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The correlation matrix for the effective delayed neutron parameters of the IPEN/MB-01 reactor

Adimir dos Santos *, Ricardo Diniz

Instituto de Pesquisas Energéticas e Nucleares – IPEN/CNEN-SP, Av. Prof. Lineu Prestes, 2242 – Cidade Universitária, SP 05508-000, Brazil

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ABSTRACT

The correlation matrix for the effective delayed neutron parameters of the IPEN/MB-01 reactor has been successfully built in this work. A standard procedure employed in several least-squares approaches was adopted to cope with this task. One of the most important applications of the effective delayed neutron parameters is to serve as input data for the relationship between reactivity and asymptotic period of a nuclear reactor given by the Inhour equation. Employing this equation, the reactivity was calculated for several periods both negative and positive. The reactivity error propagation was considered with and without the correlation matrix. The analyses of this procedure reveal that the consideration of the correlation matrix is very important. In general, the introduction of the correlation matrix reduces the overall uncertainty by a reasonable amount. There are huge cancellations in the uncertainty analyses. The analyses also reveal that the uncertainty in the reactivity will depend on the specific period range where this quantity is considered. Considering the correlation and for most of the period range considered, the reactivity uncertainties for negative periods are around 3.5% while for positive ones they are nearly 2%. If the correlation matrix is not considered the reactivity uncertainties are as high as 6%. All uncertainties considered in this work are 1-σ values. There are two extreme cases where the correlation matrix plays no role; very large negative periods and very small positive periods. In the first case, the first decay constant plays a fundamental role while in the second the prompt neutron generation is of major importance. The experiments performed at the IPEN/MB-01 reactor and evaluated for inclusion in the IRPhE handbook for the determination of the effective delayed neutron parameters will be extremely benefited considering the inclusion of their correlation matrix. The final product can be considered extremely useful to validate methods and nuclear data related to the reactivity determination of thermal reactors fueled with Uranium.

1. Introduction

The delayed neutron nuclear data enjoyed a long history in the nuclear science and technology area and their importance has been recognized since the first experimental atomic pile in Chicago, which took place on December 2, 1942 (Fermi, 1946). Since the pioneer work of Keepin et al. (1957), a worldwide effort, both experimental (Spriggs, 1993; Diniz, 2005; Diniz and Dos Santos, 2006; Dos Santos et al., 2006a,b) and theoretical (Progress in Nuclear Energy, 2002; Chiba et al., 2015; Gremyachkin et al., 2015; Foligno and Leconte, 2018), has been made in order to establish a consistent set of delayed neutron kinetic parameters. In a nuclear chain reaction, there are many fission products (approximately 270) which can be considered potential delayed neutron emitters. An experimental characterization of all these emitters is very difficult due to their low yield and/or low half-lives and also due to their very complex transmutation chains. However, for reactor calculation purposes only the effective aggregate behavior of delayed neutrons is important. This aggregate behavior is obtained in a few group model where the decay constants and abundances are mean values of various emitters with similar decay constants. These groups have no true physical basis (except the first group decay constant) but instead they are originated from the fits to measured delayed neutron decay curves following a fission pulse. A six-group model first introduced by Keepin et al. (1957) in 1957 was considered a standard for many years and these group parameters have been incorporated into several nuclear data libraries. Nowadays, however, it has been proposed an

* Corresponding author at: Instituto de Pesquisas Energéticas e Nucleares – IPEN/CNEN-SP, Centro de Engenharia Nuclear (CEN), Av. Prof. Lineu Prestes, 2242, 05508-000 – Cidade Universitária, São Paulo, SP, Brazil.

E-mail address: adsantos@usp.br (A. dos Santos).

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eight-group delayed neutron model based on a consistent set of half-lives (Spriggs et al., 2002) where the known dominant precursors (^{87}Br, ^{137}I and ^{88}Br) would have their half-lives fixed into the model. A higher-order delayed neutron model seems to be a worldwide tendency because there is a better physical basis than the old standard six-group model of Keepin, although this last one is still widely used.

The current status for the delayed neutron nuclear data for thermal reactor fueled with enriched uranium (Dos Santos and Diniz, 2014) is that an agreement can be reached in the determination of the effective delayed neutron fraction (β_{eff}) employing the recently release nuclear data libraries but there are several disagreements for the relative abundances in a six-group model as well as in the determination of the reactivity of a multiplying system. In general, it seems that there is no good agreement among the current nuclear data libraries, (Snoj et al., 2010; Meulekamp and van der Marck, 2017; Henry et al., 2015), ENDF/B-VII.1 (Kahler et al., 2011), JENDL4.0 (Shibata et al., 2011) and the JEFF3.1.1 (Santamarina et al., 2009). The parameters λ_i; the decay constant and β_i; the delayed neutron fraction both for the delayed neutron family *i* for ^{235}U thermal fission for example, are different in these libraries. The β_{eff} parameter is an exception and shows somehow a good agreement among these libraries. However as stated in (Dos Santos and Diniz, 2014) the agreement of β_{eff} is not a guarantee of a good agreement in the reactivity determination and, in fact, reactivity calculations from Inhour equation (Bell and Glasstone, 1979) using the group parameters of these libraries show a systematic deviation among them. The ENDF/B-VII.1 delayed neutron parameters underestimate the reactivity (Kahler et al., 2011) by around 12% in comparison to JENDL 4.0 although its previous release ENDF/B-VII.0 (Chadwick et al., 2006) gives better results. This improvement of the reactivity results of the ENDF/B-VII.0 library is mainly due to the first decay constant which are significantly different from its previous release. JENDL 4.0 and JEFF3.1.1 show similar results. The previous version of JENDL, JENDL 3.3 (Shibata et al., 2002) shows also excellent agreement for the reactivity determination. Although considerable improvements have been obtained in characterizing these delayed neutron parameters, some lack of consistency and/or agreement still remains.

The available experimental support to validate methods and nuclear for the determination of the effective delayed neutron parameters and consequently the reactivity is scarce and in many cases of very difficult utilization. The lack of specific benchmarks to verify the quality of the reactivity determination is a severe problem in the reactor physics area.

An attempt to partially fulfill this need was proposed and approved for inclusion in the IRPhE handbook in 2008 (Dos Santos et al., 2012). This proposal benchmark concerns thermal reactors fueled with uranium fuel slightly enriched in ^{235}U and considers several experiments performed in the IPEN/MB-01 reactor. The proposed benchmark considers all delayed neutron effective parameters measured in a six-group delayed neutron groups. The estimate uncertainty is ~6% for negative reactivity and ~4% for positive reactivity. No correlation among the parameters λ_i and β_i were considered in this evaluation.

The purposes of the present work are to determine the correlation matrix for the parameters λ_i and β_i in the IPEN/MB-01 benchmark experiment and to analyze its impact in the determination of the uncertainty in the reactivity. The analysis of the impact of the introduction of the correlation matrix in the reactivity determination will be performed employing the Inhour equation (Bell and Glasstone, 1979) Initially a brief overview of the experimental approaches developed at IPEN for the determination of the effective delayed neutron parameters is presented.

2. The delayed neutron experiments performed at the IPEN/MB-01 reactor

The experiments performed in the IPEN/MB-01 reactor are described in full details in (Diniz, 2005; Diniz and Dos Santos, 2006; Dos Santos et al., 2006a,b; Kuramoto et al., 2006). (Diniz, 2005; Diniz and Dos Santos, 2006) describe with exceptions to the first decay constant and the prompt neutron generation time, the measurements of a totally experimental effective delayed neutron data set in a six-group model employing a macroscopic noise, in a low frequency range. The quantities measured in this experiment were of the CPSD (Cross Power Spectral Density) and the APSD (Auto Power Spectral Density). (Dos Santos et al., 2006b) describes the measurements of the first decay constant employing the Spriggŝs Method (Spriggs, 1993). (Kuramoto et al., 2006) and (Dos Santos et al., 2006a) describe the measurements of the prompt neutron generation time (Λ) employing both microscopic and macroscopic noise. The totally experimental effective delayed neutron parameters are shown here considered only experimental data. No corrections of any type were applied to these final experimental values. This whole set of experiments was combined and evaluated in a single set of effective delayed neutron parameters in (Dos Santos et al., 2012).

2.1. The CPSD and APSD measurements

The major experiment performed in the IPEN/MB-01 for this work is the one considering the measurements of the delayed neutron decay constants and the abundances in a six-group model. This experiment is described in a full extent in (Diniz, 2005) and (Diniz and Dos Santos, 2006). Here, only a few details will be considered. The core configuration and the detector positions are shown in Figs. 1 and 2 respectively. The standard configuration

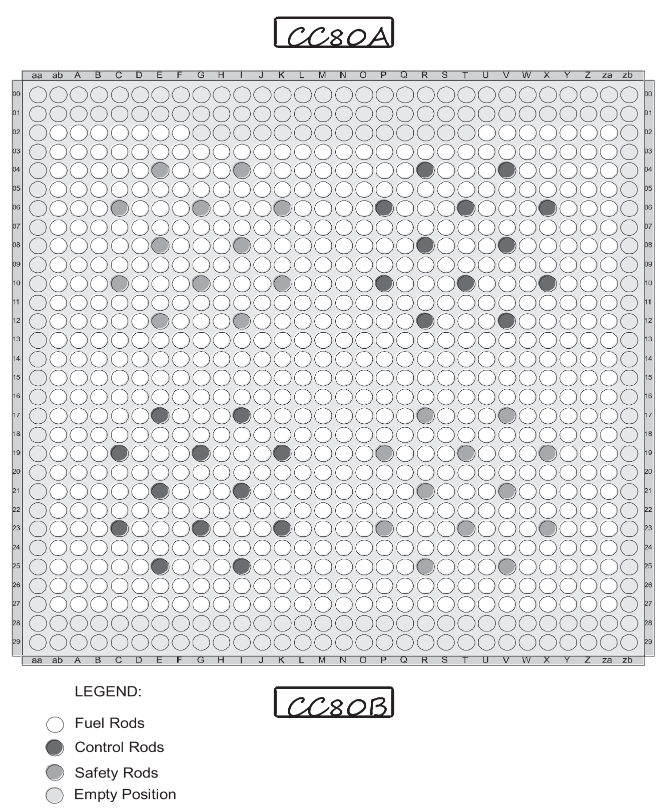

Fig. 1. Core configuration for spectral densities measurements.

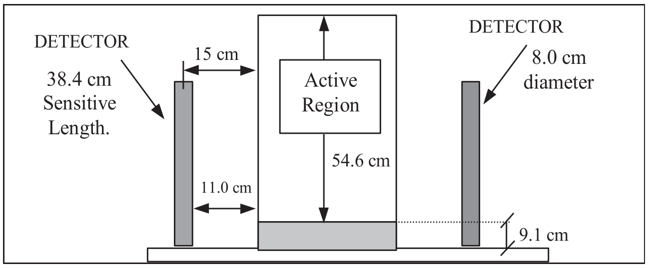

Fig. 2. Side view of the active region and the detectors positioning in the West and East faces of the core. In these conditions, the ionization chambers are in the reflector region approximately 8.0 cm away from the thermal neutron peak due to the reflector effect.

of 28 × 26 square array of UO_2 fuel rods were employed to cope with this task. Two compensated ionization chambers (CC80 from Merlin-Gerin) operating in current mode together with associated electronic equipments and a DSA (Dynamic Signal Analyzer) were employed to cope with this task. The measured quantities were the APSD for each detector and the CPSD of both detectors. The experiment was performed with the reactor as close to the critical condition and at a thermal power of 4.0 W. In the course of the data acquisition, the control rods were "frozen" in order to avoid the interference of its movement in the low frequency region. Eventually the power level may begin to change and in this case, the data acquisition is stopped and the power is restored either manually or with one of the control banks returning to the automatic mode. In both cases a waiting time of at least 2 min must be elapsed before the data acquisition starts again. Also, the ventilation system and the water level pump were turned off during the experiment in order to have a minimum electrical interference and a minimum heat exchange with the external environment.

The spectral densities measurements were performed in two steps: from 0 to 3.125 Hz with 1600 lines of resolution, and from 2 to 52 Hz with 800 lines of resolution. For the step from 0 to 3.125 Hz and 1600 lines of resolution, the first two points (1.95 mHz and 3.91 mHz) are not reliable because the low-frequency cutoff (1 mHz) of the high pass filter is not sharp, and these two points present some distortion. The third point occurs at 5.86 mHz, and thus there are only four points before the frequency associated with the first decay constant of delayed neutrons. This will impose some restriction on the first parameters (λ_i and β_i) to be fitted through the least-squares procedure.

The APSD's and the CPSD (for the 0–3.125 Hz band) were obtained with 104 sets of partial averages (each one having a different number of averages) performing a total of 1000 averages. The weighted mean of the partial sets gives the final 1000 averages spectral densities. For the 2–52 Hz band, the 1000 averages were obtained in only one step.

2.1.1. The least-squares approach

The least-square approach considered in this section was performed only for the CPSD fit. In order to get a set of totally experimental delayed neutron parameters, the first decay constant λ_i which was taken from Ref. 6 was kept fixed throughout the fitting procedure. Six groups of delayed neutrons were considered throughout of the least square process. Also, the least-squares procedure took into account the error bars of the spectral densities, which were obtained as the standard deviation of the mean,

$$\sigma\left(\bar{x}\right) = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \quad (1)$$

where n is the number of partial averages sets (104 sets for the interval from 0 to 3.125 Hz), x_i is the i-th value of the spectral density for a given set and a given frequency, and $\bar{x}$ is the final weighted mean value for the entire set and a given frequency. Thus, through Eq. (1), each point of the APSD's has an error bar of 3.4% of its value, and the CPSD an error bar of 3.7%. The same percentage values were used as error bars for the interval 2–52 Hz.

Written in FORTRAN language, the program that performs the least-squares procedure needs the partial derivatives of the spectral densities with respect to the six β_i and the six λ_i. These parameters are present only in the squared modulus of the zero power transfer function, $|G(f)|^2$. The other terms present in the spectral densities are constants.

The least-squares procedure is as follow:

1- The λ_i from the multiple transient technique experiment 9 (Dos Santos et al., 2006b) is kept fixed; λ_i = 0.012456 ± 0.0.000031 s^{-1}.
2- The procedure is iterative between the two sets $\beta_1 \ldots \beta_6$) and ($\lambda_2 \ldots \lambda_6$) with β_1 and λ_i kept fixed. The iterative process is repeated until there are no more changes in the two sets. The initial guess for ($\beta_2 \ldots \beta_6$), including β_1, and ($\lambda_2 \ldots \lambda_6$) was a simple mean of the values presented for three nuclear data libraries, namely ENDF/B-VI.8, a revised version of ENDF/B-VI.8 made at LANL (LAN review, UEVAL home page http://www.nea.fr/lists/ueval), and JENDL 3.3.
3- Next, the β_1 parameter alone is left free for fitting, with ($\beta_2 \ldots \beta_6$) and ($\lambda_2 \ldots \lambda_6$) kept fixed.
4- With the new value for β_1, the step (2) is repeated.
5- When the variations in the parameters between steps (2)–(4) no longer occur, the process is terminated.

2.2. The final values of the effective delayed neutron parameters

The final results for the inferred effective delayed neutron parameters are given in Table 1. They are a combination of the experiments described in Section 2.1 for β_1 through β_6 and for λ_2 through λ_6, the experiment for the determination of the first decay constant λ_i, described in Ref. 6 and the experiments for the determination of the prompt neutron generation time Λ described in (Dos Santos et al., 2012). All uncertainties in this table are 1-σ values and they arise from the least square approach employed in the analyses. Following the procedure described in the U.S. Guide to the Expression of Uncertainties in Measurement (ANSI/NCSL, 1997) these uncertainties can be classified as type A because the CPSD uncertainty is of statistical nature.

The data given in Table 1 are the necessary benchmark parameters needed to make in the link between reactivity and reactor period. The Inhour equation gives this relationship and when the data of Table 1 are employed the calculated results are benchmark values for the reactivity determination. Some suggested values for the reactor period are suggested in (Dos Santos et al., 2012). The user of the IPEN/MB-01 reactivity benchmark should go through the following steps. Initially, the user should select his period range of interest and calculate the benchmark reactivity values as

Table 1
The final values of the effective delayed neutron parameters.

β_i	λ_i (s^{-1})
(2.679 ± 0.023)E−4	0.012456 ± 0.000031
(1.463 ± 0.069)E−3	0.0319 ± 0.0032
(1.34 ± 0.13)E−3	0.1085 ± 0.0054
(3.10 ± 0.10)E−3	0.3054 ± 0.0055
(8.31 ± 0.62)E−4	1.085 ± 0.044
(4.99 ± 0.27)E−4	3.14 ± 0.11
Λ = 31.96 ± 1.06 µs.	

described previously. Next, the user should model the IPEN/MB-01 core and calculate the direct and adjoint neutron fluxes employing his code and related neutron data set. Next, the user should calculate the effective delayed neutron parameters employing the direct and adjoint neutron fluxes from the previous step. Sequentially, the user should calculate the reactivity in his specific period range using the Inhour equation and with his effective delayed neutron set from the previous step. The comparison of his calculated reactivity values to the respective benchmark values gives the quality of his delayed neutron set. An uncertainty analysis should be taken into consideration in these analyses.

3. The correlation matrix for the effective delayed neutron parameters

The determination of the correlation matrix for the delayed neutron parameters follows the standard procedure employed in several least-squares approaches (Smith, 1991; Bevington and Robinson, 1969). The correlation matrix is defined as:

$$Corr_{\phi_{12}}(i,j) = \frac{Cov_{\phi_{12}}(i,j)}{\sqrt{Cov_{\phi_{12}}(i,i)} \cdot \sqrt{Cov_{\phi_{12}}(j,j)}} \tag{2}$$

where $Cov_{\phi_{12}}(i,j)$ is the element (i,j) of the covariance matrix for the CPSD of detectors 1 and 2 (ϕ_{12}).

The covariance matrix is given by:

$$Cov_{\phi_{12}} = \left(S^{+} \cdot V^{-1} \cdot S\right)^{-1} \tag{3}$$

where S^{+} is the transpose of matrix S, and V^{-1}represents the inverse of the covariance matrix for the experimental data. The matrix V^{-1} is given by:

$$V^{-1} = \begin{bmatrix} \frac{1}{\sigma^2_{\phi_{12}}(\omega_1)} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma^2_{\phi_{12}}(\omega_2)} & 0 & \cdots & 0 \\ 0 & 0 & \frac{1}{\sigma^2_{\phi_{12}}(\omega_3)} & \cdots & 0 \\ \vdots & & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{\sigma^2_{\phi_{12}}(\omega_N)} \end{bmatrix}, \tag{4}$$

where $\sigma_{\phi_{12}}(\omega_i)$ represents the measured CPSD uncertainty for a generic frequency ω_i, N is the total number of frequencies considered in the CPSD measurements. The matrix S is given by:

$$S = \begin{bmatrix} \frac{\partial\phi_{12}}{\partial\beta_1}|\omega = \omega_1 & \cdots & \frac{\partial\phi_{12}}{\partial\beta_6}|\omega = \omega_1 & \frac{\partial\phi_{12}}{\partial\lambda_2}|\omega = \omega_1 & \cdots & \frac{\partial\phi_{12}}{\partial\lambda_6}|\omega = \omega_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial\phi_{12}}{\partial\beta_1}|\omega = \omega_N & \cdots & \frac{\partial\phi_{12}}{\partial\beta_6}|\omega = \omega_N & \frac{\partial\phi_{12}}{\partial\lambda_2}|\omega = \omega_N & \cdots & \frac{\partial\phi_{12}}{\partial\lambda_6}|\omega = \omega_N \end{bmatrix} \tag{5}$$

with the derivatives of the CPSD (ϕ_{12}) taken to respect of β_i and λ_i; i ranging from 1 through 6 for the abundances (β_i) and from 2 through 6 for the decay constant (λ_i).

The CPSD employed in the determination of the correlation matrix for the effective delayed neutrons is given by:

$$\phi_{12}(f) = 2D\frac{\gamma}{P\Lambda^2} I_1 I_2 |G(f)|^2 [He_1(f) He_2(f)*][Hf_1(f) Hf_2(f)*] \tag{6}$$

where D is the Diven factor (Diven, 1956), γ is the energy released per fission (3.2×10^{-11} J), I_k and I_1 are the detector currents, $G(f)$ is the zero-power reactor transfer function, $He(f)$ is the transfer function of the electrometers, $He(f)$ the transfer function of the filter-amplifiers, $\bar{q}$is the mean electric charge released per detected neutron, P is the reactor power, and Λ is the prompt neutron generation time. The symbol (*) means complex conjugation. The constant term in the Eq. (6) i.e.,$2D\frac{\gamma}{P\Lambda^2} I_k{}^2 |He_k(f) Hf_k(f)|^2$, was grouped together in a single variable. This constant term does not need to be taken into consideration in the correlation matrix since it will be cancelled in the process.

The zero-power transfer function, $G(f)$, is given by Wallerbos and Hoogenboom (1998):

$$G(f) = \frac{\Lambda}{j2\pi f\Lambda - \rho + \sum_{i=1}^{6} \frac{j2\pi f \beta_i}{(j2\pi f + \lambda_i)}} \tag{7}$$

where ρ is the reactivity, Λ is the prompt neutron generation time, f is the frequency in Hz and $j = \sqrt{-1}$.

The derivatives of the CPSD (ϕ_{12})with respect to β_n and λ_n involves only the derivative of the transfer function $G(f)$ and there are given by:

$$\frac{\partial|G(f)|^2}{\partial\beta_n} = \frac{-\Lambda^2\left\{\left[-32\pi^2 f^2 \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i^2 + 4\pi^2 f^2} \cdot D_n\right] + E_n\left[8\pi f\Lambda + 8\pi f \sum_{i=1}^{6} \frac{\beta_i\lambda_i}{\lambda_i^2 + 4\pi^2 f^2}\right]\right\}}{\left[\left(4\pi^2 f^2 \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i^2 + 4\pi^2 f^2}\right)^2 + \left(2\pi f\Lambda + 2\pi f \sum_{i=1}^{6} \frac{\beta_i\lambda_i}{\lambda_i^2 + 4\pi^2 f^2}\right)^2\right]^2} \tag{8}$$

where $D_n = \frac{\pi^2 f^2}{\lambda_n{}^2 + 4\pi^2 f^2}$ and $E_n = \frac{\pi f \lambda_n}{\lambda_n{}^2 + 4\pi^2 f^2}$, with n = 1, 2,. . .6

$$\frac{\partial|G(f)|^2}{\partial\lambda_n} = \frac{-\Lambda^2\left\{\left[-64\pi^2 f^2 \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i^2 + 4\pi^2 f^2}\right] A_n + 2\left[2\pi f\Lambda + 2\pi f \sum_{i=1}^{6} \frac{\beta_i\lambda_i}{\lambda_i^2 + 4\pi^2 f^2}\right][B_n - C_n]\right\}}{\left[\left(4\pi^2 f^2 \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i^2 + 4\pi^2 f^2}\right)^2 + \left(2\pi f\Lambda + 2\pi f \sum_{i=1}^{6} \frac{\beta_i\lambda_i}{\lambda_i^2 + 4\pi^2 f^2}\right)^2\right]^2} \tag{9}$$

where $A_n = \frac{\pi^2 f^2 \beta_n \lambda_n}{\left(\lambda_n{}^2 + 4\pi^2 f^2\right)^2}$, $B_n = \frac{2\pi f \beta_n}{\lambda_n{}^2 + 4\pi^2 f^2}$ and $C_n = \frac{4\pi f \beta_n \lambda_n{}^2}{\left(\lambda_n{}^2 + 4\pi^2 f^2\right)^2}$.

The covariance matrix for the experiment is diagonal since no correlation among the experimental points is taken into consideration. The correlation matrix for the fitted parameters is determined only for the parameters considered in the fitting process as considered in Section 2. The prompt neutron generation time (Λ) and the first decay constant (λ_1) are considered uncorrelated between themselves and among the parameters considered in the fitting process.

Table 2 shows the final result for the correlation matrix.

Some aspects must be noticed in Table 2. β_1 is anti-correlated to all other $\beta's$. With exception to β_1 the other abundances ($\beta's$) are positive correlated. With exception to β_1, the other abundances $\beta's$ and the fitted decay constants are anti-correlated. In particular the pairs (β_i, λ_i), i = 2 through 6 are totally anti-correlated. The fitted decay constants among themselves are positive correlated. These aspects will be of importance for the reactivity uncertainty analyses in the coming section.

4. The inferred reactivities and their corresponding uncertainties

One of the most important applications of the effective delayed neutron parameters is the relation between reactivity and asymptotic period of a nuclear reactor. The reactivity is an inferred quantity and the Inhour equation stablishes this relationship. The Inhour equation is given by:

$$\rho = \frac{\Lambda}{T} + \sum_{i=1}^{ND} \frac{\beta_i}{1 + \lambda_i T} \tag{10}$$

where T is the asymptotic period and ND represents the number of delayed neutron families (six in the case under consideration) and ρ

Table 2
The correlation matrix for the effective delayed neutron parameters.

	Λ	β1	β2	β3	β4	β5	β6	λ1	λ2	λ3	λ4	λ5	λ6
Λ	1	0	0	0	0	0	0	0	0	0	0	0	0
β1	0	1	−0.993	−0.846	−0.725	−0.105	−0.134	0	0.993	0.848	0.732	0.106	0.134
β2	0	−0.993	1	0.901	0.784	0.156	0.169	0	−1	−0.902	−0.79	−0.157	−0.169
β3	0	−0.846	0.901	1	0.942	0.374	0.309	0	−0.902	−1	−0.945	−0.375	−0.309
β4	0	−0.725	0.784	0.942	1	0.621	0.491	0	−0.786	−0.942	−1	−0.622	−0.491
β5	0	−0.105	0.156	0.374	0.621	1	0.89	0	−0.158	−0.374	−0.616	−1	−0.89
β6	0	−0.134	0.169	0.309	0.491	0.89	1	0	−0.17	−0.309	−0.488	−0.891	−1
λ1	0	0	0	0	0	0	0	1	0	0	0	0	0
λ2	0	0.993	−1	−0.902	−0.786	−0.158	−0.17	0	1	0.903	0.792	0.159	0.171
λ3	0	0.848	−0.902	−1	−0.942	−0.374	−0.309	0	0.903	1	0.945	0.375	0.309
λ4	0	0.732	−0.79	−0.945	−1	−0.616	−0.488	0	0.792	0.945	1	0.617	0.488
λ5	0	0.106	−0.157	−0.375	−0.622	−1	−0.891	0	0.159	0.375	0.617	1	0.891
λ6	0	0.134	−0.169	−0.309	−0.491	−0.89	−1	0	0.171	0.309	0.488	0.891	1

is the reactivity. The symbols β_i, λ_i and Λ have the same meaning as before.

The uncertainty analysis of the reactivity inferred from the Inhour equation can be performed by taking into consideration the uncertainties in the effective delayed neutron parameters as:

$$\sigma_\rho = \sqrt{\left(\frac{\partial\rho}{\partial\Lambda}\right)^2 \cdot \sigma_\Lambda^2 + \sum_{i=1}^{ND}\left(\frac{\partial\rho}{\partial\beta_i}\right)^2 \cdot \sigma_{\beta_i}^2 + \sum_{i=1}^{ND}\left(\frac{\partial\rho}{\partial\lambda_i}\right)^2 \cdot \sigma_{\lambda_i}^2 + Cov} \quad (11)$$

where σ_ρ is the uncertainty in the reactivity and:

$$\frac{\partial\rho}{\partial\Lambda} = \frac{1}{T} \quad (12)$$

$$\frac{\partial\rho}{\partial\beta_i} = \frac{1}{(1+\lambda_i \cdot T)}, \quad (13)$$

$$\frac{\partial\rho}{\partial\lambda_i} = -\frac{\beta_i \cdot T}{(1+\lambda_i . T)^2}, \text{ and} \quad (14)$$

$$Cov = 2.\sum_{i>j}^{ND}\frac{\partial\rho}{\partial x_i}.\frac{\partial\rho}{\partial x_j}.\sigma_{x_i}.\sigma_{x_j}.Corr(x_i, x_j) \quad (15)$$

where x_k represents either β_k or λ_k;and k being either i or j represents generic delayed neutron groups and $Corr(x_i, x_j)$ its correlation matrix.

The uncertainties in the effective delayed neutron parameters are given in Table 1. They were utilized in order to get the uncertainties in the inferred reactivity. Table 3 shows the final results for negative periods. The periods cover the range from −500 to −80.5 s. Also, Table 3 shows the results without and with correlations among the fitted delayed neutron parameters.

The first aspect to be noted in Table 3 is the reduction the inferred reactivity uncertainty when correlations among the fitted parameters are considered. This is due mainly to the anti-correlation among β_i and λ_i. The uncertainties increase drastically as the period approaches to its higher allowed negative value which is given by $-1/\lambda_1$; equal to −80.28 s. The reason for that will be shown in the coming table. Several terms contribute to the total reactivity uncertainty in Eq. (10). Due to the difficult to show the contribution of each term individually, the components were grouped into five broad categories: 1) the contribution of ($\beta_i's$) and the correlation among them, 2) the contribution of the correlation between β_i and λ_i, 3) the contribution of the decay constants λ_2 through λ_5 and the correlation among them, 4) the contribution of the first decay constant (λ_1), and 5) the contribution of the prompt neutron generation time (Λ). The contribution of each category was summed together and the final result was then divided by the total uncertainty. Consequently, the sum of the fractions of all category contributions gives 1.0. These data are shown in Table 4 for negative periods.

Now the analyses of the uncertainty in the reactivity become more understandable. Initially, Table 4 shows that there are huge cancellations in the uncertainty analysis. The contribution of β_i and λ_i individually are positive and the contribution of the correlation among these two quantities are negative. These explain good part of the reason why the uncertainty in the inferred reactivity is reduced when correlation is taken into account. Furthermore, the contribution of the prompt neutron generation time (Λ) to the inferred reactivity in the period range considered here is of negligible amount. Finally, as the period approaches its higher allowed value which is given by $-1/\lambda_1$, the uncertainty in the inferred

Table 3
Uncertainties in the inferred reactivity for negative periods.

Period (s)	Reactivity (pcm)	σ_ρ(pcm) No correlation	Uncertainty in units of %	σ_ρ(pcm) correlation	Uncertainty in units of %
−500	−1.97E+01	1.18E+00	−6.00EE+00	5.07E−01	−2.58EE+00
−400	−2.51E+01	1.52EE+00	−6.05EE+00	6.71E−01	−2.67EE+00
−300	−3.48E+01	2.13EE+00	−6.12EE+00	9.85E−01	−2.83EE+00
−200	−5.73E+01	3.56EE+00	−6.22EE+00	1.80EE+00	−3.14EE+00
−100	−2.01EE+02	1.05EE+01	−5.23EE+00	7.06EE+00	−3.51EE+00
−90	−3.28EE+02	1.38EE+01	−4.22EE+00	1.08EE+01	−3.31EE+00
−85	−5.71EE+02	2.53EE+01	−4.42EE+00	2.41EE+01	−4.22EE+00
−84	−6.96EE+02	3.60EE+01	−5.18EE+00	3.54EE+01	−5.09EE+00
−83	−9.11EE+02	6.24EE+01	−6.85EE+00	6.23EE+01	−6.84EE+00
−82	−1.37EE+03	1.50EE+02	−1.09EE+01	1.50EE+02	−1.09EE+01
−81	−3.12EE+03	8.43EE+02	−2.70EE+01	8.43EE+02	−2.70EE+01
−80.5	−1.00EE+04	9.12EE+03	−9.10EE+01	9.12EE+03	−9.10EE+01

reactivity is dominated by the uncertainty in the first decay constant and increases sharply. The reason for that is as the period approaches $-1/\lambda_1$ the denominator of the first term of the summation in Eq. (10) reaches very small values and its inverse higher values. Consequently, the sum in Eq. (10) is dominates by this first term. Note that in this condition the contributions of all other categories go to zero.

Table 5 shows the uncertainties for the inferred reactivity for positive periods. The uncertainties are also shown without and with correlations among the fitted effective delayed neutron parameters. Again for this case the uncertainties decrease when the correlations among the fitted parameters are taken into consideration. When the period becomes very small (<0.001 s) the prompts neutrons control the neutron reaction chain and the effect of the correlation in the uncertainty in the inferred reactivity becomes unnoticeable and practically there is no difference if the correlations are considered or not.

Similarly to the case of negative period shown in Tables 4 and 6 shows the fraction of the five categories as defined previously. The analyses of the case of positive period are very similar to the negative period. Table 6 shows that the contribution of β_i and λ_i are positive and the contribution of the correlation among these two quantities are negative. These explain good part of the reason why the uncertainty in the inferred reactivity is reduced when correlation is taken into account. The uncertainty contribution of the first decay constant is negligible, and the uncertainty contribution of the prompt neutron generation time becomes important for very small periods. Considering periods lower than 0.001 s the

Table 4
Contributions of the several components to the reactivity uncertainty for negative periods.

Period (s)	Fraction β_i's	Fraction β_i's and λ_i's	Fraction λ_i's	Fraction λ_1	Fraction Λ
−500	1.96	−6.53	5.57	0.00	0.00
−400	1.78	−6.05	5.27	0.00	0.00
−300	1.51	−5.34	4.83	0.00	0.00
−200	1.09	−4.15	4.06	0.00	0.00
−100	0.29	−1.59	2.26	0.04	0.00
−90	0.11	−0.77	1.44	0.22	0.00
−85	0.01	−0.10	0.37	0.72	0.00
−84	0.00	−0.02	0.18	0.84	0.00
−83	0.00	0.01	0.06	0.93	0.00
−82	0.00	0.01	0.01	0.98	0.00
−81	0.00	0.00	0.00	1.00	0.00
−80.5	0.00	0.00	0.00	1.00	0.00

Table 5
Uncertainties in the inferred reactivity for positive periods.

Period (s)	Reactivity (pcm)	σ_ρ(pcm) No correlation	Uncertainty in units of %	σ_ρ(pcm) correlation	Uncertainty in units of %
400	2.09E+01	1.16E+00	5.56E+00	3.76E−01	1.80E+00
300	2.72E+01	1.49E+00	5.48E+00	4.55E−01	1.67E+00
200	3.89E+01	2.07E+00	5.32E+00	5.58E−01	1.44E+00
100	6.89E+01	3.38E+00	4.91E+00	6.00E−01	8.70E−01
80	8.18E+01	3.87E+00	4.73E+00	5.62E−01	6.87E−01
60	1.01E+02	4.53E+00	4.48E+00	5.94E−01	5.88E−01
40	1.33E+02	5.48E+00	4.12E+00	1.22E+00	9.13E−01
20	2.01E+02	7.17E+00	3.56E+00	3.91E+00	1.94E+00
10	2.84E+02	9.15E+00	3.22E+00	8.19E+00	2.88E+00
1	5.82E+02	1.59E+01	2.73E+00	2.41E+01	4.14E+00
0.1	7.51E+02	1.85E+01	2.47E+00	3.14E+01	4.18E+00
0.01	1.07E+03	2.17E+01	2.04E+00	3.46E+01	3.25E+00
0.001	3.95E+03	1.08E+02	2.73E+00	1.11E+02	2.81E+00
0.0001	3.28E+04	1.06E+03	3.24E+00	1.06E+03	3.24E+00

Table 6
Contributions of the several components to the reactivity uncertainty for positive periods.

Period (s)	Fraction β_i's	Fraction β_i's and $\lambda i'$s	Fraction $\lambda i'$s	Fraction λ_1	Fraction Λ
400	4.69	−13.24	9.54	0.00	0.00
300	5.55	−15.22	10.67	0.00	0.00
200	7.81	−20.34	13.53	0.00	0.00
100	23.07	−52.18	30.11	0.00	0.00
80	38.07	−80.92	43.84	0.00	0.00
60	54.09	−104.59	51.50	0.00	0.00
40	23.68	−38.98	16.31	0.00	0.00
20	5.68	−6.63	1.96	0.00	0.00
10	2.70	−2.12	0.42	0.00	0.00
1	1.22	−0.23	0.01	0.00	0.00
0.1	1.04	−0.04	0.00	0.00	0.00
0.01	0.91	0.00	0.00	0.00	0.09
0.001	0.09	0.00	0.00	0.00	0.91
0.0001	0.00	0.00	0.00	0.00	1.00

contribution of the correlation among the fitted parameters has no importance because the neutron reaction chain is governed practically by the prompt neutrons.

5. Conclusions

The impact of the correlation matrix for the fitted delayed neutron parameters in the determination of the reactivity of a multiplying system has been successfully performed in this work. The correlation matrix for the fitted parameters is determined only for the parameters considered in the fitting process as considered in Section 2. The prompt neutron generation time (Λ) and the first decay constant (λ_1) are considered uncorrelated between themselves and among the parameters considered in the fitting process. The analyses reveal that the consideration of the correlation matrix in the error propagation of the reactivity is very important. In general, the introduction of the correlation matrix reduces the overall uncertainty by a reasonable amount. It has been observed huge cancellations in the uncertainty analysis. The analyses also reveal that the uncertainty in the reactivity depends in the specific period range where this quantity is considered. The achieved reactivity uncertainties for negative periods are around 3.5% while for positive periods they are nearly 2%. The uncertainty in the first decay constant dominates the overall reactivity uncertainty for very large negative periods. Conversely, the uncertainty in the prompt neutron generation time dominates the overall reactivity uncertainty for very small positive periods. As a general conclusion, the experiments performed and evaluated at the IPEN/MB-01 reactor can be considered a good candidate for a benchmark to validate methods and nuclear data related to the kinetic parameters of thermal reactors fueled with uranium enriched and where most of fissions occurs in ^{235}U.

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