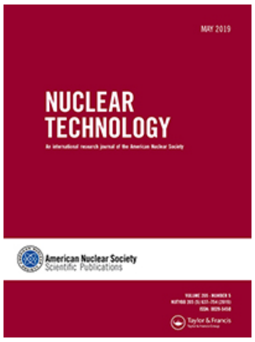

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Particle Swarm Optimization Applied to the Nuclear Fuel Bundle Spacer Grid Spring Design

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Abstract — *One of the main roles of the nuclear fuel bundle spacer grid (SG) is to safely support the fuel rods (FRs) through springs and dimples. The SG design is an important matter for nuclear power plant operation when a damaged FR could release fission products. For this work, Particle Swarm Optimization (PSO) is applied to define the geometries of the springs and dimples existing in a SG. Other algorithms had been used to optimize these geometries but not PSO. This paper proposes a PSO variable model and its fitness function in order to define an optimized geometry for the spring and the dimple so that they can provide sufficient gripping forces and minimize stresses. The implemented PSO was able to generate geometries of springs and dimples with stresses minimized and with a specific required stiffness value. The results of these two characteristics are compared with other results in the literature. For further work, PSO will be used to optimize other important design characteristics of a SG: grid-to-rod fretting, coolant flow-induced vibration, and the function of mixing coolant.*

Keywords — *Spacer grid spring optimization, particle swarm optimization, fitness function.*

Note — *Some figures may be in color only in the electronic version.*

I. INTRODUCTION

Optimization algorithms are constantly used in nuclear engineering, for example, in the nuclear reactor reloading problem. In this paper, the Particle Swarm Optimization (PSO) algorithm is used to define the geometries of springs and dimples of the nuclear fuel bundle (FB) spacer grid (SG) used in a pressurized light water reactor (PWR). Dealing with PSO has some advantages when compared with other optimization algorithms: It has fewer parameters to adjust, and such values are widely discussed in the literature.[1]

Springs and dimples are parts of the SG, which is a component in the FB. The fuel rods (FRs) are supported at some points along their lengths by SGs, which restrain the FRs laterally from bowing and vibrating.[2] Six points of contact on springs and dimples, which are part of the SG, support the FR. The magnitude of the restraining forces developed at these points on the FR must be enough to minimize possible fretting. The SG could also contain mixing vanes, which are used to promote mixing of coolant in regions of high heat flux.[2] Another important function of the SG is to allow axial thermal expansion of the FR without imposing a reaction force sufficient to damage the FR. A FB scheme is shown in Fig. 1. Figures 2a, 2b, and 2c present some details of the SG arrangement.

Figure 2c shows the point of contact between the SG and the FR. The FR is surrounded by two springs and four dimples. Each one of these is in contact with a FR, so reaction forces are provided in order to ensure the FR position. One important function of the springs and the dimples is to provide support to a FR; therefore, those

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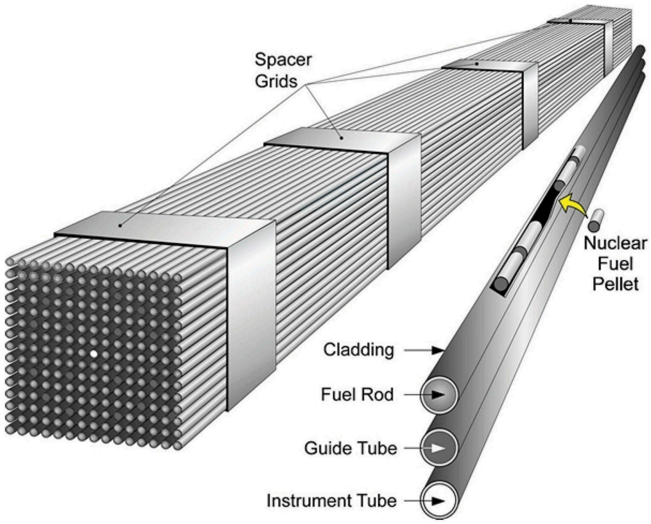


Fig. 1. Fuel bundle.

structures must have some sufficient stiffness to ensure this function. Nevertheless, such stiffness should not be able to cause damage to the FR once it could generate an exaggerated reaction force.

The definition of geometries for springs and dimples is essential to ensure the support of a FR. Defining these geometries may be a challenging engineering problem because of its curved shapes and a high sensitivity of stiffness to any length change. The present work aims to prove that the PSO algorithm is able to provide geometries of springs and dimples that have sufficient stiffness to support a FR and that have its stress intensity (SI) minimized when compressed.

Optimization algorithms have been used to optimize the geometries of springs and dimples of a SG, such as in Refs. 3, 4, and 5. In Ref. 3 the modified method of feasible directions algorithm is performed by GENESIS 7.5 (Ref. 6), but no details are given about how this algorithm works or was implemented. In Ref. 4 the optimization was not performed through an optimization algorithm but rather with a parameter procedure, and the structural calculations were performed using ABAQUS (Ref. 7). Moreover, in Ref. 5, the software GENESIS 7.5 (Ref. 6) is used to perform optimization,[4] but it was not said which optimization algorithm or approach was performed.

The present work intends to clearly show how an optimization algorithm may be implemented to define the geometries of springs and dimples of a SG. The problem considered in this paper is to define, using PSO, geometries for the springs and dimples of a SG so that theses structures have a specific stiffness value. Additionally, another aspect to be optimized in this work is to minimize the SI in these structures. Improving these characteristics, the design of the SG obtained must impose restraining forces on the FR high enough to minimize possible fretting and must allow axial thermal expansions of the FR without overstressing this last component. Note that the problem treated in this paper is simplified in order to favor dealing with PSO matters, so some simplifications were done. Radiation effects in a material are not considered; only the linear elastic behavior of the material is considered, friction forces are not evaluated, and the only load considered is the compression of a SG spring by a FR.

The PSO algorithm is implemented using MATLAB R2013a (Ref. 8), and finite element analysis (FEA) is performed with ANSYS 15.0 software.[9] Once how to implement and perform an optimization using the PSO algorithm is clearly established, further research must be done considering many other requirements for a SG, for example, minimizing the contact pressure between the spring and the FR, maximizing the resistance of a SG from external impact loads, improving its fuel rod gripping throughout the anticipated three operating cycles, and improving its mixing coolant capability.

This paper is organized as follows. Section II presents the PSO algorithm that we used. Section III discusses the considered design requirements of springs and dimples and modeling aspects. Section IV presents the application of PSO to the presented problem. Section V discusses the simulations and results. Finally, Sec. VI has conclusions and proposals for future work.

II. PARTICLE SWARM OPTIMIZATION

In 1995, the PSO algorithm was proposed by Kennedy and Eberhart[10] to optimize nonlinear continuous functions. PSO is a metaheuristic algorithm based on the concept of swarm intelligence capable of solving complex mathematics problems existing in engineering.[11]

The swarm members are called particles, and each one of them balances its individual and its swarm knowledge experience exploring or exploiting the variable searching spaces.[a] This algorithm mimics the real swarm social behavior of sharing information among its members. Such capability helps them to find points in a two-dimensional space with good surviving conditions.

[a] The term "exploring" is related to searching in large areas of the variable spaces. On the other hand, the term "exploiting" is related to a search that a particle moves beyond a local optimum region.

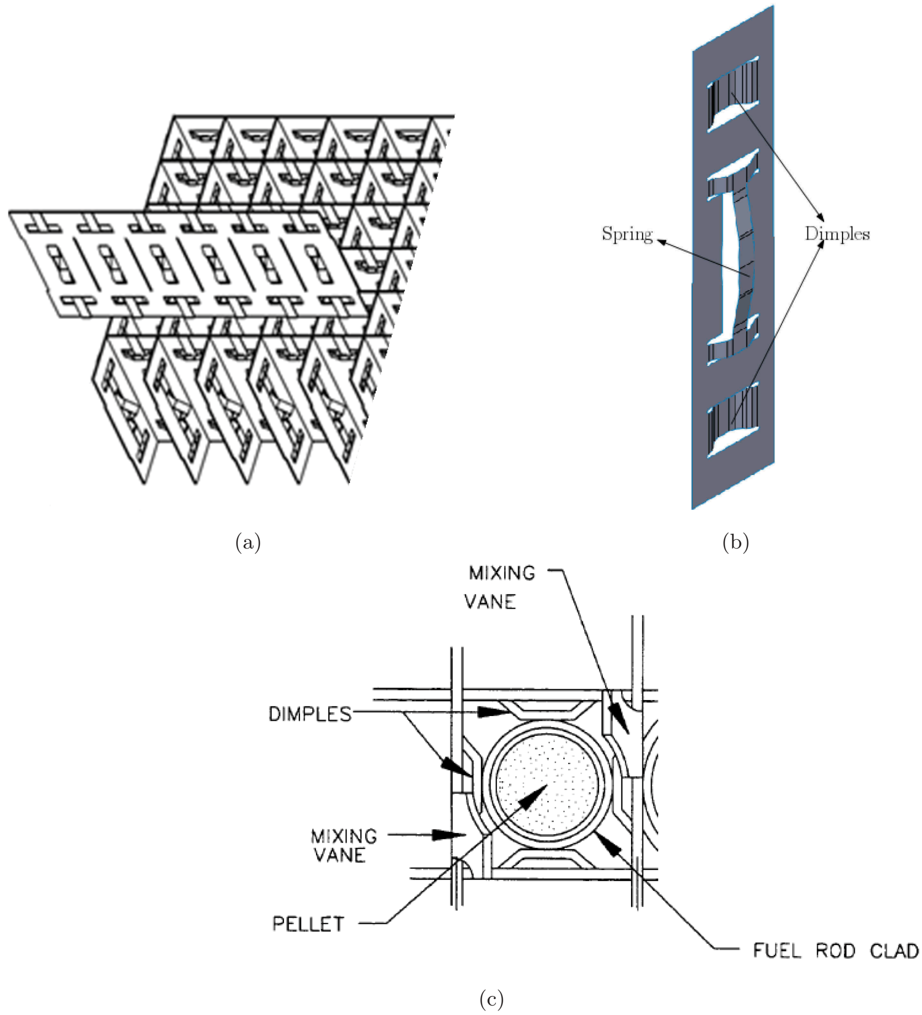


Fig. 2. Schemes showing the following aspects of the arrangement of a FR support: (a) SG comprising a series of strips interlocked with each other, (b) one spring and two dimples as parts of the strips, and (c) top view of a FR in contact with a SG by its springs and dimples.[2]

The goal of an optimization problem is to determine a variable vector $X = (x_1, x_2, ..., x_n)^T$ that minimizes[b] a function $f(X)$. In PSO, X is known as the position vector; this vector represents a variable model. The $f(X)$ function is called the fitness function, and n represents an n-dimensional search space.

Considering that a swarm is formed by P particles, there is associated with each i particle a position vector $X_i^t = (x_{i1}, x_{i2}, ..., x_{in})^T$ and a velocity vector $V_i^t = (v_{i1}, v_{i2}, ..., v_{in})^T$ at an iteration t. These vectors are updated through the dimension j according to Eqs. (1) and (2):

$$V_{ij}^{t+1} = wV_{ij}^{t} + c_1 r_1^t (pbest_{ij} - X_{ij}^t) + c_2 r_2^t (gbest_j - X_{ij}^t) \quad (1)$$

and

$$X_{ij}^{t+1} = X_{ij}^{t} + V_{ij}^{t+1} , \quad (2)$$

where $i = 1, 2, ..., P$ and $j = 1, 2, ..., n$.

Equation (1) proposes that there are three different contributions to a particle's movement, as long as Eq. (2) describes how the positions are updated. In Eq. (1), the parameter w is called the inertia weight constant, and in this paper, it is a positive constant value. This updation rule was proposed by Shi and Eberhart,[12] and it has been chosen because of its easier implementation compared to other PSO

[b] Depending on the proposed optimization problem formulation, $f(X)$ should be maximized also.

algorithm versions. This parameter is important for balancing the global search (when higher values are set) and the local search (when lower values are set). The first term of Eq. (1) represents the influence of the particle's previous motion on the current one. Thus, for example, if $w = 1$, the particle's motion is fully influenced by its previous motion, so the particle may keep going in the same direction as in its previous movement.

On the other hand, if $0 < w < 1$, such influence is reduced, which means that a particle rather goes to other regions in the search space. The parameter w is important because it interferes with the convergence of the swarm. Insofar as w is reduced, the swarm should explore more areas in the search space; hence, the chances of finding a global optimum are greater. In contrast, the number of iterations is greater also, so the simulations in such cases are more time consuming.[10]

The second term is a cognition influence, which represents the particle's own knowledge. The particle's cognition term is influenced by its own previous best position $pbest_{ij}$ once the particle compares this variable vector to the current position X_{ij}^t. Finally, the third term is the social one, representing the capability of sharing information among the particles. In this last term, the social influence is obtained by comparing the particle's current position X_{ij}^t to the best result obtained by the swarm $gbest_j$, called global best position. The parameters c_1 and c_2 are positive constants, these parameters are related to the social cognition of the particles in a swarm, and they weight the importance of the individual and the global learning, respectively. The parameters r_1 and r_2 have random values in the range [0,1] (Ref. 12). Such parameters are important because they avoid premature convergences, increasing the chances of finding the most likely global optima[c] (Ref. 10). Last, the PSO algorithm is described in Fig. 3.

Through the iterations, a particle could move beyond some of its boundary variable spaces. This scenario is forbidden in the present paper, so a special treatment was performed. In Ref. 13, Padhye et al. review popular bound handling methods combined with PSO. The random method was chosen to be used in the current work. It is a strategy simple to implement. It consists of checking for bound-violation one-by-one particle variables and modifying them if necessary. Once a boundary variable is violated, a new feasible position and velocity are then randomly created.

III. SPACER GRID SPRING DESIGN CONSIDERATIONS AND FEA MODEL

The SG is made of individual strips assembled and interlocked into a grid arrangement. These strips can be made of zirconium alloy, nickel-chromium-iron alloy, or

[c] Proving that an optimum value found by PSO is the global optimum is often impratical, so many authors refer to such values as most likely global optimums.

1. Initialization
    1.1. For each particle i in a swarm population size P:
        1.1.1. Initialize X_i randomly
        1.1.2. Initialize V_i randomly
        1.1.3. Evaluate the fitness $f(X_i)$
        1.1.4. Initialize $\boldsymbol{pbest_i}$ with a copy of X_i
    1.2. Initialize $\boldsymbol{gbest}$ with a copy of X_i with the best fitness
2. Repeat until a stopping criterion is satisfied:
    2.1. For each particle i:
        2.1.1. Update V_i^t and X_i^t according to Eqs. (1) and (2)
        2.1.2. Evaluate the fitness $f(X_i^t)$
        2.1.3. $\boldsymbol{pbest_i} \longleftarrow X_i^t$ if $f(\boldsymbol{pbest_i}) < f(X_i^t)$
        2.1.4. $\boldsymbol{gbest} \longleftarrow X_i^t$ if $f(\boldsymbol{gbest}) < f(X_i^t)$

Fig. 3. The PSO algorithm.

ZIRLO, and they are stamped and cut out creating springs and dimples.[2] Fuel rods compress these springs and dimples as they are placed through the SG producing a reaction force. The springs and the dimples must be designed so that the magnitude of the restraining forces on the FR does not cause damage to this last component. Such damage to the FR comprises development of fretting, buckling, or distortion. The springs and the dimples are also designed to allow axial thermal expansion of the FR. Combined with these reaction forces are friction forces also. These two forces, which are illustrated in Fig. 4, are responsible for supporting the FR at many loading conditions, such as earthquakes, shipping and handling, and convective forces induced by coolant flow. FEA using ANSYS was performed to calculate the spring and dimple reaction forces and their SI values when deformed by a FR.

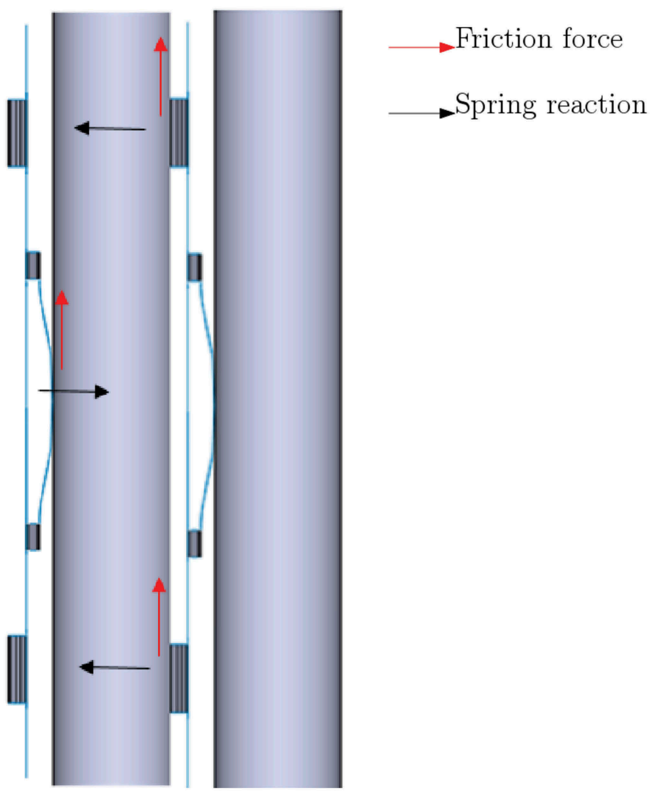


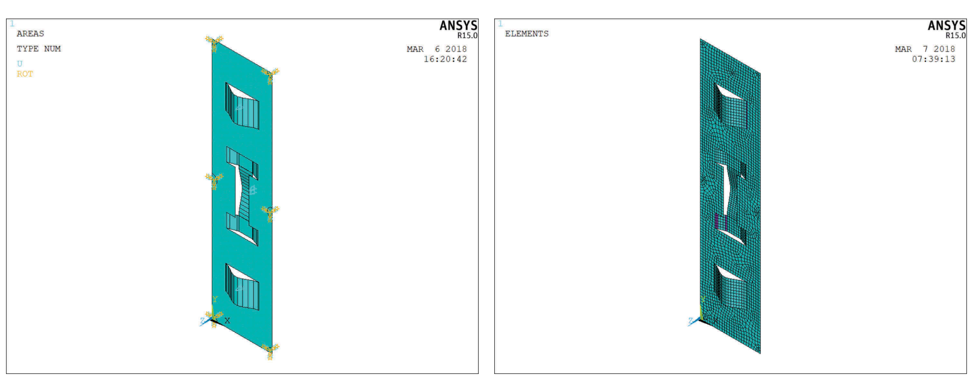
Fig. 4. The interaction between a SG with a FR produces two FR support forces: a reaction force and a friction force.

Some simulations considering many geometries of springs and dimples are performed in each PSO simulation. Figure 5 shows a generic FEA model of these simulations. The load considered is an imposed displacement produced on the spring when in contact with a FR. Boundary conditions are restrictions of translations and rotations on the welding nodes, and all of the simulations consider only the linear elastic behavior of the material. The finite element used in the analysis is SHELL181 (Ref. 9), of 0.5-mm size and 0.4-mm thickness. The material considered is Inconel 718, in which the Young's modulus equals 71 GPa (Ref. 14).

IV. PSO APPLIED TO DEFINITION OF GEOMETRY OF SG SPRING

IV.A. Variable Model

A variable model is required to be defined when the PSO algorithm is used. These variable values must be determined in order to minimize the fitness function. Figure 6 presents the variables that were chosen for this paper to optimize the geometries of the spring and of the dimples. Once this model is defined, the position vector should be written as $X_i^t = (d_{i1}, d_{i2}, d_{i3}, d_{i4}, d_{i5}, d_{i6})^T$, for



(a) (b)

Fig. 5. Generic geometry for the FEA model: (a) boundary conditions applied in the FEA model and (b) the model's mesh.

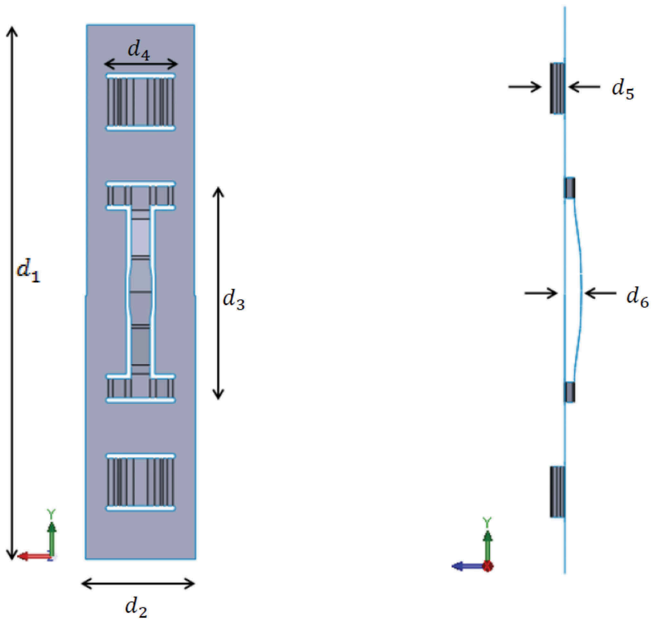


Fig. 6. Lengths used as the variable model in this paper.

each particle i of the swarm at an iteration t. The choice of these variables is related to the fact that those are the main dimensions regarding the SG shape. These measures define the stiffness of the springs and dimples and also define the SI when some load is applied. Note that other variables could also have been considered, such as the thickness of the strips; however, such measures were considered as constant parameters in order to avoid increasing the computational time.

The range of variables used for this work is established in Table I. These ranges were defined so that a wide search space of shapes could be explored by the swarms in the simulations.

TABLE I

Boundary Limits of Variables

Variable	Lower Limit (mm)	Upper Limit (mm)
d_1	50	70
d_2	10	15
d_3	5	30
d_4	5	10
d_5	1	5
d_6	1	5

Defining the fitness function is a fundamental procedure when the PSO algorithm is used to solve any engineering problem. The goal of this function is to minimize or maximize undesirable and desirable project characteristics, respectively. It is important to understand the phenomena involved in the problem to be optimized; otherwise, an improper fitness function may be implemented, which could generate meaningful solutions. The desirable characteristics for the geometries in this paper are to have a specific value of stiffness for the spring of the SG and to minimize its SI.

The idea of a position vector is to create FEA models considering many geometries. The applied displacement in each spring of these FEAs is equal to the difference between the space available for the passage of the FR in the SG without the FR, regarding some X_i, and the diameter of the FR. The FR diameter considered is 9.7 mm. Typically, this value is set around 1 mm, such as in Ref. 15.

IV.B. Fitness Function

The fitness function created to perform PSO is presented in Eq. (3):

$$f(x) = \begin{cases} \sigma + c_K(K_{calculated} - K_{reference}), & \text{if } displacement \geq 0.4\text{mm} \\ 1\ 000\ 000, & \text{otherwise} \end{cases} . \tag{3}$$

The fitness function to be implemented does not exist in the literature; thus, this function must be created. The idea behind the optimization in this work is to increase characteristics in a design through the minimization of a fitness function. This work intends to optimize two characteristics: to minimize the SI and to create a design in which the stiffness of a SG spring can be as close as possible to a reference value $K_{reference}$. Therefore, the proposed fitness function has two terms: One is related to the SI, and the other one is related to the stiffness of the SG spring. The option for combining these two terms in a linear manner was made in order to simplify the interpretation of results. Once a nonlinear relation had been established, it would be harder to understand the influence of these terms on the results.

There are two terms in Eq. (3). The first one is the SI calculated by the FEA model, and the second one is the influence of stiffness. The swarm must find a geometry that produces the minimum value for this function in order to optimize the problem. To minimize the first term, represented by σ, the swarm must find a geometry that minimizes the

SI. On the other hand, the second term is influenced by the difference between the stiffness calculated through the FEA model $K_{calculated}$ and a reference stiffness $K_{reference}$. To minimize this term, the swarm must find a geometry with a calculated stiffness equal to the established value of the referenced stiffness. The constant c_K may fit the order of magnitude between the first and the second terms. Note that the fitness function does not have dimensional consistency, while the unit of the first term σ is in megapascal and the unit of the second term is in newton per millimeter. The fitness function does not require unit consistency once its value is only a mathematical abstraction. This function does not represent any physical entity but rather a value that can measure how much a possible solution is better than another one.

If a particle has its geometry producing a lower displacement than 0.4 mm on the spring, caused by the FR contact, this particle is badly evaluated according to Eq. (3), with the value of 1 000 000. This was established because the SG must be designed to not develop large deformations, to prevent the FR from bowing or moving. In references such as Refs. 3, 4, 5, and 16, the value of 0.4 mm is considered for this same purpose.

Note that a fitness function must represent in its terms all of the characteristics wanted to be optimized in a problem. In the present problem, the SI and the stiffness are the goals to be optimized, and as was explained, each one of these characteristics is represented by a term in the fitness function. The same thing could be done in order to take into account many other desirable characteristics to be optimized, for example, to consider thermal-hydraulic conditions. This characteristic could be represented in the fitness function by a third term, such as $c_H(h_{calculated} - h_{reference})$. In this term, c_H is a constant that may fit the order of magnitude between the other terms of the equation, just like c_K. The value of $h_{calculated}$ could represent a calculated heat transfer convective coefficient in a high-heat-flux region, considering a given geometry of SG, while the value of $h_{reference}$ could represent a desired heat transfer convective coefficient for that region. The term characterized by the difference of $h_{calculated}$ and $h_{reference}$ should develop the same functionality of the stiffness term explained before.

IV.C. Parameter Values

All simulations performed for this work considered $P = 100$ particles in the swarm. Furthermore, the constants c_1 and c_2 were set to be 1.8 and 2.2, respectively. According to Trelea,[17] if the sum of c_1 and c_2 is equal to 4.0, a global attractor is formed in the searching space for the particles, justifying the chosen values presented. That is an important fact in optimization once the chances of the swarm to find a global optimum are improved. Another assumption made for all simulations was if any improvement of *gbest* had not been achieved in the ten last iterations, the simulation would be interrupted.

The PSO algorithm applied to the optimization of the geometry of a SG spring presented in this work depends upon the definition of two parameters: c_K, which belongs to the fitness function, and w, which is the parameter responsible for weighting the influence of a particle's previous movement on the current one, existing in Eq. (2). Hence, finding the most likely global optimum depends on the values of these two parameters.

The parameter c_K from Eq. (3) is important to fit the order of magnitude between the first and the second terms of Eq. (3). A series of simulations with different values of c_K was performed in order to calibrate it. The results from these simulations are presented and discussed in Sec. V. Once defined, a calibrated value for c_K simulations should produce results with stiffness close to $K_{reference}$. This last parameter had its value chosen to be 27.2 N/mm, in accordance with Ref. 16.

V. SIMULATIONS AND RESULTS

Seven simulations were performed for this paper with many c_K and w values, which are summarized in Table II. The five former simulations were performed to calibrate the value of c_K. Once its value had been determined, the value of w was decreased through simulations 6 and 7. As explained in Sec. II, the possible values of w must be between 0 and 1 (Ref. 10) in such a way that the lower this value is, the higher are the chances of finding a global optimum solution. Whereas the values of w are known, the value of c_K must be determined once this parameter belongs to the fitness function originated in this work. The purpose of simulations 1 through 5 is not to find the most likely global optimum solution but rather to define a value of c_K that yields spring geometries with its stiffness $K_{calculated}$ close to $K_{reference}$. On the other hand, the purpose of simulations 6 and 7 is to increase the chances of finding the most likely global optimum, and this is done by reducing the value of w. Note that in order to calibrate the value of c_K in simulations 1 through 5, a higher value of w was used once; in this way the computational time is

TABLE II

Simulations Performed

Simulation	w	c_K	Computational Time (h)
1	0.9	1	Did not converge
2	0.9	20	Did not converge
3	0.9	40	4.0
4	0.9	60	4.0
5	0.9	80	4.0
6	0.7	60	6.5
7	0.5	60	8.1

reduced, as explained in Sec. II. Another remark to be made is about the computational time of simulations 1 and 2, as these simulations did not converge to any solution. The reason for this is that once the parameter c_K had not been set yet, the fitness function of such simulations could not appropriately evaluate the candidates; hence, the PSO could not converge to any solution.

Table III shows the results from simulations 1 through 7. As c_K varies as presented in Table II, the result of $K_{calculated}$ presented in Table III also does. For example, in simulation 1, $c_K = 1$, and the result of $K_{calculated} = 70.3$ N/mm was calculated, while in simulation 2, using $c_K = 20$, $K_{calculated} = 26.7$ N/mm was obtained. A good value for c_K to be picked up is one that has its response of $K_{calculated}$ equal to $K_{reference}$ once it is the desirable value of stiffness. One may conclude that in simulation 4, where $c_K = 60$, this role is played well; then, the value to be picked for this parameter is $c_K = 60$.

As expected, as the w value decreases, the chances of finding the most likely optimum global solution increases, and the computational time increases. Simulation 7 converged to the very best solution archived in this work. Figure 7 presents the fitness improvements of this simulation, and Fig. 8 shows the SI result of this solution.

TABLE III

Calibration of Coefficient c_K

Simulation	σ (MPa)	$K_{calculated}$ (N/mm)
1	620	70.3
2	242	26.7
3	234	27.1
4	287	27.2
5	241	27.3
6	207	27.2
7	196	27.2

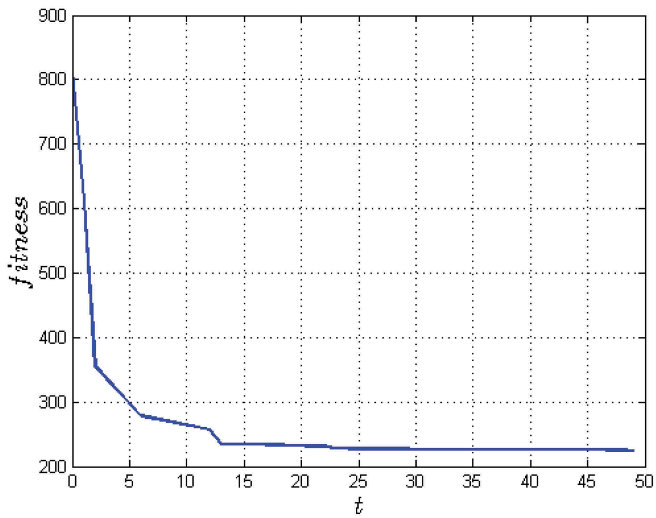


Fig. 7. Fitness improvements of simulation 7.

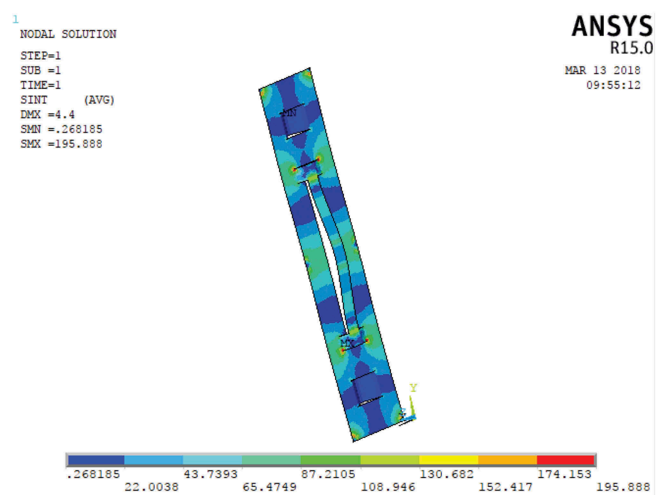


Fig. 8. Nodal SI plot from simulation 7 result.

V.A. Comparison of Results

In Ref. 16, Waseem et al. perform a FEA and an experiment to measure stress and stiffness for Chashma Nuclear Power Plant Unit 1's (CHASNUPP-1's) SG spring, which is currently in operation under the same conditions presented in this paper. CHASNUPP-1 is a nuclear power plant that works with a PWR. In Ref. 16, Waseem et al. used a FEA to calculate a SI of 816 MPa and a spring stiffness of 27.2 N/mm.

The present paper shows that it is possible to obtain a geometry of the SG spring in the same conditions as in Ref. 16 using the PSO algorithm. The SI calculated of the very best geometry found by PSO in this work is equal to 196 MPa, and the calculated stiffness of such a spring is equal to 27.2 N/mm. The PSO algorithm was able to generate a spring geometry with the same stiffness and with a lower SI than in Ref. 16. Comparing these results, one may conclude that by using the PSO, it was possible to obtain a spring geometry with the same stiffness but with a lower SI than the SI of a geometry developed without the use of an optimization algorithm.

VI. CONCLUSION AND FUTURE WORK PROPOSALS

This paper discusses the application of the PSO algorithm to the optimization of the geometry of parts of a SG. The results demonstrate that PSO is an optimization metaheuristics algorithm to be applied to the FR support problem. Creating a fitness function and setting the problem's parameters have been proved to be fundamental when dealing with the optimization of the considered characteristics. As also discussed, the fitness function must represent all the characteristics to be optimized. In this context, it is important to note that the more terms that are placed in this function, the more constants will appear, and then more simulations will be required to calibrate these parameters, like the procedure that was performed in this work.

Further research using the PSO algorithm must be done to optimize characteristics such as the contact pressure of springs on FRs, the SG resistance considering a shock load condition, the function of mixing the coolant by the SG, and the reduction of the vibration induced on the SG by the coolant flow.

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References

1. S. SARKAR, A. ROY, and B. S. PURKAYASTHA, "Application of Particle Swarm Optimization in Data Clustering: A Survey," *Int. J. Comput. Appl.*, **65**, *25*, 38 (2013).
2. "Westinghouse AP1000 Design Control Document," Final Safety Analysis Report, Westinghouse Electric Company (2011).
3. M. K. SHIN et al., "Optimization of a Nuclear Fuel Spacer Grid Spring Using Homology Constraints," *Nucl. Eng. Des.*, **238**, *10*, 2624 (2008); https://doi.org/10.1016/j.nucengdes.2008.04.003.
4. S. LEE, Y. KIM, and K. SONG, "Parameter Study for a Dimple Location in a Space Grid Under the Critical Impact Load," *J. Mech. Sci. Technol.*, **22**, *11*, 2024 (2008); https://doi.org/10.1007/s12206-008-0620-5.
5. K. J. PARK et al., "Design of a Spacer Grid Using Axiomatic Design," *J. Nucl. Sci. Technol.*, **40**, *12*, 989 (2003); https://doi.org/10.3327/jnst.40.989.
6. "GENESIS Users Manual," Version 7.5., Vanderplaats Research and Development Inc. (2004).
7. "ABAQUS 6.11 Analysis User's Manual," Dassault Systèmes (2011).
8. "MATLAB and Statistics Toolbox Release 2013a," The MathWorks, Inc. (2013).
9. "ANSYS User's Manual for Revision 5.0," Swanson Analysis Systems, Inc. (2013).
10. J. KENNEDY and R. C. EBERHART, "Particle Swarm Optimization," *Proc. Int. Conf. Neural Networks*, Perth, Australia, November 27–December 1, 1995, Institute of Electrical and Electronics Engineers (1995).
11. A. A. M. MENESES, M. D. MACHADO, and R. SCHIRRU, "Particle Swarm Optimization Applied to the Nuclear Reload Problem of a Pressurized Water Reactor," *Prog. Nucl. Energy*, **51**, 319 (2009); https://doi.org/10.1016/j.pnucene.2008.07.002.
12. Y. SHI and R. C. EBERHART, "A Modified Particle Swarm Optimizer," *Proc. Int. Conf. Evolutionary Computation*, Anchorage, Alaska, May 4–9, 1998, Institute of Electrical and Electronics Engineers (1998).
13. K. PADHYE, N. DEB, and P. MITTAL, "Boundary Handling Approaches in Particle Swarm Optimization," KanGAL-2012014, Computational Optimization and Innovation Laboratory (2012).
14. *ASME Boiler and Pressure Vessel Code*, Sec. II, Part D, "Properties (Materials)," ASME International, New York (2010).
15. G. S. WEIHERMILLER and W. B. ALLISON, "LWR Nuclear Fuel Bundle Data for Use in Fuel Bundle Handling," Pacific Northwest Laboratory (1979).
16. W. WASEEM et al., "Fuel Rod-to-Support Contact Pressure and Stress Measurement for CHASNUPP-1 (PWR) Fuel," *Nucl. Eng. Des.*, **241**, *1*, 32 (2011); https://doi.org/10.1016/j.nucengdes.2010.11.004.
17. C. I. TRELEA, "The Particle Swarm Optimization Algorithm: Convergence Analysis and Parameter Selection," *Inf. Process. Lett.*, **85**, *6*, 317 (2003); https://doi.org/10.1016/S0020-0190(02)00447-7.