

# Theoretical and experimental study of supersonic gas jet targets for laser wakefield acceleration

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**Abstract**—This work reports a theoretical and experimental study of supersonic gas jets to be used as targets in laser wakefield acceleration. A comparison between theoretical and experimental results is made in order to estimate the results of a de Laval nozzle manufactured in our laboratory, used to produce a N<sub>2</sub> gas target. The comparison between the results shows that Computational Fluid Dynamic simulations are able to describe the phenomena of the supersonic gas jet.

**Keywords**—de Laval nozzles, CFD simulation, interferometry, gas density, laser wakefield acceleration

## I. INTRODUCTION

The use of laser wakefield acceleration systems (LWFA) has grown rapidly in the scientific community as a new technology for particle accelerator systems [1, 2]. This technique relies on the generation of acceleration gradients above 100 GV/m, which are currently limited to the order of 100 MV/m [3] in conventional RF accelerators due to materials breakdown. It also reduces the size and complexity, and consequently the costs, of particle accelerators. Moreover in this type of acceleration systems, usually renewable and fluid targets are used to create the plasma and stimulate the generation of a longitudinal electric field (wakefield) capable of accelerating electrons [4]. The interaction of particles (electrons) with matter allows the development of compact and tunable sources, which can be used in many applications as new imaging techniques [5] and production of medicinal radiocompounds [3, 6, 7].

Usually, lasers with peak powers ranging from TW to PW are used to accelerate electrons [8, 9], but recent advances were obtained in the acceleration with sub-TW pulses [10-12]. To achieve acceleration with these lower powers, gas jet targets with  $\sim 100 \mu\text{m}$  thickness and densities between  $10^{18}$  and  $10^{21}$  atoms/cm<sup>3</sup> are needed to ensure that Self-Modulated Laser Wakefield Acceleration (SM-LWFA) [8, 13, 14] occurs, and generate highly nonlinear waves of plasma even for mJ-class laser pulses [7]. Gas targets in the form of supersonic jets are necessary to provide new, reproducible conditions fast as possible to each laser pulse.

De Laval nozzles [15, 16] are convergent-divergent nozzles used to accelerate fluids to supersonic speeds (high

Mach numbers). With this kind of nozzle, sharp jet-vacuum boundaries are created and homogeneous flat-top gas density profiles are typically obtained. These characteristics can simplify the design and control of laser-plasma experiments by enhancing the laser interaction with the fluid [17].

High-intensity laser experiments currently using de Laval nozzles involve laser frequency upshift and high-harmonic generation, laser filamentation, relativistic self-focusing, laser-driven electron acceleration, THz generation, extreme ultraviolet and x-ray beam generation, laser pulse compression, among others [8]. Infrastructures [17, 18] are being prepared to produce particle beams to be used in medical applications like x-ray and electron therapy, nuclear reactions and, eventually, proton therapy [18].

We are working to implement LWFA in our laboratory, which demands the use of gas jet laser targets to accelerate electrons. Those targets are being generated by homemade nozzles and compared to theoretical analysis made by Computational Fluid Dynamic (CFD) simulations, with a Finite Element Method (FEM) in the Ansys software, version 19, Academic. Supersonic gas jets are generated, and their experimental analyses are made by interferometer and Schlieren images and compared to the theoretical results to provide feedback to improve the nozzles and targets.

De Laval nozzles [15, 19] are designed to generate supersonic gas jets, by constricting the flow in a throat and then allowing its free expansion. A scheme of a typical de Laval nozzle is represented in Fig. 1, where  $P_0$ ,  $T_0$ ,  $\rho_0$  and  $P_1$ ,  $T_1$ ,  $\rho_1$  are the pressure, temperature, and density for the backing and experimental chambers, respectively,  $r_t$  is the throat radius and  $r_e$  is the radius of the nozzle exit.

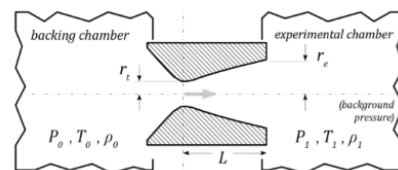


Fig. 1. Sketch of a typical de Laval nozzle [20].

A simple quasi-1D [21] model can estimate the gas jet Mach number  $M$  based only on the ratio of the exit and throat areas,  $A_e$  and  $A_t$ , respectively, and the type of gas used:

$$\frac{A_e}{A_t} = \frac{1}{M} \left[ \frac{2 + (\gamma - 1)M^2}{\gamma + 1} \right]^{\frac{\gamma + 1}{2(\gamma - 1)}}, \quad (1)$$

where  $\gamma$  is the gas heat capacity ratio, or adiabatic index. This relation gives a starting point to design nozzles with a desired Mach number, providing the sizes of the nozzle throat and exit. As an example, Fig. 2 presents eq. (1) graphically, depicting the ratio of the nozzle throat and exit areas as a function of the Mach number for nitrogen gas.

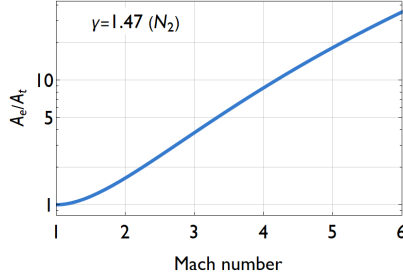


Fig. 2. Dependence of the exit and throat areas ratio on the Mach number for  $N_2$ .

This quasi-1D model disregards the exact value of the background pressure into which the jet expands, the presence of shock waves, as well as the nozzle length and other geometrical features like the length of the converging region, shapes and curvatures, all these affecting the jet discharge and speed (and pressure) as second-order corrections. To better understand and design nozzles, more sophisticated models or numerical simulations, such as the one used in this work are needed.

## II. METHODOLOGY

We assembled a Mach-Zehnder interferometer with Schlieren imaging, shown in Fig. 3, to characterize the profile and density of the gas jets generated by the homemade de Laval nozzles. A polarized green HeNe CW beam is spatially filtered by a pinhole and collimated to a 1 cm radius beam. This beam is split in a Mach-Zehnder interferometer, and one of its arms contains the gas jet. After the beam exits the Mach-Zehnder, a 10 cm converging lens is positioned in the final of optical axis and 12 cm away from the jet, projecting a  $5\times$  magnified image of the jet in a CCD surface. A knife-edge is positioned on the lens focus to produce a Schlieren image in the same CCD, when blocking the beam in the arm without the jet. Furthermore, we have manufactured a small vacuum chamber [20], with Brewster windows, which can be fitted to the nozzle. The jet can be characterized either in atmosphere or inside the small vacuum chamber when applying a nitrogen backing pressure up to 50 bar. At this high backing pressure, the residual pressure inside the chamber is  $\sim 20$  mbar.

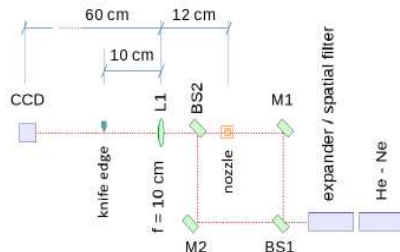


Fig. 3. Setup for measurement of the gas jet through Mach-Zehnder interferometer and Schlieren images.

The phase-shift caused by the gas jet is made visible by the bending of the fringes in the interferogram [22]. Optical path differences due to gas density variations result in deformations of the fringes, to which the phase-shift is proportional.

The Computational Fluid Dynamic (CFD) simulations were used as a tool to understand the flow and all phenomena that can happen when a gas is forced through a home-made de Laval nozzle. The simulated nozzle was etched in a  $600 \mu\text{m}$  thick alumina ( $Al_2O_3$ ) plate by an ultrashort pulse machining system in our lab. Its exit and throat diameters are  $190 \mu\text{m}$  and  $86 \mu\text{m}$ , respectively, defining a Mach number of  $\sim 3$  to nitrogen gas, according to the graph shown in Fig. 2.

The fluid simulated in this work is ideal  $N_2$ , with a CFX module from Ansys 19, Academic version, and the gas flow is described by the Navier-Stokes differential equations, which are solved using the upwind scheme discretization method. The turbulence is modeled by the K- $\epsilon$  method, which uses two equations based on the turbulent viscosity concept. This parameter of viscosity inside the flow is an unknown term that needs to be modeled numerically. This viscosity may be written as a function of K and  $\epsilon$ , which are, respectively, the turbulent kinetic energy and the viscosity dissipation of this energy. The backing pressure is 50 bar and the background pressure is determined with the exhaust pressure of 20 mbar.

## III. RESULTS

### A. Experimental Analysis

Fig. 4 shows an interferogram of the Mach 3 manufactured nozzle [20], using  $N_2$  with a backing pressure of 50 bar. The interferogram was measured inside the vacuum chamber, with 20 mbar of background pressure. The red rectangle shows the fringes deformation, indicating the region with the gas jet axis.

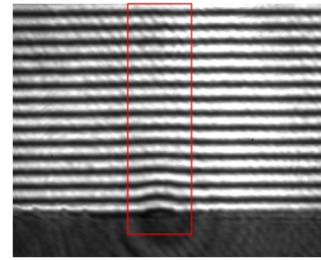


Fig. 4. Image for interferometer Mach-Zehnder of a gas jet ( $N_2$ ) flowing through a de Laval nozzle in vacuum (20 mbar background pressure).

Fig. 5.a) presents the phase-shift map obtained from the interferogram shown in Fig. 4, and the corresponding density map, (b), derived from it considering the gas refractive index induced phase-shift:

$$n = 1 + \Delta\phi \lambda / (2\pi l), \quad (2)$$

where  $\Delta\phi$  is the measured phase-shift,  $\lambda = 543 \text{ nm}$  is the interferometer laser wavelength, and  $l$  is the gas jet diameter, which was estimated from the phase map to be  $190 \mu\text{m}$  assuming a cylindrical symmetry of the jet and a background refractive index of 1 (ideal vacuum). From the refractive index 2D map and eq. (2) we also can estimate the gas density distribution using the simple expression of the Lorentz-Lorenz relation [23, 24]:

$$\rho = 3/(4\pi\alpha) \times (n^2 - 1)/(n^2 + 2), \quad (3)$$

where  $\alpha$  is the molecular polarizability, which for  $N_2$  is  $1.710 \text{ \AA}^3$  [25].

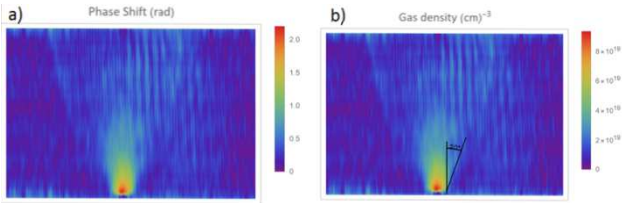


Fig. 5. a) Phase-shift map of a gas jet flowing from the De Laval Nozzle. b) Gas density map obtained by the phase-shift map.

From the density map we can see that the jet divergence angle is  $\sim 20^\circ$ . The simplified quasi 1-D model provides a simple expression, [21] to estimate the relation between the jet Mach number,  $M$ , and its divergence,  $\alpha$ :

$$\alpha = 1/M, \quad (4)$$

and from this equation we can estimate  $M \approx 2.8$ .

### B. Theoretical Analysis

The de Laval nozzle that was simulated has a conical geometry, with cylindrical transversal section. Its exit diameter is  $190 \mu\text{m}$  and the throat diameter is  $86 \mu\text{m}$ , with the length of the nozzle being  $600 \mu\text{m}$ , with low roughness walls. Those dimensions are the same that the nozzle used in our experiments. Fig. 6 shows the Mach (speed) distribution of the gas jet flow through the de Laval nozzle and after its exit. It can be easily seen that the maximum speed occurs near the nozzle exit, with a Mach number of  $\sim 2.5$ . The discrepancy between this Mach number and the value of 3 obtained from Fig. 2 comes from the simplified 1D model that provides the curve shown, which disregards the nozzle length and shape, and also the 20 mbar background pressure.

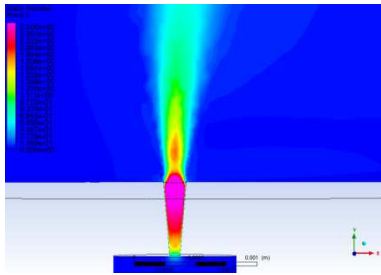


Fig.6. Mach number distribution of the gas jet flowing through the de Laval nozzle and exiting into the vacuum chamber. The Mach number distribution is represented by a color map.

Fig. 7, Fig. 8 and Fig. 9 show the molecules density distribution on the gas jet in different planes. From these pictures, the cylindrical symmetry of the jet can be seen, and it is also possible to verify the sharp transversal density increase in the target, characteristics that are desired for the LFWA. The density decreases along the nozzle as the gas jet velocity grows, reaching a minimum value at the exit, then increasing again

The inhomogeneities in Fig. 7, Fig. 8 and Fig. 9 are simulation artifacts arising due to non-ideal discretization of the space, arising from restrictions on the grid due to the academic license (limitation on the maximum number of nodes and elements of the grid) that result in a simulation

that does not describe the physical system adequately. Because of the relatively small number of space elements allowed, it is necessary to decrease the simulation volume, while keeping it large enough to minimize interferences of the volume walls in the flow. The number of mesh elements influences the spatial resolution of the simulation. Another consequence of the finite volume of the vacuum chamber is the residual background pressure. An exhaustion duct was created to extract the gas from the simulation volume, but the flow is not enough to keep a low vacuum, resulting in a background pressure that limits the gas expansion. These simulation limitations are responsible for the discrepancies between the simulations and the results experimental results, and studies are being done in order to further reduce the background pressure and improve the simulation volume discretization, thus bringing the theoretical results closer to the experimental ones.

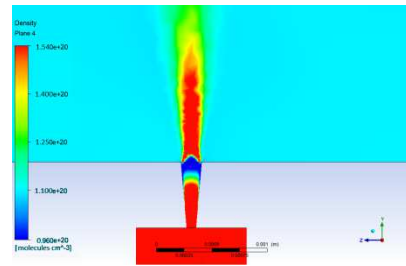


Fig. 7. Density distribution in ZY plane of the gas jet flowing through the De Laval nozzle and exiting into the vacuum chamber.

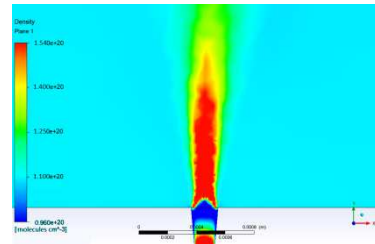


Fig.8. Density distribution in XY plane of the gas jet flowing through the de Laval nozzle and exiting into the vacuum chamber.

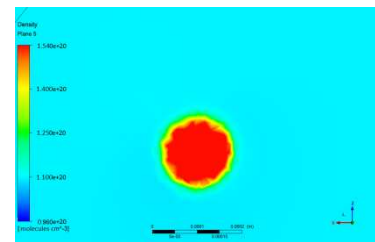


Fig.9. Density distribution in XZ of the gas jet flowing through the de Laval nozzle at the entrance of the vacuum chamber.

### C. Comparison between theory and experiments

It is possible to verify that the simulations describe the geometry of the gas jet closely to the experimental results, mainly the obtained Mach number and density close to the nozzle exit orifice [21], although the experimental jet is more divergent than the theoretical prediction. In Fig. 7 and Fig. 8 is possible to see a region with a great density variation near the nozzle exit. There are also other discrepancies when comparing results, like the jet Mach number determined from the interferograms, which is 2.8 while the maximum Mach from the simulation is 2.5. This difference is due to

the simplified model [21] used in the experimental data, which disregards the nozzle length and background pressure. Despite these differences, the simulation provides a good estimate from a simple model. For the density distributions shown in figures 5b, 8 and 9, there is a difference by a factor 2 since the peak experimental density is  $\sim 9 \times 10^{19}$  molecules/cm<sup>3</sup>, while the theoretical peak density reaches  $\sim 1.5 \times 10^{20}$  molecules/cm<sup>3</sup>. This value discrepancy probably arises from considering the gas density constant along the laser path in the interferogram. A more realistic density estimate would need to recover the gas transversal profile considering a cylindrical symmetry, which would increase the experimental retrieved peak density by a factor 2-3. This retrieval can be done by an Abel transform [26, 27], and we are working to implement this technique. With all these considerations, the experimental and theoretical results are close, indicating that the numerical simulations are representing the real phenomena.

#### IV. CONCLUSIONS

In this work, we presented the results of numerical simulations that describe experimental results of gas jet flows fairly close. The prediction obtained from the simulations can simplify the process of creating different gas target geometries from the design of the de Laval nozzles.

We are working to improve our simulations, and increase their resolution, aiming to get more realistic results that will allow the design of better nozzles and gas targets, optimized for electron acceleration with our laser.

With the knowledge that we are acquiring in this study, it will be possible to perform simulations of more complex systems, like asymmetrical nozzles [28], which produce more complex gas distributions that can improve the electron acceleration process, and simulate other mixtures of gases that also enhance the acceleration, contributing to our experimental developments.

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