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Consistency test of coincidence-summing calculation methods for extended sources

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ABSTRACT

An internal consistency test of the calculation of coincidence-summing correction factors F_C for volume sources is presented. The test is based on exact equations relating the values of F_C calculated for three ideal measurement configurations. The test is applied to a number of 33 sets of F_C values sent by 21 teams. Most sets passed the test, but not the results obtained using the quasi-point source approximation; in the latter case the test qualitatively indicated the magnitude of the bias of F_C.

1. Introduction

Coincidence-summing effects represent one of the factors affecting the calibration of the full energy peak (FEP) efficiency for high efficiency HPGe detectors. These effects are enhanced in present day measurements, due to the desire and possibility to achieve high efficiency measurement conditions.

In view of the importance of the coincidence-summing corrections and of the complex procedures applied for evaluating them, the Gamma-Ray Spectrometry Working Group (GSWG) of the International Committee for Radionuclide Metrology (ICRM) carried out a series of actions intended to test the quality of these procedures (Lépy et al., 2010, 2012a; Vidmar et al., 2014, 2016). As a continuation, this work presents the results of an action proposed in the meeting of the GSWG organized during the ICRM Conference in Buenos Aires (ICRM GSWG webpage, 2018). It has the specific task of testing the *internal self-consistency* of the

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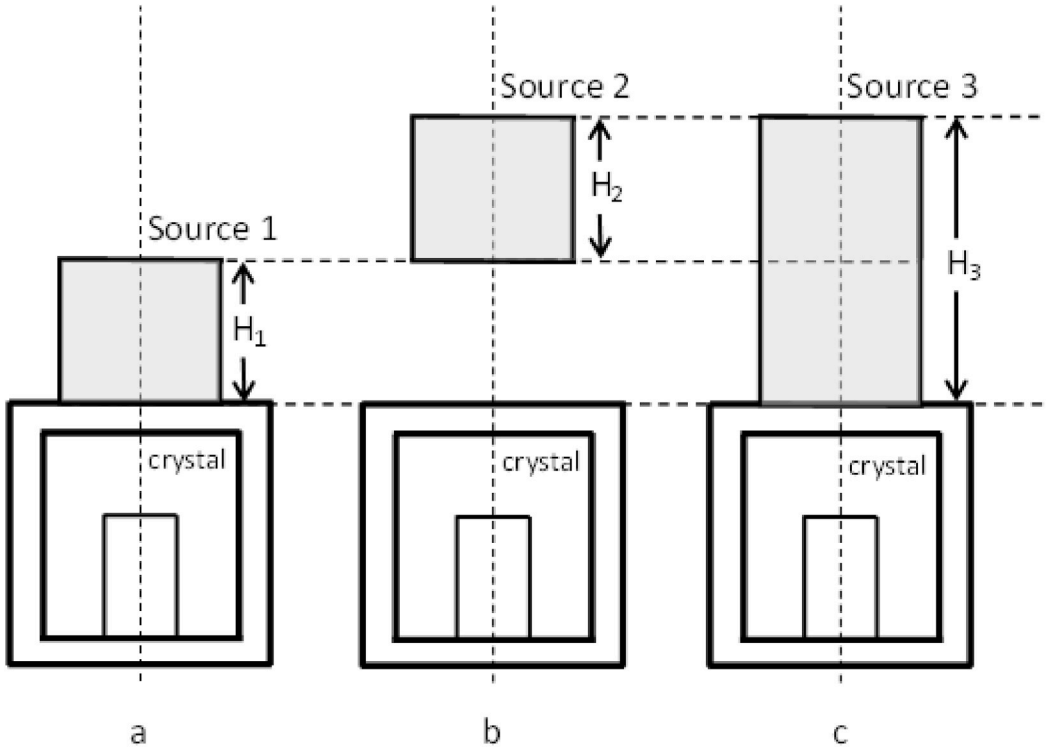

Fig. 1. Measurement configurations.

methods applied for the computation of coincidence-summing corrections *for volume sources*. The proposed test is based on exact relations that should be fulfilled in certain ideal measurement configurations. More precisely, the results obtained using any computation method for one such configuration should be related by exact equations to the results given by the same method for other configurations. Thus, this test does not require experimental data (avoiding the problem of experimental uncertainties and the conformity of the detector model used in the computations with the actual one) or comparisons of a method with other methods (avoiding the debate concerning the selection of a particular reference method).

2. Theoretical background

From the point of view of activity measurement by gamma-ray spectrometry, the occurrence of coincidence-summing has two types of effects: coincidence-losses from existing peaks and coincidence summing-in effects (Debertin and Helmer, 1988; Gilmore, 2008). The latter add counts to existing peaks or produces peaks (pure sum peaks) which cannot be assigned to any photon emitted by the decay of the nuclide.

In the absence of coincidence-summing effects, the net count rate (corrected, if necessary, for background peak count rate) in every peak (energy E_i) is proportional with the nuclide activity A and the photon emission probability $p_i = I_\gamma(E_i)$:

$$N(E_i) = \varepsilon(E_i) \cdot p_i \cdot A \tag{1}$$

In a given measurement configuration, the FEP efficiency $\varepsilon(E_i)$ depends only on the energy of the photon. In the presence of coincidence-summing effects, the probability of recording a count in the peak of energy E_i depends also on the probability that other photons (energy E_j) interact with the detector simultaneously. The count rate equation can be written similarly to Eq. (1), by introducing the apparent efficiency $\varepsilon^{app}(E_i)$:

$$N(E_i, X) = \varepsilon^{app}(E_i, X) \cdot p_i(X) \cdot A(X) = \varepsilon(E_i) \cdot F_C(E_i, X) \cdot p_i(X) \cdot A(X) \tag{2}$$

In this equation $F_C(E_i, X)$ represents the coincidence-summing correction factor for the peak of energy E_i of the nuclide X. In the case of a point source and neglecting the angular correlation effects, it can be written as (Sima and Arnold, 2000):

$$F_C(E_i, X) = 1 - \sum_j \frac{p_{ij}}{p_i} \cdot \eta(E_j) + \sum_{j,k} \frac{p_{ijk}}{p_i} \cdot \eta(E_j) \cdot \eta(E_k) - \ldots + \sum_{p,q} \frac{p_{pq}}{p_i} \cdot \frac{\varepsilon(E_p) \cdot \varepsilon(E_q)}{\varepsilon(E_i)} - \sum_{p,q,r} \frac{p_{pqr}}{p_i} \frac{\varepsilon(E_p) \cdot \varepsilon(E_q) \cdot \eta(E_r)}{\varepsilon(E_i)} + \ldots \tag{3}$$

The first sum describes the coincidence losses from the peak of energy E_i due to the simultaneous detection of photon E_j (pair coincidences), summed over all photons emitted in cascades with the photon of energy E_i; the second sum describes triple coincidences and so on. The next sums from the equation (which are the only terms contributing in the case of pure sum peaks) describe the summing-in contribution, when relevant; the first term corresponds to complete energy deposition of the photons of energies E_p and E_q, with $E_p + E_q = E_i$, summed over all possible combinations, whereas the second term takes into account the events when some counts of this type are lost from the peak due to coincidence losses produced by the detection of another photon E_r. Note that the magnitude C of the coincidence effects is given by the sum terms from Equation (3), i.e. $C = 1\text{-}F_C$ in the case of usual peaks (irrespective to the fact that only coincidence losses or both losses and summing in contribute) and $C = F_C$ in the case of pure sum peaks. In Eq. (3), ε represents the FEP efficiency, η the total efficiency, and p_{ij} and p_{ijk}, …are the probabilities of simultaneous emission of the groups of (i,j) and (i,j,k), … photons.

In the case of volume sources, the terms become more involved. For example, $\eta(E_j)$ from equation (3) should be replaced by the quantity

Table 1
Procedures applied by the participants for the calculation of coincidence-summing corrections. In the Details column, only specific information is listed, standard features (all coincidence orders included, pure sum peaks evaluated, K X-rays included) are not mentioned. In the last column the name of the participant and the code of the set of results (in parentheses) are listed.

Efficiencies computation	Volume source effect	Coincidence formalism[a]	Decay data	Details	Participant (code of the results)
MCNPX or MCNP6	Monte Carlo integration	MCNP-CP D.S.	ENSDF	No pure sum peaks; F_C from spectra with and without coincidences;	De Vismes Ott (10); Thanh (11); Ferreux (12);
PENELOPE	Monte Carlo integration	PENNUC; D. S.	DDEP	β particles included; F_C from spectra with and without coincidences;	Peyres (20)
PENELOPE	Monte Carlo integration	PENNUC; D. S.	DDEP	β particles included; No pure sum peaks; F_C from spectra with and without coincidences;	Lépy (21)
GEANT 4	Monte Carlo integration	GEANT4; D. S.	ENSDF	β particles included; F_C from spectra with and without coincidences;	Hurtado (31)
TRIPOLI-4	Monte Carlo integration	TRIPOLI-4	ENSDF	Applied only for the 1173 and 1332 keV peaks of ^{60}Co	Lee (33)
MCNP	Eff. solid angle; ETNA Integration	ETNA A. I.	DDEP	Pair coincidences only; No pure sum peaks; Reference for efficiency transfer: point (16), target (volume) (17) sources	Thanh (167 Jodlowski (17)
MCNPX	Volume split in domains: 80 (*a*), 32 (*b*), 112 (*c*)	MATLAB; A. E.	DDEP	Pair coincidences only; Spectra evaluated separately for each domain; merged together by MATLAB F_C from spectra with and without coincidences	Solc (13)
MCNP	Volume split in domains: 80 (*a*), 32 (*b*), 112 (*c*)	A. E.	DDEP	Pair and triple coincidences; K_α and K_β X-rays7 Efficiencies and F_C evaluated independently for each domain, then averaged over the domains	Dryak (14)
MCNP6	Averages over random points	SUMCOR A. I.	DDEP	Efficiencies and F_C evaluated independently for each of the 200 (*a* & *b*) or 400 (*c*) random points, then averaged	Dias et al.[b] (15)
MCNP6	Random points and LS formalism	SUMCOR A. I.	DDEP	Efficiencies evaluated independently for 200 (*a* & *b*) or 400 (*c*) random points, then LS formalism and F_C	Dias et al.[b] (18)
MCNP6	Quasi-point source	SUMCOR A. I.	DDEP	Volume source efficiencies (peak & total): MCNP6. F_C by SUMCOR (no volume integration)	Dias et al.[b] (19)
PENELOPE	Averages over random points	SUMCOR A. I.	DDEP	Efficiencies and FC evaluated independently for 200 (*a* & *b*) or 400 (*c*) random points, then averaged	Dias et al.[b] (22)
PENELOPE	Random points and LS formalism	SUMCOR A. I.	DDEP	Efficiencies evaluated independently for 200 (*a* & *b*) or 400 (*c*) random points, then LS formalism and F_C	Dias et al.[b] (23)
PENELOPE	Quasi-point source	SUMCOR A. I.	DDEP	Volume source efficiencies (peak & total): PENELOPE F_C by SUMCOR (no volume integration)	Dias et al.[b] (24)
PENELOPE	Eff. solid angle; ETNA Integration	ETNA A. I.	DDEP	No pure sum peaks; Point source efficiencies (peak & total): PENELOPE; Eff. transfer, integration and F_C by ETNA	Lépy (25)
PENELOPE	Quasi-point source	ETNA A. I.	DDEP	No pure sum peaks; Volume sources efficiencies (peak & total): PENELOPE; F_C by ETNA (no integration over volume)	Lépy (26)
PENELOPE	Quasi-point source	TrueCoinc A. I.	ENSDF	No pure sum peaks; Volume sources efficiencies (peak & total): PENELOPE; F_C by TrueCoinc (no integration over volume)	Karfopoulos (27); Savva (28)
GESPECOR	Monte Carlo integration	GESPECOR A. E.	KORDATEN (from DDEP)	Standard GESPECOR: FC evaluated in a single run; Correlated sampling; variance reduction techniques	Gurau (1); Yucel (2); De Vismes Ott (3); Luca (4); Ferreux (5); Laubenstein (6); Sima(7)
GESPECOR	Monte Carlo integration	GESPECOR A. E.	KORDATEN (from DDEP)	Angular correlation included; everything else as in standard GESPECOR	Sima (8)
GESPECOR	Quasi-point source	GESPECOR A. E.	KORDATEN (from DDEP)	Volume sources efficiencies (peak & total): GESPECOR; FC: analytical formulas (no integration over volume);	Sima (9)
EFFTRAN	Eff. solid angle; LS formalism	EFFTRAN A. I.	KORDATEN	K X-Rays and L X-Rays included; No pure sum peaks; Reference for efficiency transfer: point source (29), volume source (30); Volume integration: LS formalism	Vidmar (29); Nikolic et al.[c] (30)
ISOCS	ISOCS	ISOCS	ISOCS	Coincidences up to 3 photons; No pure sum peaks; Efficiencies calculated by MCNP	Tyminski et al.[d] (32)

[a] Acronyms: D. S. = full decay simulation; A. I. = Implicit analytical formulas; A. E. = Explicit analytical formulas.
[b] Group participant = Dias, Semmler, Menezes, Moreira and Koskinas.
[c] Group participant = Nikolic and Vukanac.
[d] Group participant = Tyminski, Saganowski and Tyminska.

$$\eta(E_j) \rightarrow \eta^{eff}(E_j; E_i) = \frac{\int \varepsilon(E_i, \vec{r}) \cdot \eta(E_j, \vec{r}) dv}{\int \varepsilon(E_i, \vec{r}) \cdot dv} \quad (4)$$

where the integration is carried out over the source volume, dv being the volume element around $\vec{r}$. In the presence of angular correlations, the dependence of the efficiency on the direction of emission of the photon should be introduced, the angular correlation function should be included in the integrand and integrations should be carried out over the directions of emission of the photons (the same procedure is valid also in the case of point sources). The expression in equation (4) can be viewed as an effective total efficiency (Arnold and Sima, 2001) for photon E_j, depending also on the energy E_i. The terms involving two efficiencies in equation (3) should be replaced by similar integrals of products of three efficiencies, and so on. Note that for the evaluation of F_C, the variation of the efficiencies as a function of the emission point within the volume source is needed. Thus, contrary to the case of point sources, the efficiency related quantities required for the computation of F_C do not have a directly measurable correspondent. Furthermore, due to the dependence of the effective total efficiency on E_j and E_i, not only on E_j, the number of quantities that should be evaluated is higher. Consequently, it is clear that compared to the case of point sources, the computation of F_C for volume sources is more difficult and can take much longer time. The

angular correlation effects further complicate the evaluation of F_C.

In view of the complexity of the evaluation of F_C in the case of volume sources, in some codes explicit or hidden approximations are applied. The uncertainties associated with some of these approximations can be partly revealed by the test described below.

3. Self-consistency test

Consider three sources, S1, S2 (identical with S1) and S3, with negligible container walls and filled with air (or vacuum). The dimensions of the sources satisfy $R_1 = R_2 = R_3 = R$, $H_2 = H_1$, $H_3 = H_1 + H_2$. The sources are measured in the configurations a, b and c presented in Fig. 1. Note that configuration c corresponds to mounting the first two sources one upon the other on the detector. Each configuration is placed in vacuum (or in air) and there are no materials in the vicinity of the detector and sources. In these conditions the interactions within the sources, in the container walls or outside the sources and detector, are negligible. Then the properties of the S3 source measured in configuration c should be identical with the properties of the composite source, obtained by placing S2 above S1.

For example, the contribution to the count rate of the decays from each of the two parts of the composite source is equal to the contribution to the count rate of the sources 1 and 2 measured independently in geometries a and b. This is true both in the case of the peak count rate (either in the absence or in the presence of coincidence-summing effects) and in the case of the total count rate. Therefore the following relations should hold (ICRM GSWG webpage, 2018):

$$\varepsilon_{comp} = \frac{\varepsilon_a + \varepsilon_b}{2} \tag{5}$$

$$\eta_{comp} = \frac{\eta_a + \eta_b}{2} \tag{6}$$

$$F_{Ccomp} = \frac{F_{Ca} \cdot \varepsilon_a + F_{Cb} \cdot \varepsilon_b}{\varepsilon_a + \varepsilon_b} \tag{7}$$

$$\varepsilon_{comp}^{app} = \frac{1}{2}\left(\varepsilon_a^{app} + \varepsilon_b^{app}\right) \tag{8}$$

Denote by ε_c, η_c, F_{Cc} and ε_c^{app} the respective quantities computed directly for source S_3, measured in geometry c. If the procedure used to compute these quantities is self-consistent, then their values should be equal, within the range of their uncertainties, with ε_{comp}, η_{comp}, F_{Ccomp} and ε_{comp}^{app}.

The participants to the test were requested to compute the coincidence-summing correction factors F_C with a statistical uncertainty better than 1%, for several peaks of ^{60}Co (1173.23, 1332.49, 2505.75 keV), ^{134}Cs (569.33, 604.72, 795.86, 1400.59 keV), ^{133}Ba (276.40, 356.01 keV) and ^{152}Eu (121.78, 1408.01 keV), for the three configurations a, b, c. The parameters of the sources were R = 2 cm, $H_1 = H_2 = 2.5$ cm, $H_3 = 5$ cm. A simplified model of an n-type HPGe detector (Vidmar et al., 2014) with radius RD = 3 cm, length LD = 6 cm, cylindrical inner hole (RH = 0.5 cm, LH = 4 cm) and negligible dead layer thickness was assumed. The parameters of the Al endcap were: thickness 1 mm, radius 4 cm, distance from the crystal to the endcap 0.5 cm.

The participants were asked to report also the values of the full energy peak efficiency and to provide information on the method applied. The use of nuclide data recommended by the ICRM, based on DDEP (DDEP, 2018), was suggested.

4. Participants

A number of 21 individual or group participants took part in this action. Several participants have sent more than one set of results, obtained using different computation methods. Thus, 1 participant has sent 6 sets, 2 participants – 3 sets, 3 participants – 2 sets, resulting in a total number of 33 sets of results being received.

The main characteristics of the procedures applied, as well as the codes of the results received, are presented in Table 1. The procedures differ in the method applied for the computations of the efficiency (including volume source effect), of the required decay data parameters and in the way in which the efficiencies and the decay data are combined (Sima, 2012).

General purpose programs such as MCNP (Briesmeister, 2000; Pelowitz, 2013; Goorley et al., 2013), PENELOPE (Salvat, 2015) or GEANT 4 (Agostinelli et al., 2003; Allison et al., 2006) have been used by several participants for the computation of coincidence-summing corrections (Thanh et al., 2018; García-Toraño et al., 2017; Hurtado et al., 2004). In all the cases the value of F_C is obtained by running the code to obtain the peak efficiencies from the simulated spectra both in the presence and in the absence of coincidence emission. MCNP-CP (Berlizov, 2012) and MCNPX, PENELOPE with PENNUC (García-Toraño et al., 2017) and GEANT 4 with the Radioactive Decay Module (Hauf et al., 2013) were used, the decay being fully simulated using ENSDF (MCNP-CP and GEANT 4) or DDEP (PENNUC) decay data.

Another generalist code, developed primarily for applications in reactor physics, (TRIPOLI-4 webpage), was applied for the computation of F_C values for the two gamma photons of ^{60}Co.

MCNP and PENELOPE were also used for providing input values of the efficiencies for other software applied for the computation of F_C.

Thus, MCNP was used in combination with ETNA (Lépy et al., 2004, 2012b) by two participants. The complete version of ETNA was used for the evaluation of the efficiencies dependence on the position of the emission point in the volume source, using the efficiency transfer from a reference geometry, for which the efficiency was computed by MCNP. One participant defined as the reference geometries the configurations a, b and c, which were considered also the target geometries. Using the efficiencies dependence on position, ETNA performs a numerical integration for obtaining F_C.

In the application of the software developed at CMI (Dryak et al., 2012) (2 participants), the volumes of the sources were decomposed in a number of small domains. One participant evaluated the spectra in the presence/absence of coincidences for each domain and then merged them together. F_C was obtained from the ratio of the peak count rates in the combined spectra. The other participant used MCNP to compute ε, η and F_C for each domain and then averaged the values. The joint emission probabilities were evaluated using a MATLAB script or numerically.

SUMCOR (Dias et al., 2018) coupled with MCNP code was used in 3 sets of results. In the first, the efficiencies were computed for a number of 200 (geometries a and b) or 400 (geometry c) randomly sampled points and a Semkow type procedure (Semkow, 1990) was applied to obtain individual F_C values, that were finally averaged. In the second set, the same point source efficiencies were used for the evaluation of the LS curve, that was further applied for F_C computation via a Blaauw and Gelsema (2003) type formalism. The quasi-point source approximation using the efficiencies computed by MCNP for the complete sources was also applied in SUMCOR.

PENELOPE was used for providing input values of the efficiencies for 7 sets of F_C results obtained with other software, not with PENNUC. SUMCOR was applied with input from PENELOPE in the same way as with input from MCNP (3 sets of results). The complete version of ETNA for volume sources was also applied with input efficiencies for the reference point source evaluated by PENELOPE; in this case ETNA provided the values of elementary efficiencies for emission points within the volume sources and then performed a numerical integration for obtaining F_C. In addition, the efficiencies for the complete volume sources were computed using the transfer method from the point source reference (with efficiencies evaluated by PENELOPE) and the FAST version of ETNA (quasi-point source approximation) was applied for obtaining F_C. PENELOPE was also used by 2 participants for the computation of the efficiencies required in the input of the TrueCoinc software (Sudar, 2002).

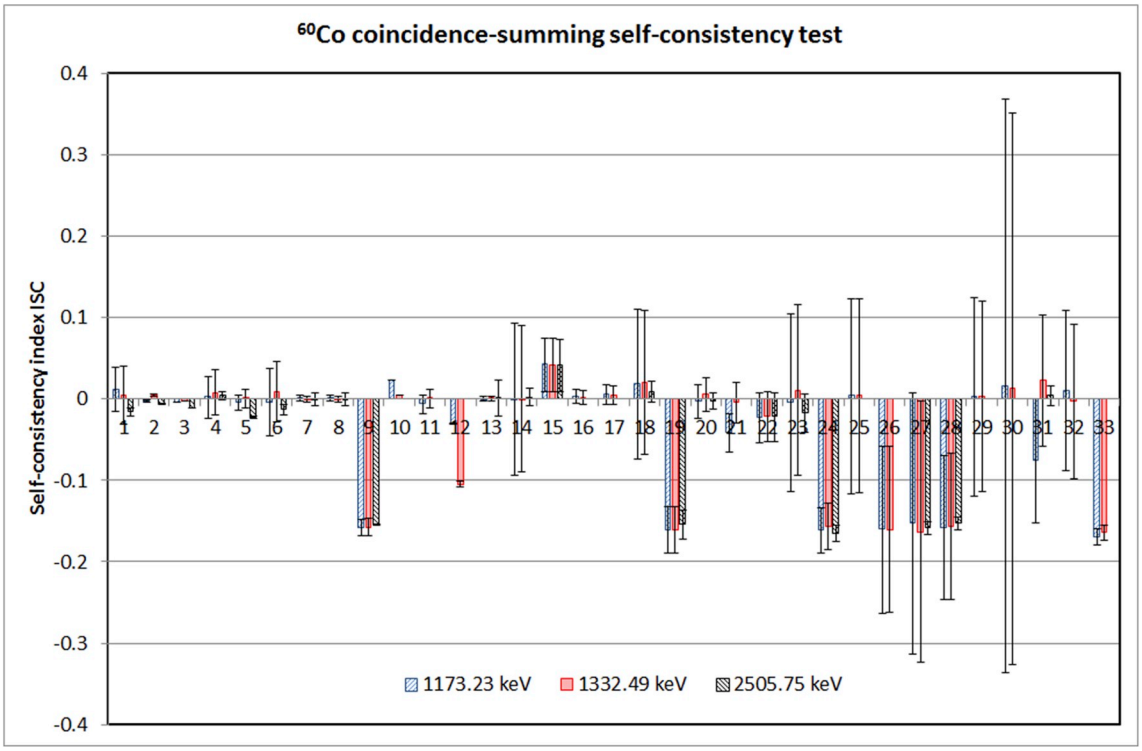

Fig. 2. Self-consistency index for ^{60}Co peaks. The values of ISC(NP) for the 1173.23 keV and 1332.49 keV peaks, and of ISC(SP) for the 2505.75 keV peak are displayed in function of the code of the participants. The uncertainty bars correspond to 1 σ.

The standard version of GESPECOR (Sima et al., 2001; Sima and Arnold, 1996; Arnold and Sima, 2004, 2006) was applied by 7 participants. In this code the joint emission probabilities of groups of photons, evaluated analytically (Sima and Arnold, 2008), are combined with the corresponding integrals of products of efficiencies, obtained by Monte Carlo simulation. The GESPECOR version including the angular correlations was also applied (1 set). Finally, one set of results obtained with GESPECOR deliberately applied the quasi-point source approximation.

2 participants reported results obtained using the EFFTRAN software (Vidmar, 2005; Vidmar and Likar, 2005; Vidmar and Korun, 2006; Vidmar et al., 2011). In EFFTRAN the effective solid angle method is used for obtaining the total efficiency for the sample as a whole, in combination with an approximate way of accounting for the contribution from the scattering within the sample itself (Vidmar and Likar, 2005). The spatial dependence of the efficiency within the volume sources is treated with the LS-curve formalism introduced by Blaauw and Gelsema (2003), but contrary to his original approach, the LS factors are obtained by pure calculation, rather than based on a measurement (Vidmar and Korun, 2006; Vidmar and Kanisch, 2010). To arrive at the full-energy-peak efficiencies the effective solid angle values are multiplied with a peak-to-total ratio of a point source obtained from a deterministic model based on (Vidmar et al., 2001). Finally, the Andreev recursive formulae (Andreev et al., 1972) are applied, using the peak and total efficiencies multiplied by the value read off of the LS curve at a corresponding gamma-ray energy.

The ISOCS procedure included in the CANBERRA software was used by one participant. The coincidence summing corrections including up to 3 photons are evaluated using decay data from an internal library (Zhu et al., 2009). In the case of extended sources, the effects are evaluated for points located inside of the volume source and then are averaged (Kolotov et al., 1996).

In most of the cases, DDEP data were used (DDEP, 2018). Exceptions were the applications of MCNP-CP, GEANT 4, TrueCoinc and ISOCS.

In several calculations, only pair coincidences were included, in others all possible coincidences with γ photons were considered. There were also differences in the procedures applied to deal with X-Rays. Several participants included the contribution of beta rays and of bremsstrahlung radiation to coincidence summing, but the effect is very small and most results reported did not include these effects. With one exception, angular correlations were neglected.

Several participants did not report the uncertainties of all the computed quantities.

Concerning the completeness of the results sent, it should be mentioned that not every set received included all the requested results. In fact, only the F_C values for 1173 and 1332 keV were reported in every set. From the 33 sets of results received, only 19, respectively 21, included results for the sum peaks with energy 1400 keV (^{134}Cs) and 2505 keV (^{60}Co); apparently in the standard versions of MCNP, ETNA, EFFTRAN, ISOCS and TrueCoinc it is not possible to make calculations for these pure sum peaks, but a trick applied by several participants allowed obtaining results for pure sum peaks using these codes (see Table 1).

5. Results

According to the principle of the proposed self-consistency test, the value of the quantity x (F_C or the efficiency) computed directly for the geometry c, x_c, should be equal to the value of the same quantity computed considering the geometry c as composed from a and b, x_{comp}. For testing purposes, an index of self-consistency is defined by:

$$ISC = \frac{x_c - x_{comp}}{x_{comp}} \quad (9)$$

The value of ISC can be interpreted as follows. If the procedure applied for the computation of the quantity x would be absolutely correct in the case of geometries a and b, then the relative bias of the quantity x calculated with that procedure for geometry c is equal to the value of ISC. On the other hand, if also the values x_a and x_b computed with that procedure are biased, then the value of ISC is no longer equal to the relative bias of x_c, the relation between the two depending on the

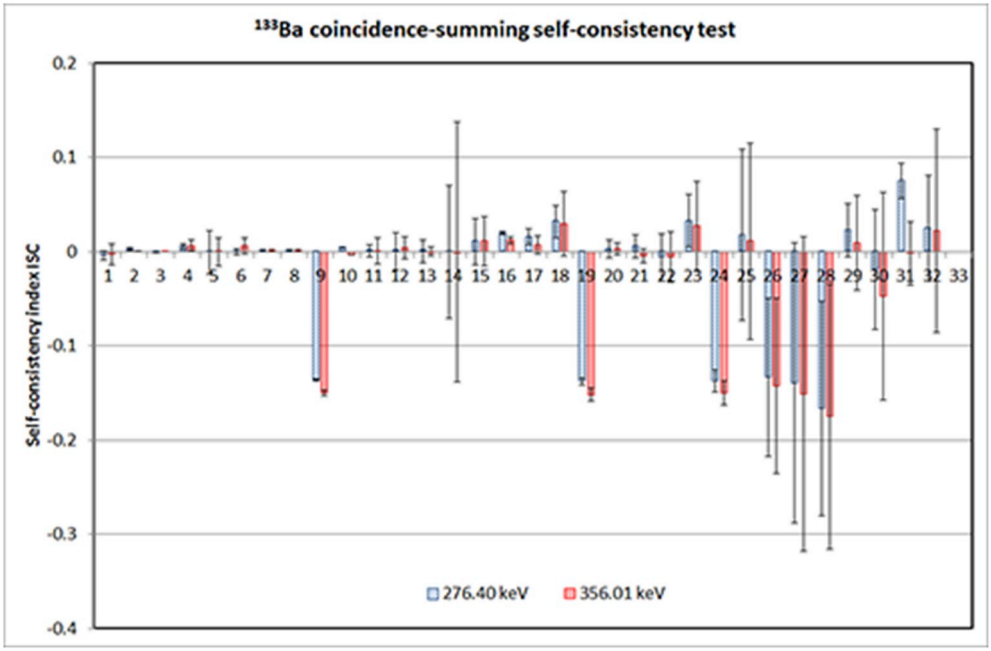

Fig. 3. Self-consistency index for ^{133}Ba peaks of energy 276.40 keV and 356.01 keV. The values of ISC(NP) are displayed in function of the code of the participants.

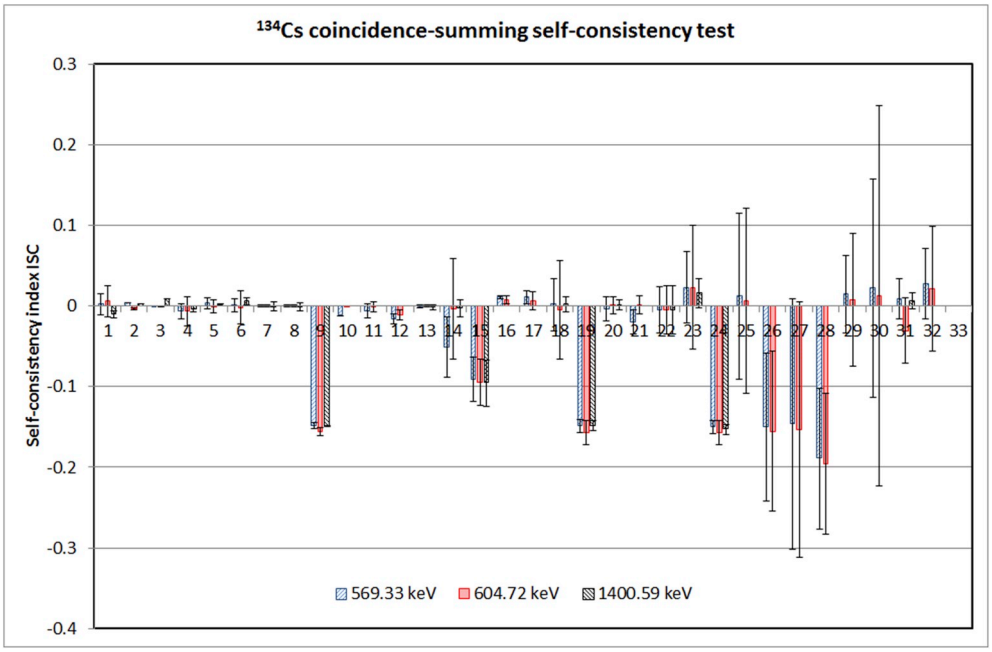

Fig. 4. Self-consistency index for several peaks of ^{134}Cs. The values of ISC(NP) for the 569.33 keV and 604.72 keV peaks, and of ISC(SP) for the 1400.59 keV peak are displayed in function of the code of the participants.

specific case and in fact the relative bias of x_c can be larger than ISC.

In the following, the values of the self-consistency index for coincidence summing effects, ISC(NP) for usual peaks ($x = 1\text{-}F_C$) and ISC(SP) for pure sum peaks ($x = F_C$), as well as the index ISC(AE) for the apparent efficiency ($x = \varepsilon^{app}$), will be presented.

In Figs. 2–5, the distribution of the self-consistency index is displayed for selected peaks of each nuclide considered. Most values of the self-consistency index are around 0, but there are several values around −0.15, clearly violating the self-consistency. According to Equations (7) and (9), ISC(NP) and ISC(SP) depend both on the values of F_C and on the values of the peak efficiencies. However, the evaluation of the self-consistency index for the peak efficiencies demonstrates that with the exception of the set 30, for which the index is around 0.034 in the energy range from 121 to 1408 keV, in all the other cases the values of the peak efficiencies are self-consistent to a high degree. Thus, if ISC(NP) and ISC (SP) are significantly different from zero, this is due to the method applied for the evaluation of F_C, and not of the efficiencies. It should be mentioned that the sets with code numbers 9, 19, 24, 26, 27 and 28 were obtained using the quasi-point source approximation, either deliberately, or because the software requires simply the efficiency values for the complete volume source. In Table 2 the magnitude C of the coincidence-summing effects obtained when the quasi-point source approximation was deliberately applied (specific versions of GESPECOR, ETNA and SUMCOR) are compared with the results obtained

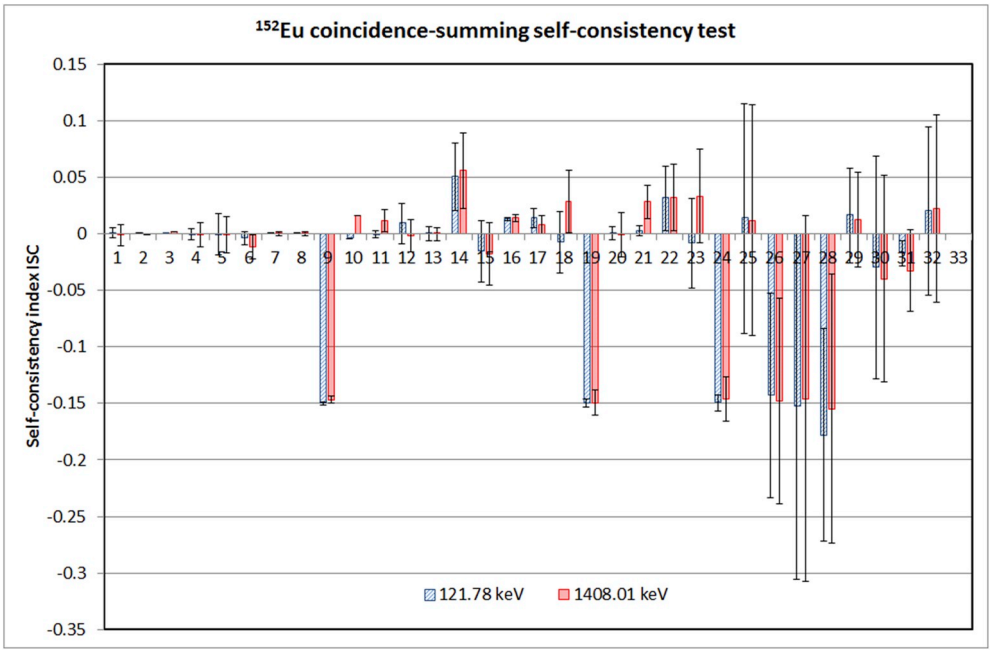

Fig. 5. Self-consistency index for ^{152}Eu peaks of energy 121.78 keV and 1408.01 keV. The values of ISC(NP) are displayed in function of the code of the participants.

Table 2
The magnitude of the coincidence-summing effects C(QPS) calculated using the quasi-point source approximation compared with the values (C) obtained by applying the volume averaging of the effects. The values displayed are (C(QPS)-C)/C, where C(QPS) and C are computed with the same software: GESPECOR, ETNA (with efficiencies evaluated by PENELOPE), SUMCOR with efficiencies evaluated by MCNP (SMC1) and SUMCOR with efficiencies evaluated by PENELOPE (SMC2). C = 1-F_C in the case of the usual peaks, C=F_C in the case of pure sum peaks.

Nuclide	E_i(keV)	Geometry a				Geometry b				Geometry c			
		GESP	ETNA	SMC1	SMC2	GESP	ETNA	SMC1	SMC2	GESP	ETNA	SMC1	SMC2
^{60}Co	1173.23	−0.100	−0.099	−0.069	−0.129	−0.043	−0.043	−0.017	−0.053	−0.237	−0.238	−0.245	−0.245
^{60}Co	1332.49	−0.100	−0.101	−0.069	−0.132	−0.044	−0.045	−0.017	−0.055	−0.236	−0.239	−0.244	−0.244
^{134}Cs	569.33	−0.088	−0.095	−0.118	−0.093	−0.044	−0.045	−0.041	−0.040	−0.218	−0.231	−0.166	−0.220
^{134}Cs	604.72	−0.096	−0.098	−0.128	−0.102	−0.046	−0.046	−0.042	−0.041	−0.232	−0.236	−0.178	−0.232
^{134}Cs	795.86	−0.096	−0.100	−0.130	−0.104	−0.043	−0.046	−0.043	−0.043	−0.233	−0.238	−0.180	−0.235
^{133}Ba	276.40	−0.070	−0.077	−0.075	−0.045	−0.045	−0.048	−0.058	−0.046	−0.195	−0.209	−0.209	−0.171
^{133}Ba	356.01	−0.082	−0.105	−0.087	−0.054	−0.048	−0.049	−0.061	−0.048	−0.216	−0.232	−0.232	−0.191
^{152}Eu	121.78	−0.080	−0.084	−0.055	−0.062	−0.049	−0.049	−0.038	−0.059	−0.215	−0.221	−0.182	−0.226
^{152}Eu	1408.01	−0.081	−0.087	−0.057	−0.064	−0.046	−0.048	−0.036	−0.057	−0.212	−0.224	−0.181	−0.224
^{60}Co	2505.75	−0.099		−0.054	−0.122	−0.040		−0.026	−0.040	−0.232		−0.228	−0.241
^{134}Cs	1400.59	−0.094		−0.122	−0.098	−0.051		−0.050	−0.046	−0.224		−0.164	−0.226

with the same software using the averaging over the volume of the sources. As expected, the relative difference between the values calculated using the two procedures is highest in the case of the source with larger dimensions (geometry c) and smallest in the case of the source in geometry b (same volume as in geometry a, but a weaker relative dependence of the efficiencies on the position of the emission point due to longer distance to the detector). In the case of the sets of results obtained using the quasi-point source approximation, the self-consistency index is around −0.15, but the bias (with respect to volume integration) of the results for geometry c is around −0.23; thus, the fact that also the results for geometries a and b are biased, results in a bias higher than ISC in geometry c. The conclusion that the true bias of the results in geometry c is larger than ISC is valid, even if the reference values from Table 2 might also have a small bias.

In a correct approach to the computation of F_C the self-consistency index should be zero. If this condition is not met, certainly there should be some approximations and inconsistencies in the computational method. Therefore it is interesting to correlate the computed magnitude of the coincidence-summing effects (*1-F_C* for usual peaks and F_C for pure sum peaks) for the case of configuration (a) with the self-consistency index. Several examples are given in Figs. 6–8. It can be observed that generally many points belong to one of two groups, the first around ISC = 0 (self-consistency fulfilled) and the other around ISC = −0.15 (quasi-point source approximation). In the group around ISC = 0, the magnitude of coincidence-summing effects for a given energy is roughly the same, whereas in the group around ISC = −0.15, the values are spread over a wider range. In addition, there are also points that do not belong to any of these two groups, as evident in Fig. 7. In these cases the decay schemes (^{133}Ba and ^{152}Eu) are more complex and X-rays contributions to the coincidence effects are significant. The differences between specific computation codes in the description of the X-rays contributions, as well as the possible differences in the decay data libraries, might be responsible for these points being located apart from the two groups.

From the point of view of activity calculation, the most important quantity characterizing the measurement is the apparent full energy peak efficiency ε^{app}. Generally, for usual peaks the index ISC(AE) is not higher than 4–5%; in most cases it is close to 0. This is due to the fact that ε^{app} for normal peaks is equal to the product $F_C\varepsilon$. Because F_C is relatively small with respect to 1, even if the relative bias of *1-F_C* is about 15%, as in the case of quasi-point source approximation, the effect on F_C is much smaller and consequently, the relative bias introduced in the value of

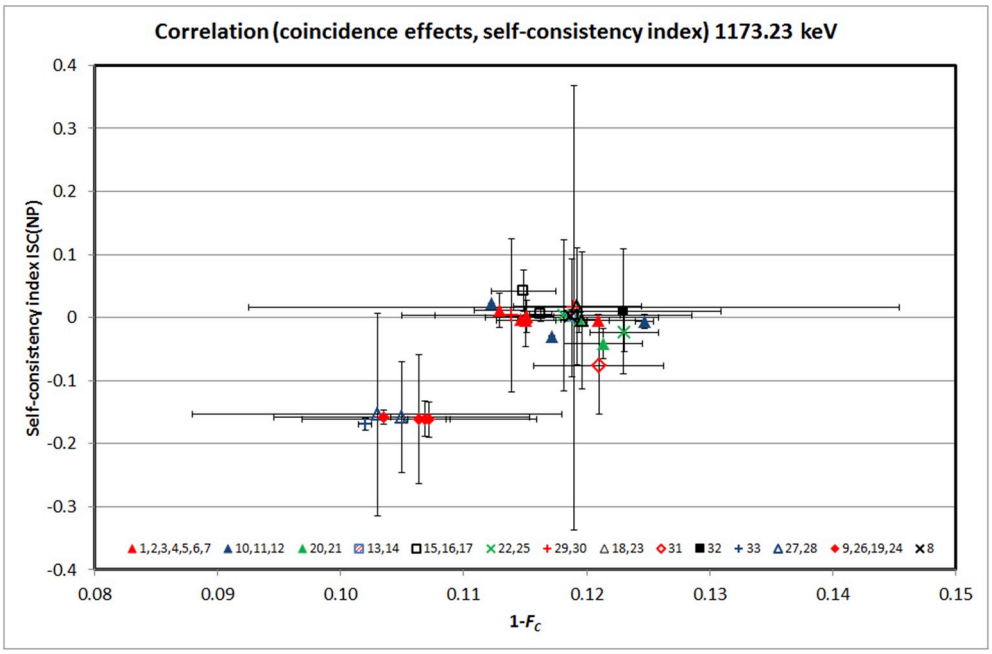

Fig. 6. The distribution of the self-coincidence index as a function of the magnitude C of coincidence summing effects in configuration (a) for the 1173.23 keV peak of ^{60}Co.

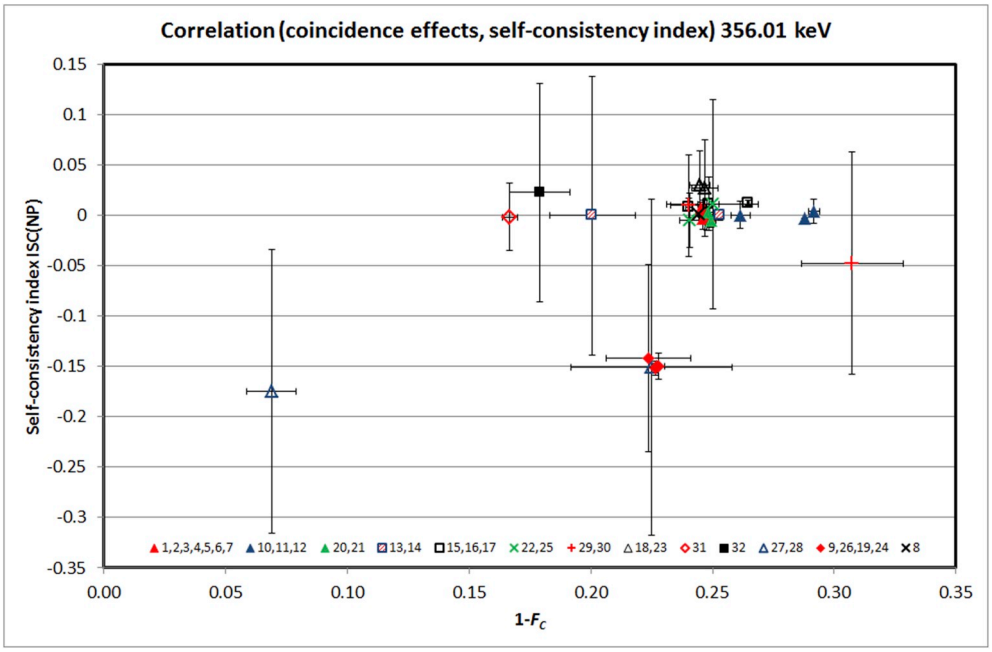

Fig. 7. The distribution of the self-coincidence index as a function of the magnitude C of coincidence summing effects in configuration (a) for the 356.01 keV peak of ^{133}Ba.

ε^{app} is also small. Thus F_C and ε^{app} for usual peaks are weakly sensitive to the approximations applied in the computation. This is in contrast with the case of pure sum peaks, for which the value of ISC(AE) is about the same as ISC(NP), around −0.15, when the quasi-point source approximation is applied in the computation. In Fig. 9 the self-consistency index of the apparent efficiencies is represented as a function of energy, but keeping only the results of the sets for which at least one absolute value of ISC(AE) is higher than 0.05.

Concerning the uncertainties represented in the figures, they correspond to one standard deviation. The values were computed on the basis of the data reported by the participants using uncertainty propagation formula. In this context, the uncertainties of ISC(NP) and ISC(SP) should be interpreted with some caution, because they were computed assuming that the values of F_C and of ε are not correlated. This is true in the case of some sets of reported results, but not in all cases; however, as the covariance between F_C and ε was not provided by the participants, it was always disregarded.

The proposed self-consistency test may reveal the existence of some problems in the method for the evaluation of the coincidence-summing effects. However, it may be insensitive to some features of the computation method, even if they imply specific approximations and led to biased results. A simple example is the following: if all efficiencies are

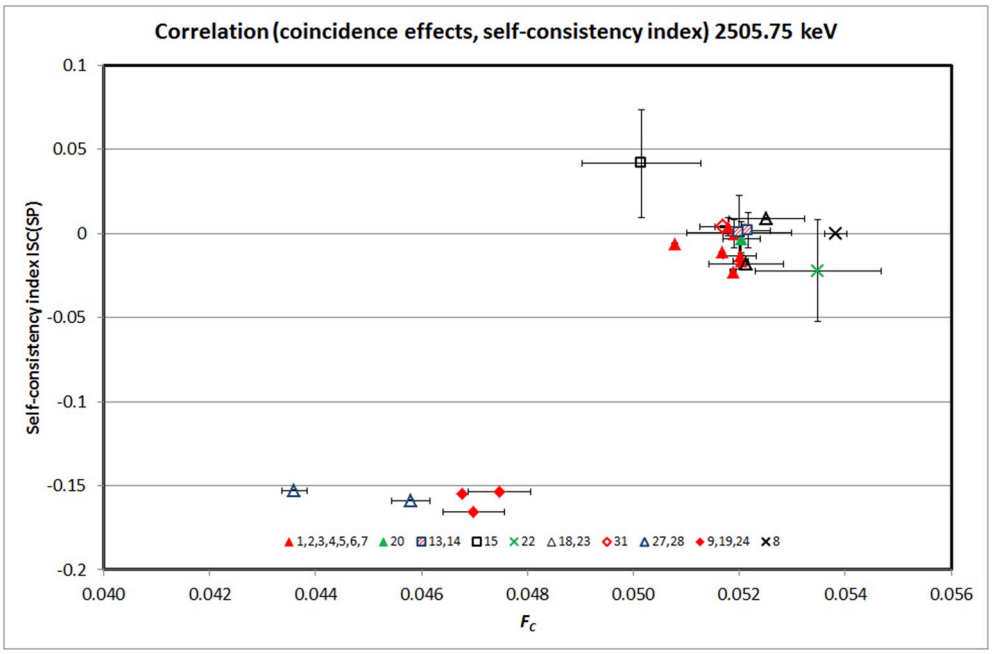

Fig. 8. The distribution of the self-coincidence index as a function of the magnitude C of coincidence summing effects in configuration (a) for the pure sum peak of ^{60}Co at 2505.75 keV.

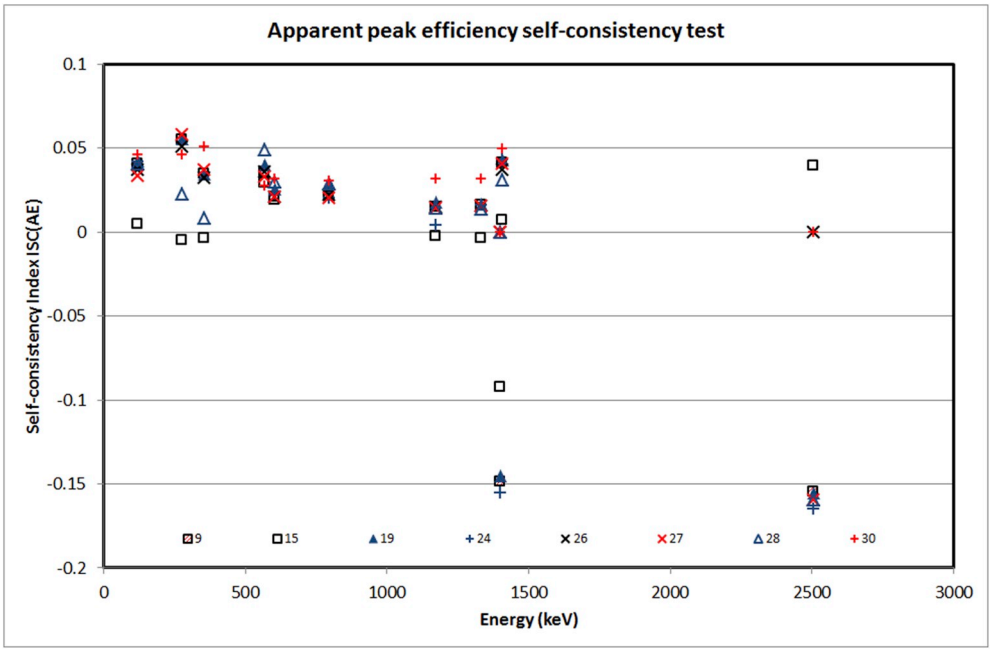

Fig. 9. The self-consistency index for the apparent peak efficiency for the sets of results in which at least one absolute value of ISC(AE) > 0.05. The uncertainty bars were removed for avoiding a too clumsy figure.

multiplied by the same scale factor, the value of ISC for the peak efficiency remains unchanged. Also, ISC(NP), ISC(SP) and ISC(AE) are insensitive to a scale factor applied to all efficiencies if only pair coincidences contribute to summing effects. A more complex example is provided by the angular correlation effects. These effects were included in one set of results. This set passed the self-consistency test, as well as many sets in which angular correlations were neglected. In Table 3 the effect of angular correlations on F_C is presented. Of course, the discrepancies with respect to isotropy are higher in the case of geometry b, corresponding to the smallest solid angle. In the case of pure sum peaks, it is higher than 7% in geometry b, but it is not negligible also in the other geometries. The effect on F_C for usual peaks is much lower, but the effect on $1\text{-}F_C$ for these peaks is almost the same as in the case of pure sum peaks. The small difference between the effect on $1\text{-}F_C$ for 1173 and 1332 keV and on F_C for 2505 keV, all subjected to the same angular correlations, is due to the integration over a larger solid angle in the first case (higher effective solid angle for any interaction than for complete energy deposition in the detector).

6. Summary and conclusions

A self-consistency test to check some features of several procedures

Table 3
Impact of neglecting angular correlations. F_C(ISOT) calculated assuming isotropy of each radiation, F_C(AC) calculated including the angular correlation of radiations emitted in cascade.

Nuclide	E_i(keV)	F_C(ISOT)/F_C(AC)-1			(1-F_C(ISOT))/(1-F_C(AC))-1		
		Geometry a	Geometry b	Geometry c	Geometry a	Geometry b	Geometry c
^{60}Co	1173.23	−0.004	−0.003	−0.004	0.032	0.073	0.037
^{60}Co	1332.49	−0.004	−0.003	−0.004	0.032	0.073	0.037
^{134}Cs	604.72	−0.006	−0.005	−0.006	0.032	0.080	0.038
^{134}Cs	795.86	−0.007	−0.005	−0.006	0.036	0.084	0.042
^{60}Co	2505.75	0.037	0.076	0.042			
^{134}Cs	1400.59	0.036	0.075	0.041			

applied for the evaluation of coincidence-summing corrections in the case of extended sources was proposed. The test is based on exact relations between the values of F_C and of the peak efficiencies calculated for three ideal measurement conditions. These relations should be valid in the case of correct procedures for the evaluation of F_C. The test parameter proposed is the index of self-consistency ISC, equal to 0 if the relations are valid. A value of ISC significantly different from 0 suggests that there are some problems in the computation of the coincidence-summing effects and indicates qualitatively the magnitude of the bias of the results.

The main purpose of this GSWG action was to provide to each user of gamma-ray spectrometry a simple and convincing test of the quality of the procedure applied in his laboratory for the computation of coincidence-summing corrections. The test is particularly useful for emphasizing the limitations of the quasi-point source approximation in the calculation of coincidence-summing corrections for extended sources. In the test, vacuum (or air) sources are considered, whereas actual matrices of the sources present non-negligible photon interaction probabilities; in such measurements the expected bias is higher, because efficiency dependence on the position of the emission point should be stronger than in the case of vacuum sources (Arnold and Sima, 2001).

As with any test, the hypothesis of correctness of the procedure can be ruled out if ISC significantly differs from 0, but the fact that ISC is close to 0 is not a proof of the correctness of the procedure. Therefore the proposed test should be complemented by other tests, including experimental validation of the results.

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