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Mu-negative metamaterials seen as band-limited non-Foster impedances for magnetic coupled systems

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Abstract

In the last decade, various works have demonstrated that a class of artificial material called metamaterials (MTM) can synthesize μ-negative (MNG) media capable of evanescent-wave focusing which largely enhances the magnetic coupling between coils, which is the basic mechanism of Inductive Power Transmission (IPT) systems. In the present work, MTM-enhanced coupling in IPT systems is examined through analytical and numerical results, which are validated by experimental data. Adopting a transmission-line based approach to describe the general MTM-enhanced IPT system, it is evidenced that MNG MTMs can be interpreted as a negative impedance from a circuital point of view.

1. Introduction

The exploitation of evanescent waves in the near field for power transfer purpose has gained a lot of interest recently. In order to prevent radiation, the drivers are high-Q electrically small antennas (ESA). Most of their energy remains stored in the surrounding near field and power transfer occurs primarily via induction. Nonetheless, such inductive power transmission is efficient only for distances smaller than the diameter of the antennas being also quite sensible to misalignment. Electromagnetic MTM slabs presenting effective negative permeability can be used as near-field lenses to improve the overall efficiency of such inductive power transmission systems by enhancing the inductive coupling between the antennas. It has been called MTM-enhanced coupling [1] [2].

This paper describes a transmission-line based approach of the interaction between a MNG MTM and a magnetic link. By introducing the concept of *virtual magnetic transmission line* (VMTL) to describe the magnetic circuit and by calculating an equivalent virtual impedance to represent the MTM slab, the gain mechanism of those lenses can be regarded as a non-Foster impedance matching. Foster's reactance theorem states that any lossless passive one-port network has a reactance (or susceptance) derivative that increases with frequency [3]. According to [4], non-Foster circuits can provide negative resistance, negative inductance and negative capacitance (meaning that their slope varies decreases with frequency). As it will be further shown, a passive MNG MTM behaves as a band-limited negative impedance described by a negative resistor in series with a negative inductor. The negative inductance implies on reduction of the amount of net energy stored in the magnetic circuit while the negative resistance implies on potential gain across the MTM.

This work is organized as follows: In section 2, a generalized theory of transmission lines is presented in order to include a magnetic coupled system. In section 3, an equivalent virtual impedance for the proposed MTM slab is obtained. In section 4, it is demonstrated by means of analytic calculations and numerical results that a magnetic link can be satisfactorily described as virtual transmission line and that the gain introduced by the MTM is due to improvements in the impedance matching between the drivers. The results are supported by experimental evidence.

2. Generalized theory of transmission lines

2.1. Physical and virtual transmission lines

Most of the classical literature on transmission line (TL) concentrates exclusively on electric transmission lines (ELTL) or the guidance of electromagnetic energy using two or multiple electric conductors (two-wire TL, coaxial cable, stripline and so on). Only recently, more general concepts of TLs have been proposed. According to [5], a time-varying magnetic flux can guide EM fields as electric currents in conventional TLs. In [6], it is proposed a generalization for TL theory by introducing the concept of *virtual TLs*.

Since electric and magnetic fluxes have the same dimension of electric and magnetic charges, respectively, a time-varying flux acts as a sort of "virtual current" driving the fields.

It can be easily shown that a time-varying flux is a current:

$$j\omega\psi_m = j\omega \iint \boldsymbol{B} \cdot d\boldsymbol{s} = \iint j\omega\boldsymbol{B} \cdot d\boldsymbol{s} = \iint \boldsymbol{J}_{m,d} \cdot d\boldsymbol{s} = I_{m,d} \quad [V] \qquad (1)$$

According to the Collins Dictionary [7], the word *virtual* means *"sonnearlyntrue that for most purposes it can be regardednasntrue",n"havingnthenessencenornneffectnbutnnotnthen appearancenornformnof"* or *"beingnsuchnpracticallynorninn* *effect, although not in actual fact or name”*. Thus, as fluxes produce *virtually* the same effect as physical charges concerning guidance, the phenomena of electric and magnetic induction can be effectively described through TL equations by taking the displacement currents as *virtual currents*, the fluxes as *virtual charges* and the reluctance to the flux of the medium as *virtual impedances*.

Hence, physical TLs are defined as the ones where EM fields are guided by means of conduction currents, and virtual TLs, as the ones where EM fields are guided by means of displacement currents.

2.2. Propagation equations for virtual magnetic transmission lines

The virtual magnetic transmission line (VMGTL) is defined as a magnetic circuit formed between two insulated drivers (no physical charge is flowing from one driver to the other) coupled by an intervening quasistatic magnetic field. The transmitting driver is the magneto-motive force (MMF) source generating the time-varying magnetic flux (magnetic displacement current) that links the terminals. The receiving driver is the transducer connected to the load that “converts” the virtual magnetic charges (the magnetic flux) to physical ones (see Fig. 1).

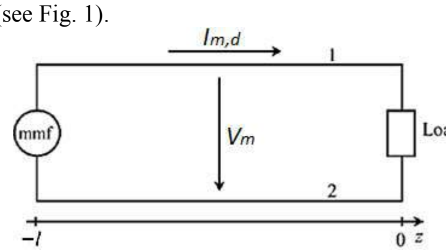

Figure 1: Schematic of a virtual magnetic transmission line.

The total MMF of the virtual line is a real magnetic potential V_m because fluxes going in opposite directions possess inverted magnetic potentials. Consequently, there must be a uniform magnetic field stored between them:

$$MMF = \psi_m \mathcal{R}_m = \int_a^b \boldsymbol{H} \cdot d\boldsymbol{l} = V_m \quad [A] \tag{2}$$

As it is shown in Fig. 2, power flow in a VMGTL is the product of this magnetic field stored between the fluxes and the electric field generated by the time-varying flux in the form of electric-field circulation [8]:

$$P = j\omega \mathcal{R}_m \psi_m^2 = V_m I_{m,d} \quad [W] \tag{3}$$

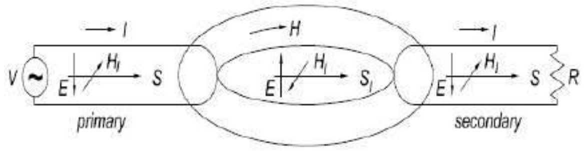

Figure 2: The transversal magnetic field component is responsible for power flow, not the magnetic field parallel to the flux.

Making the simplifying hypothesis that the magnetic flux is mostly confined in a magnetic circuit of fixed transversal section, the virtual TLs can be said to obey propagation equations in terms of potential and current similar to the ones followed by classical ELTLs:

$$\frac{d}{dz} V_m(z) = \left(\frac{\mathcal{R}'_m}{j\omega} + G'_e + j\omega C' \right) I_{m,d}(z) = Y_m I_{m,d}(z) \tag{4}$$

$$\frac{d}{dz} I_{m,d}(z) = (G'_m + j\omega L') V_m(z) = Z_m V_m(z) \tag{5}$$

where Z_m, Y_m, $\mathcal{R}'_m$, G'_e, G'_m, L' and C' denote the series impedance, the shunt admittance and the magnetic reluctance, the electrical conductance, the magnetic conductance, the inductance and the capacitance per unit length, respectively.

Its characteristic impedance Z_0 is given by:

$$Z_0 = \frac{I_{m,d}(z)}{V_m(z)} = \sqrt{\frac{Z_m}{Y_m}} = \sqrt{\frac{G'_m + j\omega L'}{\frac{\mathcal{R}'_m}{j\omega} + G'_e + j\omega C'}} \tag{6}$$

The term $\frac{\mathcal{R}'_m}{j\omega}$ can be said to be a virtual admittance because it neither stores nor dissipate any real power. Like the radiation resistance of an antenna, which is also a virtual impedance [9], it has no relationship with the thermal equilibrium of the circuit that is generating the flux.

The complex propagation constant γ, the load reflection coefficient Γ and the input impedance Z_{in} are given by:

$$\gamma = \sqrt{Z_m Y_m} = \sqrt{\left(\frac{G'_m}{j\omega} + L' \right) \left(-\frac{\mathcal{R}'_m}{\omega^2} + \frac{G'_e}{j\omega} + C' \right)} \tag{7}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{8}$$

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} \tag{9}$$

where Z_L is the load impedance.

The magnetic potential and the virtual current can be identified and converted into the physical electric potential V_e and the physical electric current I_e in the terminals through:

$$V_m(-l) = V_m^{in} = I_e^{in} \tag{10}$$

$$I_{m,d}(-l) = I_{m,d}^{in} = V_e^{in} \tag{11}$$

$$V_{m,v}(0) = V_m^{out} = I_e^{out} \tag{12}$$

$$I_{m,d}(0) = I_{m,d}^{out} = V_e^{out} \tag{13}$$

Note that the magnetic and electric potentials and currents are dimensionally inverted.

Since VMGTLs support the transverse electromagnetic (TEM) mode [5], propagation is described by a onedimensional Helmholtz equation [10]. Its generic solution is given by:

$$I_m(z) = I_m(0)e^{-\gamma z}(1 + \Gamma e^{2\gamma z}) \tag{14}$$

$$V_m(z) = \frac{I_m(0)}{Z_0}e^{-\gamma z}(1 - \Gamma e^{2\gamma z}) \tag{15}$$

2.3. Propagation equations for virtual magnetic transmission lines in free space

If there is no ferromagnetic material channeling the flux from the source to the load, the propagation equations of the virtual line must be adapted to compensate the flux leakage. Supposing that the reflection coefficient Γ must increase inversely with the coupling between the terminals of the TL:

$$I_m(z) = I_m(0)e^{-\gamma z}\left(1 + \frac{\Gamma_m}{\kappa_m}e^{2\gamma z}\right) \tag{16}$$

$$V_m(z) = \frac{I_m(0)}{Z_0}e^{-\gamma z}\left(1 - \frac{\Gamma_m}{\kappa_m}e^{2\gamma z}\right) \tag{17}$$

where κ_m is the magnetic coupling coefficient:

$$\kappa_m = \frac{L_m}{\sqrt{L_1 L_2}} \tag{18}$$

3. Equivalent virtual impedance for a μ-negative metamaterial

Table 1: The constitutive parameters of the MTM slab

L_{cell}	$240\ nH$
f_0	$32.4\ MHz$
$C = \frac{1}{\omega_0^2 L}$	$100pF$
Q_{cell}	245
Cell number	49
Periodicity	2.3 cm

The proposed MTM to synthesize a MNG slab is the same model exploited in [1]. Its main characteristics are presented in Table 1.

The Lorentz model can be used to estimate the relative permeability μ_r of the MTM [11]:

$$\mu_{r,MTM} = 1 + \frac{F\omega^2}{\omega^2 - j\omega\xi - \omega_0^2} \tag{19}$$

where F is the coupling coefficient between adjacent cells of the lattice and ξ is the damping ratio of the system.

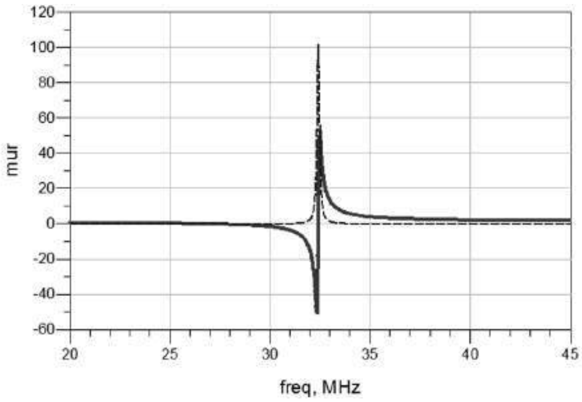

Figure 3: Estimation of the real (solid) and imaginary part (dash) of μ_r.

As it can be seen in Fig. 3, μ_r becomes negative around the resonance frequency ($f_0 = 32.4\ MHz$):

$$\mu_{r,MTM_{analytical}} = -51 - j101.6 \tag{20}$$

The equivalent inductance of the MTM can be determined from its effective parameters:

$$L_{MTM} = \mu_{r,MTM}\mu_0 a = \left(1 + \frac{F\omega^2}{\omega^2 - j\omega\xi - \omega_0^2}\right)\mu_0 a \tag{21}$$

Then, the MTM equivalent virtual impedance is given by:

$$Z_{m,v}(\omega) = jX_{MTM}(\omega) = j\omega L_{MTM}(\omega) \tag{22}$$

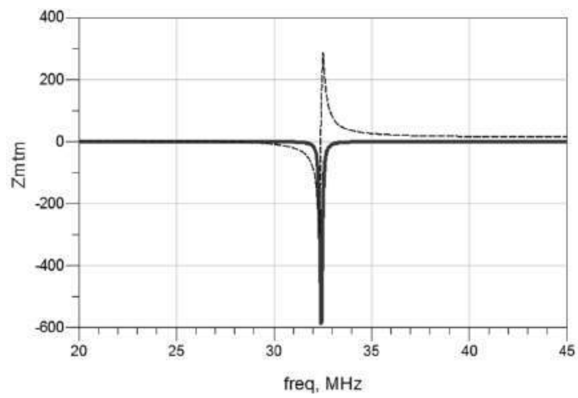

Figure 4: MTM equivalent impedance Z_{MTM} as a function of frequency.

Since the slope of Z_{MTM} does not increases monotonically with the frequency, it can be said to possess a band-limited non-Foster behavior. As it is shown in FIG, the MTM is perceived as negative resistance and a negative reactance around the resonance ($Z_{MTM} \in \mathbb{R}^-$). At the range of sub-resonance frequencies, it is completely invisible to the virtual line ($Z_{MTM} = 0$). And at the over-resonance one, it is seen as a positive reactance ($Z_{MTM} \in Im^+$).

As a negative resistance it acts in the sense of enhancing (restoring) the magnetic current (and consequently the magnetic flux) of the virtual line:

$$\frac{d}{dz} I_{m,d}(z) = -|Re\{Z_{MTM}(\omega_0)\}| V_{m,v}(z) \tag{23}$$

As a negative reactance, it becomes equivalent to a negative inductor and converts the stored energy in the magnetic field into the complementary electric one, reducing the total reactance of the circuit and enhancing the electric potential. On the other hand, as a positive reactance the MTM becomes equivalent to an inductor and stores energy in the surrounding magnetic field. The energy stored by the MTM from higher frequency modes is the energy it uses to amplify the resonance mode.

4. Application

The proposed free-space VMGTL consists of two magnetically coupled loop antennas of radius $r = 5\,cm$ made of copper wire with diameter $p = 1\,mm$ assisted by the MTM described in section 3. The loops operate far from self-resonance. It is assumed that the electrical resistance of wires, the capacitance C and the radiation resistance of the antennas as well as the internal resistance of the source are all negligible. The secondary driver is connected to a load $R_L = 50\,Ohms$. The drivers are separated by a distance $D = 15\,cm$. The amplitude of the tension source is $V_{e,c}^{in} = 1\,V$. The source loop is assumed to have a small internal resistance $R_{source} = 0.5\,Ohms$.

The analytical results are a direct application of the TL model for coupled systems presented in section 2. The numerical results were achieved by the FDTD method on Agilent EMPro and CST studio. The experimental data were obtained using a Vector Network Analyzer.

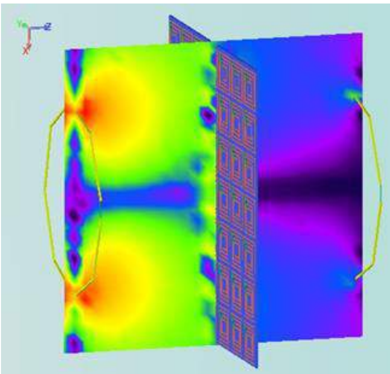

Figure 5: Magnitude of H_x out of the resonance at the plane XZ

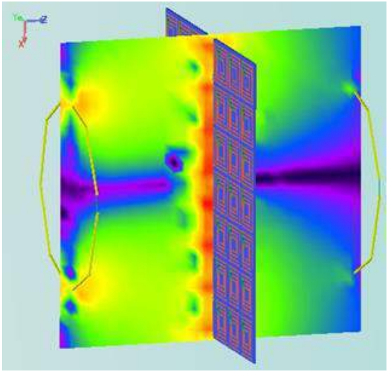

Figure 6: Magnitude of H_x at the resonance at the plane XZ

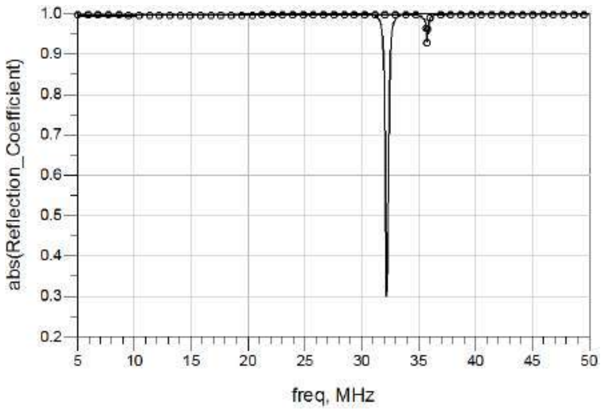

Figure 7: Analytical (solid) and numerical (circle) results for the magnitude of Γ with the MTM slab

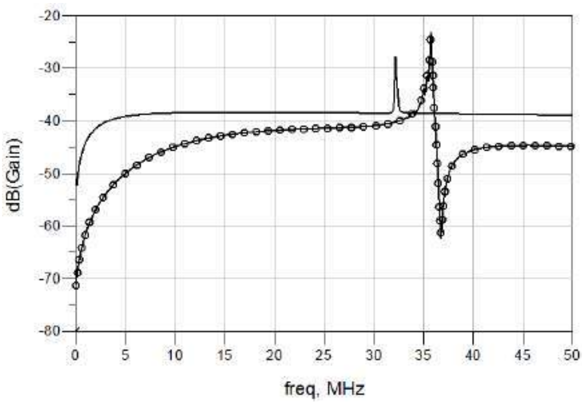

Figure 8: Analytical (solid) and numerical (circle) results for the gain G with the MTM slab inside of the magnetic link

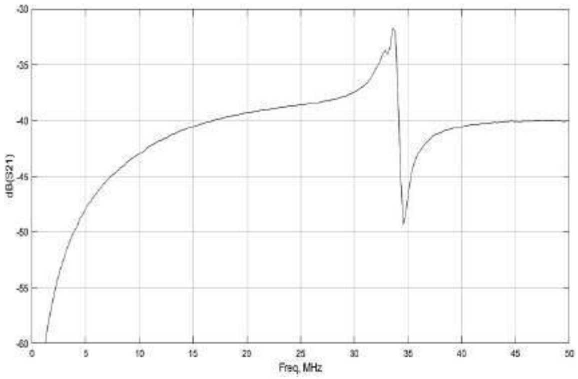

Figure 9: Experimental result for the gain G with the MTM slab

5. Discussion and conclusion

As it is shown in Fig. 5 and Fig. 6, there is a component H_x of the magnetic field inside the link because of the magnetic potential difference V_m between the incoming and the ongoing fluxes trespassing the drivers. It can be clearly seen that the distribution of H_x (thus, the distribution of V_m) is quite improved along the line when the MTM is activated, making the system to behave more similarly to a confined VMGTL with improved transmission. The improvement of the potential distribution along the magnetic link is equivalent to improve the impedance matching of the virtual line as shown in Fig 7.

Table 2: Comparison between the predicted gains and the experimental one

	Gain
Analytical	10 dB
Simulated	15 dB
Experimental	9 dB

As it can be seen in Table 2, the analytical and experimental results for the MTM gain at the resonance present a greater agreement than the one predicted by the numerical simulation, despite the fact that the curve behavior obtained by simulation is more similar to the one obtained with experiment (see Fig. 8 and Fig. 9). The discrepancy between the analytical and experimental curves comes from the simplifying hypotheses concerning the magnetic flux distribution employed by the model.

Finally, one important aspect of the considered system must be pointed out: the relative position of the MTM to the source does not interfere significantly with the gain, which reinforces that such magnetic link has a TL-like behavior.

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