

Nuclear power plants must operate with minimal risk. The nuclear power plants licensing process is based on a paired model, combining probabilistic and deterministic approaches to improve fuel rod performance during both steady state and transient events. In this study, performance fuel codes were used to simulate the test rod IFA-650-4, with a burnup of 92 GWd/MTU within a Halden reactor. In a loss-of-coolant test, the cladding failed within 336 s after reaching a temperature of 800 °C. Nuclear systems work with many imprecise values that must be quantified and propagated. These sources were separated by physical models or boundary conditions describing fuel thermal conductivity, fission gas release, and creep rates. These factors change output responses. Manufacturing tolerances show dimensional variations for fuel rods, and boundary conditions within the system are characterized using small ranges that can spread throughout the system. To identify the input parameters that produce output effects, we used Pearson coefficients between input and output. These input values represent uncertainties using a stochastic technique that can define the effect of input parameters on the establishment of realistic safety limits. Random sampling provided a set of runs for independent variables proposed by Wilks' formulation. The number of samples required to achieve the 95th percentile, with 95% confidence, depending on verifying the confidence interval to each output. The FRAPTRAN code utilized a module to reproduce the plastic response, defining the failure limit of the fuel rod.

## 1. INTRODUCTION

Sensitivity analysis has long been the key issue for improving nuclear reactor design and safety. Nuclear fuel systems have at least three uncertainty sources that must calculate results with variations. First imprecision source is the manufacturing tolerances regarding core components. Second cause arises from environment determining boundary conditions. The last source created by inaccuracy in the evaluation of physical models adopted. In recent decades, progress regarding uncertainty and sensitivity analysis (UASA) has been used for risk management. Uncertainty analyses follow the same stages as best-estimate models. First, they must identify key features of the input of the systems, which generate a global influence on output. Initially, this identification is based on the nuclear plant type and the licensing code. The computed uncertainty of inputs differs from the elements considered as output items, and a large amount of code is run to identify the inner propagation models (using the Monte Carlo simulation). The key features of the uncertainty source must be identified, such as manufacturing tolerances, initial conditions, and the aggregated effect spread through inner physical models.

In this study, the major goals were to measure the uncertainty for light water reactors (LWRs), do a fuel analysis during transients (such as during a loss-of-coolant accident or LOCA), and test reactivity initiated accidents (RIAs). We also describe a method for estimating more realistic safety bands based on those uncertainties. To do this, it was necessary to quantify the uncertainty contribution of each input, producing a sensitivity analysis for the effect reduction of the largest contributors. By examining the correlation indices, we can reduce the impact on changes in fuel designs.

Building a UASA consists of measuring and quantifying inaccuracies. This stage utilized a probability density function (PDF), where uncertainties are spread from inputs into the nuclear system. Gaussian functions represent the deviances of the input parameter, as derived from empirical experiments. The interpretation of results was based on a simulation using fuel code coupled as the optimization software. The last phase sought to discover dependencies between the input and output using the Pearson or Spearman correlation indices.

## **1.1 Reactor Safety Regulation and Probabilistic Methods**

The safety assessment process used by nuclear plants estimates the failure of structural elements. Safety thresholds were developed empirically, based on conservative analyses. These rules defined the criterion for the Emergency Cool Cooling System (ECCS) guidelines, which is the most critical safety system for nuclear plants. ECCS shows functions predominantly for guaranteeing the integrity of the reactor core in case of accidents. In 1974, the emergency cooling criteria was used to create the Design Bases Accident (DBA), which sets forth the minimum RIA a facility must be able to withstand. The DBA, however, has drawbacks due to its conservative rules [1]. Nuclear systems are extremely nonlinear, and the assumptions adopted helped solve this problem in a simple manner. In last decade, the rules have been revised to allow for practical methods of complementing risk assessments using a purely deterministic approach. The amended Code of Federal Regulations (10-CFR50.46) adopted Best Estimate (BE) models in its search for realistic models for LOCA accidents due to cold leg rupture of pressurized water reactors [2].

## **1.2 Best Estimate**

USNRC proposed the first initiatives to define uncertainty analysis in 1988. New rules set to the licensing process of LWRs based on the code scaling, applicability, and uncertainty (CSAU) methodologies [3]. Worldwide, many models were created for risk analyses as applied to the licensing of nuclear units. In Italy, the University of Pisa created the Based-on-Accuracy Extrapolation (UMAE) [4], and UMAE-CSAU is considered as percussion models for licensing [5]. Use of the Best Estimate Plus Uncertainty (BEPU) model introduced between 1974 and 1986 [6]. Currently, CFR guidelines allow the use of coupled models that use best estimate methods coupled with the conservative models. BE methods are used for licensing codes for the calculation of nuclear transients in a more realistic way, versus the classical style. A risk analysis must identify all parameters that could affect a system's response. The fuel code can calculate uncertainties using physical models such as the thermal conductivity of the fuel. These rules define the safety criterion and margins only by deterministic limits. The risk assessment may make estimates based on a combination of both conservative and probabilistic models [7]. The treatment of uncertainty aims for an increased power output by the reactors, with a lower investment.

## 1.2 Fuel Code Performance

In 1975, the first version of fuel performance code was presented in A Computer Code for the Calculation of Steady State Thermal-Mechanical Behavior of Oxide Fuel Rods (FRAPCON) [8]. The system code simulates fuel performance for the licensing process used by the USNRC guideline Standard Review Plan Section 4.2. The fuel code is an analytical tool that calculates behavior under the irradiation of an LWR fuel rod. The code is applied when power variations and boundary conditions are slow enough so that the term permanent regime is applied. Also, it can include many situations (such as long periods at a constant power, or slow power ramps) which are typical of normal operations within a nuclear reactor. The fuel code calculates the time variances for all significant variables of the fuel rod, including; fuel temperatures, deformations, densification, swelling, and fission gas release (FGR). These fuel codes use a finite difference heat conduction model within the mechanical model (FRACAS), and for the solution of thermal equations in a transient. The physical models must consider a loss of mechanical propensities by the irradiation. Capabilities for modeling uncertainty were also incorporated into the physical models of FRAPCON. The FRAPTRAN (fuel rod analysis program transient) is used to calculate the thermal and mechanical fuel response to a regime of LWR transients [9].

## 1.3 DAKOTA Toolkit

In 1996, work began on a software (developed at Sandia National Laboratories or SNL) for global optimization of complex systems. The Design Analysis Kit for Optimization and Terascale applications (DAKOTA) uses the C++ object-oriented programming language, and currently produces a flexible interface for iteration at a systems level. The computational models used in DAKOTA were evaluated based on a system's structural mechanics. It also checks for heat transfer, fluid mechanics, physical shock, and nuchal systems and makes it possible to predict the behavior of complex systems.

Often, DAKOTA simulations are used as virtual prototypes to obtain acceptable or optimized designs for a given system. Its interface is well generalized, encompassing fuel performance codes such as FRAPCON. The analysis kit contains several algorithms capable of implementing optimization models and quantifying uncertainty [10]. The toolkit has several resources allowing for complex projects that lead to a better understanding of the behaviors of the system. It was created for uncertainties treatment, having the ability to perform an analysis of complex systems. It also implements sampling models such as Monte Carlo and Latin Hypercube Sampling (LHS). The fuel codes, along with DAKOTA, help perform the sensitivity and variance analyses.

## 2. OVERVIEW OF UNCERTAINTY ANALYSIS METHODOLOGIES

### 2.1. Sample Size Determination for Tolerance Limits

The Wilks' formulation serves to determine the minimum number of code runs needed to reach significance, combined with the desired confidence level within tolerance bounds. Wilks proposed a way for estimating sample sizes as a function of the tolerance limits using Static-Order [11]. The formula proves that a random sample of a population between two different

orders do not show any dependence on the population tested [12]. Eqs. (1) and (2) describe the Wilks' formulation:

$$1 - \gamma^n = \beta \quad (1)$$

$$1 - \gamma^n - n(1 - \gamma)\gamma^{n-1} = \beta \quad (2)$$

where  $\beta$  is the expected tolerance interval,  $\beta = 0.95$ ,  $\gamma$  is the confidence level ( $\gamma=0.95$ ),  $\alpha$  is the quantile, and  $N(\alpha, \beta)$  describes the sample size [13]. Since 1990, the Wilks' formulation has been used in the determination of nuclear safety. The minimum number of trials for a given 95% quantile, and a confidence level of 95% is shown in Table 1 [14].

**Table 1: Number minimum of trials**

| Confidence level<br>$\beta \backslash \alpha$ | One-side |      |      | Two-side |      |      |
|-----------------------------------------------|----------|------|------|----------|------|------|
|                                               | 0.90     | 0.95 | 0.99 | 0.90     | 0.95 | 0.99 |
| 0.90                                          | 22       | 45   | 230  | 38       | 77   | 388  |
| 0.95                                          | 29       | 59   | 299  | 46       | 93   | 473  |
| 0.99                                          | 44       | 90   | 459  | 64       | 130  | 662  |

## 2.2 Latin Hypercube Sample

Latin Hypercube Sample (LHS) is a way to perform random sampling (such as a Monte Carlo sampling) resulting in a robust analysis. Uncertainty is represented by a probability distribution using LHS and executed with the DAKOTA package. The LHS technique requires that all uncertain input variables are independent. Nuclear units show increased risks depending on the lifetimes of operations, using a random sample according to the combined probability Gaussian distributions. Nuclear fuel tolerances are shown as normal distributions, based on their mean and standard deviations. The uncertainty modeling consists of the dimension of variation used for each uncertain parameter, quantified by a probability density function (PDF). The deviations used are epistemic, and the uncertainty space created from input parameters are specified by their possible deviations.

## 2.3 Uncertainty Propagation

Monte Carlo simulations utilizing FRAPCON have a high computational cost, but are widely used. The fuel code produces the propagation of uncertainties, calculated using all physical models defined by the performance codes, based on the Monte Carlo method. For each input (constructed by a Gaussian distribution) different values are repeatedly calculated for each of the uncertainty parameters. The quality of sensibility founded is independent from the number of parameters used. The UASA related to accidents have been applied to over 50 input items, and an estimated five outputs. The spread of uncertainties influences the results produced in the simulations performed by fuel codes [15], [16]. DAKOTA makes it possible to simulate input variabilities and models, to identify parameters, to quantify uncertainty and its propagation, and to control parameters of governing equations of the system. Summary, the propagation is calculated by the FRAPCON and FRAPTRAN codes.

## 2.4. Sensitivity Analysis

Global sensitivity analysis was used to measure how the output uncertainty of a model could be assigned to distinct sources of uncertainty by the model input. A local analysis assumes that a single model assessment measuring the local impact of uncertainty items is spread into the model. There are several methods to measure sensitivity, such as a Pearson product, Spearman rank, and Kendall rank.

The sensitivity analysis calculates how the degree of input uncertainty acts on the results from a given model. Two correlation coefficients (namely Pearson and Spearman) were used as measures of the relationship between input and output. Eq. (3) expresses the Pearson correlation, and eq. (4) describes the Spearman rank correlation:

$$r_s = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sqrt{\sum x_i^2 - \frac{(\sum x_i)^2}{n}} \sqrt{\sum y_i^2 - \frac{(\sum y_i)^2}{n}}} \quad (3)$$

where  $x_i$  and  $y_i$  are values of  $n$  pairs of two variables of interest.

The Pearson correlation detects a linear dependence amongst variables, while the Spearman index defines a monotonic relationship. The correlations obtained by the Spearman or Pearson methods can produce better, more precise variability bands:

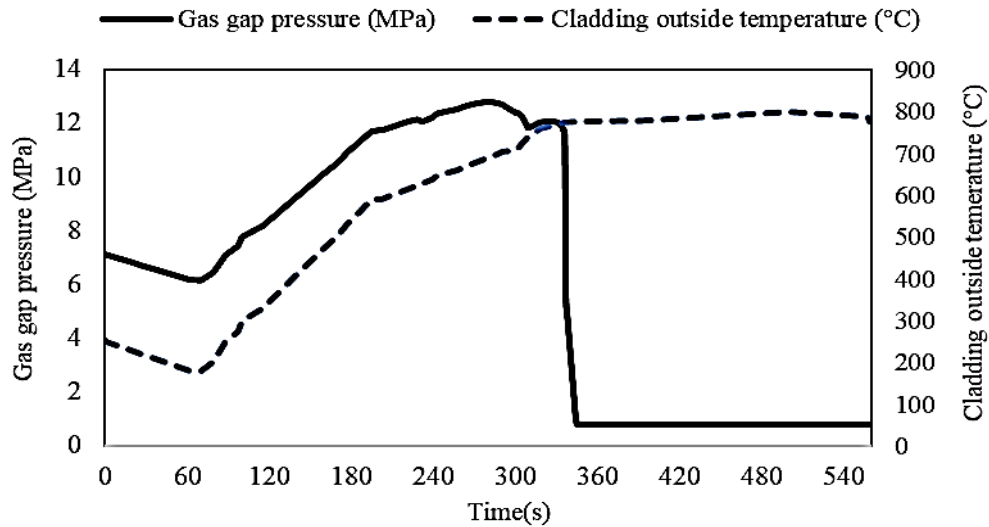
$$r_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)} \quad (4)$$

where  $r_s$  is the coefficient of rank correlation, and  $d_i$  is the difference in rank between paired values of  $X$  and  $Y$  (calculated as  $[\text{rank of } X_i] - [\text{rank of } Y_i]$  .)  $n$  is the sample size or number of pair values ( $X$  and  $Y$ ) within the selected sample.

However, to improve the statistical output results, 200 samples were used. The uncertainty quantification and sensitivity models have a complex scheme that is often executed with a set of random samples while varying the input parameters.

## 3. THE HALDEN LOCA TEST IFA-650 SERIES

In last few decades, the Halden reactor project (HRP) in Norway has conducted several fuel rod experiments for light water reactors. The IFA-650 series target provides the better understanding of fuel behaviors, such as fragmentation and relocation of the fuel pellet. The IFA-650.4 rod experiment was performed in April of 2006 by HPR. Its irradiation cycle was conducted for 2305 days having a burnup of 92 GWd/MTU, with a short section of 50 cm cut from the fuel rod and irradiated by the fuel rod of a commercial LWR plant. The maximum cladding temperature was 800 °C, and was subject to severe deformation. The balloon area was 434 mm<sup>2</sup>, and strain of 62% occurred at a temperature of 770–780 °C [16].



**Figure 1: Plenum pressure and cladding outside temperature.**

### 3.1. Fuel Uncertainties

We had to evaluate the sources of uncertainties arising from fuels using a statistical distribution. Gaussian curves are preferred for modeling because they faithfully reproduce mechanical tolerances. Distributions were centered on an average, and combined with their tolerances. For chosen sources of uncertainty, possible variations in the theoretical density of the fuel were evaluated, as well as deviations of the internal pressure on the gas and the rates of enrichment [19], [20]. In Table 2, exposed uncertainties parameters, which must spread through system.

**Table 2: Uncertainty parameters**

| Uncertainty parameter        | $\mu$ (mean) | $\sigma$ (sigma) |
|------------------------------|--------------|------------------|
| Pellet outside diameter (mm) | 8.1890       | 0.1638           |
| Cladding thickness (mm)      | 0.6299       | 0.0126           |
| Gap thickness (mm)           | 0.0851       | 0.0017           |
| Fuel density (%)             | 96.00        | 7.6800           |
| Cold plenum length (mm)      | 219.99       | 4.4000           |
| Cladding roughness (mm)      | 0.00050      | 0.0000075        |
| Fuel roughness (mm)          | 0.00199      | 0.0000299        |
| Increasing density (%)       | 100          | 1.00             |
| Enrichment (%)               | 3.50         | 0,175            |
| Fill gas pressure (MPa)      | 4.00         | 0.020            |

### 3.2 Treatment of Uncertainty Boundary Conditions

Following the hierarchy of causes to the uncertainty within fuel behavior, we must quantify the effects of the initial conditions. Nuclear reactors have several control parameters used to solve the physical models that contribute to improving incorrect responses. The linear heat rate is a power profile that explains boundary conditions for predicting fuel performance. All deviations

from the initial states of the burn cycle can be represented by uncertainties introduced via boundary conditions. In this case, the initial states available to the corrosion model and hydrogen pickup applied to the cladding. In Table 3, shown uncertainties due to boundary conditions.

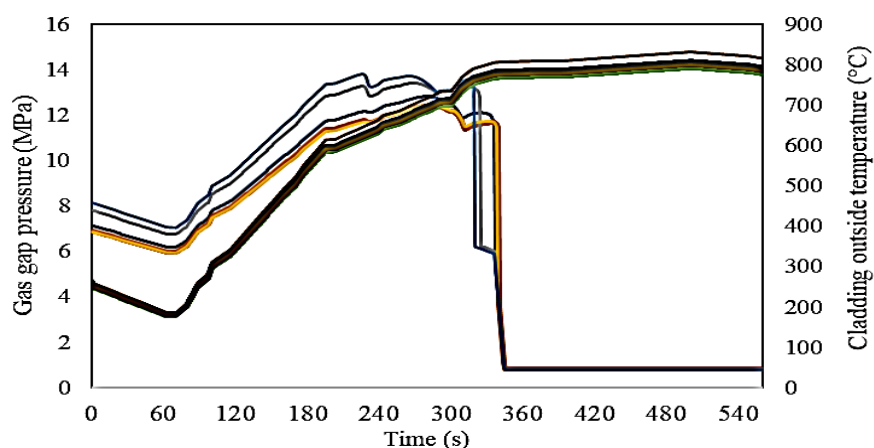
**Table 3: Boundary conditions used of Halden LOCA for IFA-650-4 rod**

| Parameter          | Range        | Distribution |
|--------------------|--------------|--------------|
| Outlet pressure    | $\pm 1.0\%$  | Normal       |
| Mass flow rate     | $\pm 1.0\%$  | Normal       |
| Inlet temperature  | $\pm 1.5 \%$ | Normal       |
| Wall roughness     | $\pm 5.0\%$  | Normal       |
| Hydraulic diameter | $\pm 1.0\%$  | Normal       |
| Flow area          | $\pm 1.0\%$  | Normal       |

At the beginning of the blowdown phase, the linear heat rate of the fuel was 0.93 kW/m, and that of the heater was 1.5 kW/m. During the heat-up stage, heater power was regularly maintained. Subsequently (following the blowdown phase) the inlet cladding temperature reached 800 °C, and was maintained for five minutes. The coating failure occurred after blowdown at 366 s at a temperature of 785 °C. The temperature increase rate was 2 °C/s. In this sequence, water spray was turned on after cladding burst.

#### 4. RESULTS AND DISCUSSION

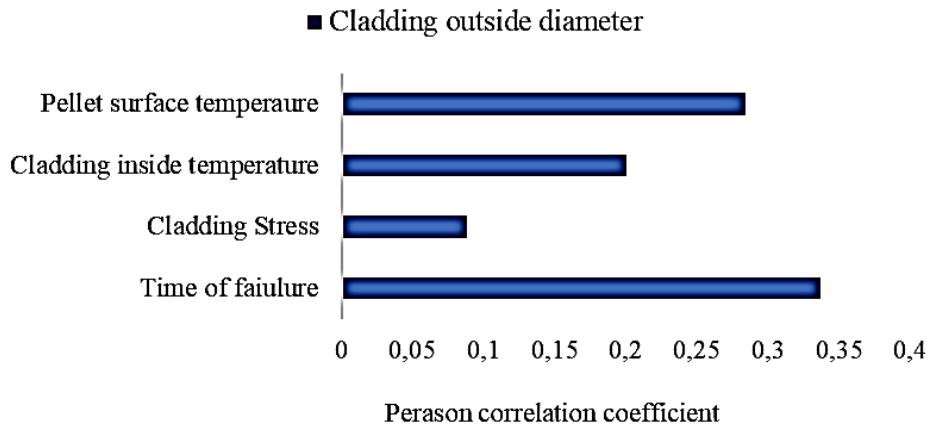
Applying the UASA methodology after defining the uncertainties, the contour conditions offered by FRAPCON and FRAPTRAN were the simulations of type Monte-Carlo. Sensitivity was measured using the optimization package for the calculation of the coefficients which expresses the sensitivity analysis of the simulations. Fig. 2, shows uncertainties as time-of failure of fuel rod. The minimum number of run code adopted was 83 simulations.



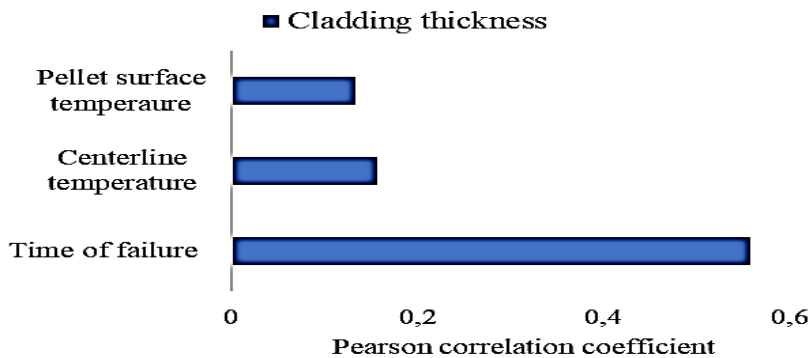
**Figure 2: Plenum pressure for run code to propagate the uncertainty**

The values obtained are the results of a single case which exhibits an extended burn cycle, which produces degradation of materials as fuel and cladding. The sources of uncertainties used within normal distributions and standard deviation were practically limited to 1% of the

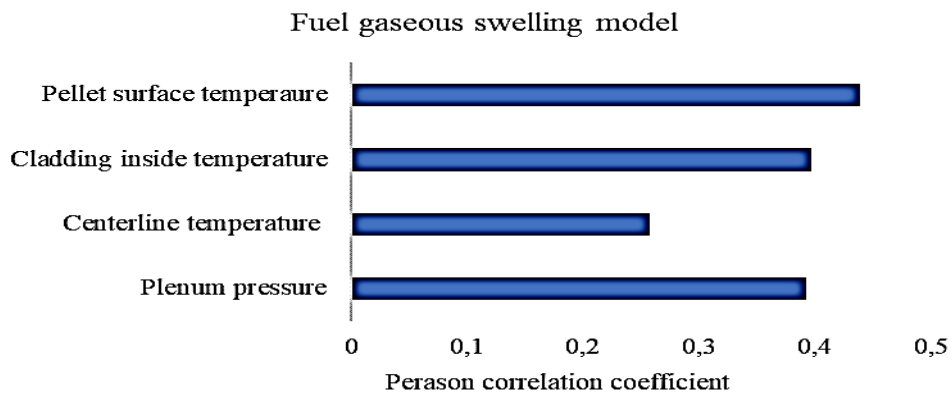
values. Nominal variables provide a table of priority variables in the simulated case. The results help to indicate the influence of the uncertainty sources of the input through the Pearson correlation coefficient. Fig. 3, shows sensitivity analyses for cladding outside diameter. Fig. 3, indicates the sensibility of cladding thickness. Fig. 4, presents of effects of uncertainty models of fuel performance code in safety parameters used in accident analysis. Fig. 5, indicates the influence of fission gas release model and fuel swelling.



**Figure 3: Influence of cladding outside diameter over safety parameters**



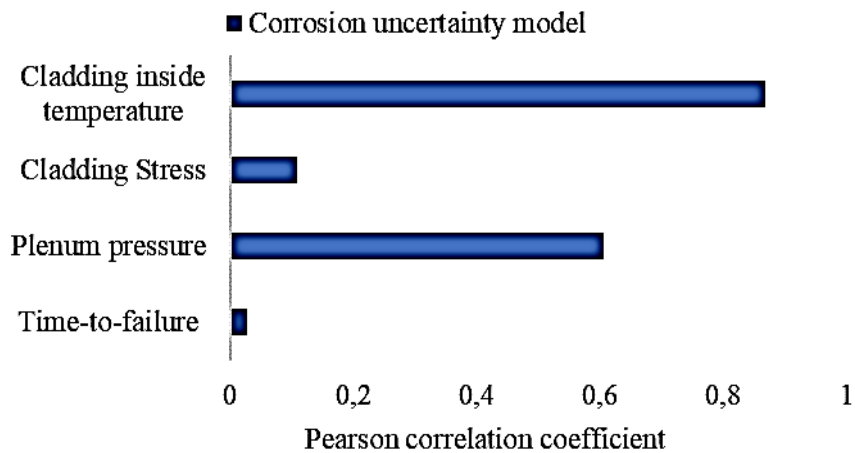
**Figure 4: Influence of cladding thickness over safety parameters**



**Figure 5: Sensitivity analysis of fuel gaseous swelling model**



The mechanical tolerances also offer a considerable influence for IFA-650-4. Fig. 6, shows the power of corrosion model influencing safety parameters.



**Figure 6: Sensitivity analysis of corrosion model over safety parameters**

The internal standards of FRAPCON permit constructed sensitivity analysis based on thermal conductivity, thermal fuel expansion, fission gas release, fuel swelling, irradiation creep, cladding thermal expansion, and cladding corrosion cladding hydrogen pickup. The methodology, when applied, can predict the fuel rod experiment ifa-650-5, and the results were presented.

#### 4. CONCLUSIONS

The recommended models for measuring uncertainties and performing sensitivity analyses help with the integration of fuel performance systems with the DAKOTA toolkit. Here, we investigated a loss-of-coolant accident by integrating the sensitivity analysis directly, using the Spearman index. DAKOTA statistical distributions implemented the model based on the mean and deviation of the variables.

The licensing code FRAPCON, combined with FRAPTRAN, was used for two hundred simulations. In past versions, FRAPCON helped develop the SA using internal models for uncertainty propagations. The effects produced by manufacturing tolerances, combined with boundary conditions, are the basis for the sampling used. The models that describe the thermal conductivity of the fuel, FGR, and the creep rates are large sources of uncertainty in a sensitivity analysis. The methodology presented here was used to predict the results of the fuel rod experiment IFA-650-4. The results are presented.

#### ACKNOWLEDGMENTS

The authors acknowledge and appreciate the support of the Nuclear and Energy Research Institute IPEN/CNEN/SP – Brazil, without which the present study could not have been completed.

## REFERENCES

1. US Nuclear Regulatory Commission, “Best-Estimate Calculations of Emergency Core Cooling System Performance”, *Regulatory Guide 1.157*. Washington, DC, May (1989).
2. C. Frepoli, K. Ohkawa, “Westinghouse Experience in Licensing and Applying Best-Estimate LOCA Methodologies within the Industry”: *Past, Present and Future*. NEA-CSNI-R--2013-8, (2013).
3. H. Zhao; V. A. Mousseau, “Use of forward sensitivity analysis method to improve code scaling, applicability, and uncertainty (CSAU) methodology”. *Nuclear Engineering and Design*, **vol. 249**, pp: 188-196, (2012).
4. B. E. Boyack, I. Catton, R. B. Duffey, K. R. Katsma, G.S. Lellouche, S. Levy, N. Zuber, “Quantifying reactor safety margins part 1: an overview of the code scaling, applicability, and uncertainty evaluation methodology”. *Nuclear Engineering and Design*, **vol 119(1)**, pp: 1-15, (1990).
5. F. D’auria, N., Debrecin, N., G.M., GA Iassi, “Outline of the uncertainty methodology based on accuracy extrapolation”, *Nuclear technology*, **vol 109(1)**, pp: 21-38, (1995).
6. M. Y. Young, S. M., Bajorek, M. E., Nissley, L. E., Hochreiter,” Application of code scaling applicability and uncertainty methodology to the large break loss of coolant”. *Nuclear Engineering and Design*, **vol 186(1)**, pages 39-52, (1998).
7. D’Auria, C. Camargo, O. Mazzantini, “The Best Estimate Plus Uncertainty (BEPU) approach in licensing of current nuclear reactors”. *Nuclear Engineering and Design*, **vol. 248**, pages 317-328, (2012).
8. K. J. Geelhood, W. G. Luscher, P.A Raynaud, I.E. Porter. “A Computer Code for the Calculation of Steady-State, Thermal-Mechanical Behavior of Oxide Fuel Rods for High Burnup. *Nuclear Regulatory Commission*, (2014).
9. J. M., Cuta, K. J., Geelhood, W. G., Luscher, C. E., Beye, “Integral assessment. Pacific Northwest National Laboratory Report”, (2011).
10. B.M. Adams, M.S. Ebeida, M.S. Eldred, G. Geraci, J.D. Jakeman, K.A. Maupin, J.A. Monschke, L.P. Swiler, J.A. Stephens, D.M. Vigil, T.M. Wildey, Dakota, A Multilevel Parallel Object-Oriented Framework for Design Optimization, Parameter Estimation, Uncertainty Quantification, and Sensitivity Analysis: Version 6.6 User’s Manual, Technical Report SAND2014-5015, Sandia National Laboratories, Albuquerque, NM, Updated May (2017).
11. A. Guba, M. Makai, L Pál, Statistical aspects of best estimate method—I. *Reliability engineering & system safety*, **vol 80(3)**, pages: 217-232, (2003).
12. J.C. Helton, F. J., Davis Latin hypercube sampling and the propagation of uncertainty in analyses of complex systems. *Reliability Engineering & System Safety*, **vol 81(1)**, pages 23-69, (2003).
13. S. W., Lee, B. D., Chung, Y. S., Bang, S. W., & Bae, ANALYSIS OF UNCERTAINTY QUANTIFICATION METHOD BY COMPARING MONTE-CARLO METHOD AND WILKS’FORMULA. *Nuclear Engineering and Technology*, **vol 46(4)**, pp: 481-488, (2014).
14. A. Saltelli, M., Ratto, T. Andres, F. Campolongo, J., Cariboni, D. Gatelli, S. Tarantola. *Global sensitivity analysis: the primer*. John Wiley & Sons. (2008)
15. E. De Rocquigny, *Modelling under risk and uncertainty: an introduction to statistical, phenomenological and computational methods*. John Wiley & Sons, (2012).
16. D. S., Gomes, A. T., Silva, “Nuclear Fuel Safety Threshold Determined by Logistic Regression Plus Uncertainty. *World Academy of Science, Engineering and Technology*,

- International Journal of Mechanical, Aerospace, Industrial, Mechatronic and Manufacturing Engineering*, **vol. (3)**, pages 611-617, (2017).
17. A. Bouloré, C. Struzik, F. Gaudier, Uncertainty and sensitivity analysis of the nuclear fuel thermal behavior. *Nuclear Engineering and Design*, **vol. 253**, pages: 200-210, (2012).
  18. T. Manngård, A., Massih, J. O., Stengård, “Evaluation of the Halden IFA-650 loss-of-coolant accident experiments 2, 3 and 4”, publisher kerhetsmyndigheten (SSM), (2014).
  19. G. Pastore, L.P. Swiler, J. D. Hales, S. R. Novascone, D. M., Perez, B. W, Spencer, R. L. Williamson, “Uncertainty and sensitivity analysis of fission gas behavior in engineering-scale fuel modeling”. *Journal of Nuclear Materials*, **vol 456**, pp: 398-408, (2015).
  20. J.R. Wang, J.H. Yang, H.T. Lin, C. Shih, “Uncertainty Analysis for Maanshan LBLOCA by TRACE and DAKOTA”. *US Nuclear Regulatory Commission, Institute of Nuclear Energy Research*, Atomic Energy Council Taiwan (2014).