



# Population inversion between the $^3H_4$ and the $^3F_4$ excited states of $Tm^{3+}$ investigated by means of numerical solutions of the rate equations system in $Tm^{3+}$ -doped and $Tm^{3+}$ , $Ho^{3+}$ -codoped fluoride glasses

A.F.H. Librantz<sup>a</sup>, L. Gomes<sup>b,\*</sup>

<sup>a</sup> Department of Sciences, UNINOVE, São Paulo, SP 02111-030, Brazil

<sup>b</sup> Center for Lasers and Applications, IPEN/CNEN-SP, P.O. Box 11049, São Paulo, SP 05422-970, Brazil

## ARTICLE INFO

### Article history:

Received 16 May 2008

Accepted 6 November 2008

Available online 19 November 2008

### PACS:

78.50

78.55

71.55

### Keywords:

1.5  $\mu$ m-Laser emission

Energy-transfer constant rate

Numerical simulation

Rate equations system

Thulium-doped fluoride glass

## ABSTRACT

Population inversion between the  $^3H_4$  and the  $^3F_4$  excited states of  $Tm^{3+}$  ions responsible for the 1.5  $\mu$ m emission in  $Tm^{3+}$  singly doped (0.5%) and  $Tm^{3+}$ ,  $Ho^{3+}$ -codoped fluoride (ZBLAN) glasses and its dependence on the  $Ho^{3+}$  concentration ( $x = 0.2$ –1%) was investigated by means of numerical solution of the rate equations system for continuous pumping at 797 nm. Mean lifetimes of donor and acceptor states were evaluated by using the integration method applied to the best fitting of fluorescence curves previously reported. Lifetime values were used to obtain the rate constants of all non-radiative energy-transfer processes involved and a complete set of rate equations better describing the observations was given. The rate equations were solved by numerical method and the population inversion between the  $^3H_4$  and the  $^3F_4$  excited states of  $Tm^{3+}$  was calculated to examine the beneficial effects on the gain associated with  $Ho^{3+}$  codoping. The results have shown that  $Tm^{3+}$  population inversion is reached only for high  $Ho^{3+}$ -codoping ( $\geq 0.3$  mol%). Highest population inversion ( $\sim 1.6 \times 10^{18} Tm^{3+}$  ions  $cm^{-3}$ ) was obtained in  $Tm(0.5\%)$ ,  $Ho(1\%)$ -codoped (ZBLAN) pumped by  $2.8 kW cm^{-2}$ . This population inversion density is  $\sim 6.4$  times higher than that one observed in  $Tm:Tb:GLKZ$ ,  $Tm:Tb:Ge-Ga-As-S-CsBr$  and  $Tm:Ho:Ge-Ga-As-S-CsBr$  for a similar pumping condition ( $\sim 2.5 \times 10^{17} cm^{-3}$ ). In addition,  $Tm(0.5\%):Ho(1\%):ZBLAN$  presents the highest population inversion that linearly increases with the pumping intensity; this behavior does not show saturation effect at least for the maximum intensity of  $12 kW cm^{-2}$  employed. The use of 1 mol% of  $Ho^{3+}$ -codoping maximizes the potential gain of  $Tm^{3+}$ -doped (0.5%) ZBLAN to produce stimulated emission near 1.5  $\mu$ m, making this material suitable for using it as fiber optical amplifier and/or fiber laser operating in 1.4–1.5  $\mu$ m region of the spectrum.

© 2008 Elsevier B.V. All rights reserved.

## 1. Introduction

In the last years,  $Tm$ -doped glasses have been investigated for using as fiber lasers and optical amplifiers [1] based on the  $^3H_4 \rightarrow ^3F_4$ -stimulated transition of  $Tm^{3+}$ . However, two intrinsic properties of  $Tm^{3+}$  single-doped materials act detrimentally, reducing its capacity of light signal amplification in S band (1.45–1.53  $\mu$ m) in many oxide glasses, in which the  $^3H_4$  level easily decay to the next lower-level  $^3H_5$  via multiphonon relaxation. That is not the case of the fluorozirconate (ZBLAN) glass used in this work. Nevertheless, the lifetime of the lower  $^3F_4$  level involved in the 1.47  $\mu$ m emission is considerably longer than that of the upper  $^3H_4$  level ( $\sim 6$  times is observed in  $Tm:ZBLAN$  and  $Tm:Ge-Ga-As-S-CsBr$ , and  $\sim 8.3$  times in germanate glasses). To

\* Corresponding author. Tel.: +55 11 38169319; fax: +55 11 38169315.

E-mail addresses: [librantz@uninove.br](mailto:librantz@uninove.br) (A.F.H. Librantz), [lgomes@ipen.br](mailto:lgomes@ipen.br) (L. Gomes).

realize population inversion in  $Tm^{3+}$  ions,  $Ho^{3+}$  (0.15%) or  $Tb^{3+}$  (0.15%) ions have been used to codope the  $Tm$ -doped material to depopulate the  $^3F_4$  level of  $Tm^{3+}$  in fluoroindate [2], fluorozirconate (ZBLAN) [3,4], fluorogermanate [5], tellurite [6] and chalcogenide [7] glasses. Because the energies of  $^5I_7$  and  $^7F_{0,1,2}$  levels of  $Ho^{3+}$  and  $Tb^{3+}$ , respectively, coincide with the energy of the  $^3F_4$  level, holmium and terbium ions may actuate as deactivator of the  $^3F_4$  lowest excited state of  $Tm^{3+}$  to improve the population inversion of  $Tm^{3+}$  ions in many luminescent solid materials. Besides the  $^3H_4$  and  $^3F_4$  fluorescence decay effects of  $Tm^{3+}$  caused by  $Ho^{3+}$  ions have been investigated in fluorozirconate [3] and more recently in fluorogermanate glass [5], a detailed investigation of the mechanisms of  $^3H_4$  and  $^3F_4$  deactivation are still lacking, as well the extension to high holmium doping effects. Also, a better approach of calculating the population inversion in  $Tm:Ho$  system needs to be performed based on the numerical solutions of a realistic set of rate equations, which can describe the system behavior under continuous pumping of the  $^3H_4$  state. In this work, we investigated the population inversion of  $^3H_4 \rightarrow ^3F_4$

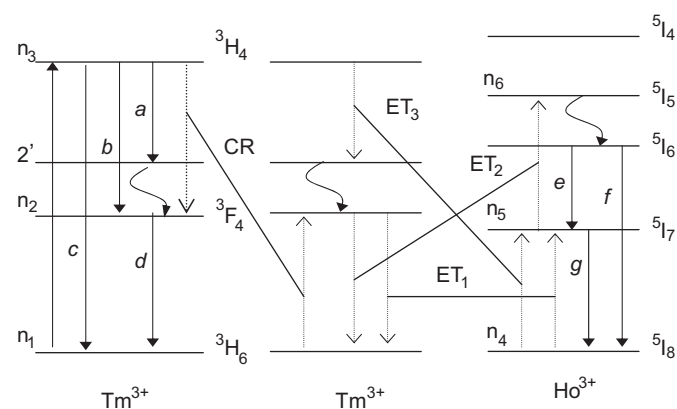
transition of  $\text{Tm}^{3+}$  in  $\text{Tm}^{3+}$ -doped and  $\text{Tm}^{3+}$ ,  $\text{Ho}^{3+}$ -codoped ZBLAN glasses due to its favorable optical properties such a wide transmission window (typically 300–5000 nm), good corrosion resistance and mechanical stability, and low cut-off phonon energy among glass materials ( $800\text{ cm}^{-1}$ ). Also this material has high solubility for rare earth doping and low fusion temperature. A depopulation of  ${}^3\text{F}_4$  state of  $\text{Tm}^{3+}$  by the energy transfer to  $\text{Ho}^{3+}$  ions is an important phenomena in solids that must be investigated among different host materials to establish a wide comparison of the physical effects in the upper and lower excited states of the  ${}^3\text{H}_4 \rightarrow {}^3\text{F}_4$  transition to determine the best deactivator concentration to maximize the population inversion and consequently the gain for light signal amplification at  $1.47\ \mu\text{m}$ . This population inversion was calculated using the numerical solutions obtained using the Runge–Kutta method applied to the rate equations system. This method has been recently used to investigate the population inversion of  ${}^3\text{H}_4 \rightarrow {}^3\text{F}_4$  transition of  $\text{Tm}^{3+}$  in  $\text{Tm}:\text{Tb}$  [4] and  $\text{Tm}:\text{Eu}:\text{germanate}$  [8] glasses giving an important contribution for understanding the potential and limitations of  $\text{Tm}^{3+}$  and  $\text{Tm}$ -doped materials for light signal amplification.

## 2. Experimental procedure

Best fitting parameters of  ${}^3\text{H}_4$  excited level luminescence decay of  $\text{Tm}^{3+}$  (at 1470 nm) and best fitting energy-transfer parameters of  ${}^5\text{I}_7$  (at 2000 nm) excited level luminescence transient of  $\text{Ho}^{3+}$ , previously reported for  $\text{Tm}:\text{Ho}:\text{ZBLAN}$  [3], were used in this work to obtain the mean lifetime by applying the integration method. The transfer rate constant was obtained using the mean lifetime value for each related energy-transfer process and it was used to solve the rate equations system to perform the population inversion investigation.

## 3. Results and discussion

Fig. 1 shows the simplified energy levels scheme used for the  $\text{Tm}:\text{Ho}:\text{ZBLAN}$  system used for continuous laser pumping of  ${}^3\text{H}_4$  ( $\text{Tm}^{3+}$ ) level ( $n_3$ ) at 797 nm. The  $n_1$ ,  $n_2$  and  $n_3$  represent the population of the  ${}^3\text{H}_6$ ,  ${}^3\text{F}_4$  and  ${}^3\text{H}_4$  levels of  $\text{Tm}^{3+}$ , and  $n_4$ ,  $n_5$  and  $n_6$  are the populations of  ${}^5\text{I}_8$ ,  ${}^5\text{I}_7$  and  ${}^5\text{I}_6$  levels of  $\text{Ho}^{3+}$ , respectively.



**Fig. 1.** A schematic energy levels diagram used for the  $\text{Tm}:\text{Ho}:\text{ZBLAN}$  considered for continuous laser pumping at 797 nm (solid up-arrow).  $n_1$ ,  $n_2$ ,  $n_3$  represents the  $\text{Tm}^{3+}$  populations and  $n_4$ ,  $n_5$ ,  $n_6$  are the  $\text{Ho}^{3+}$  populations. Radiative transitions are indicated as:  $a = 2300\text{ nm}$ ,  $b = 1470\text{ nm}$  ( $\beta_{32} + \beta_{32'} = 0.1$ ),  $c = 800\text{ nm}$  ( $\beta_{31} = 0.9$ ),  $d = 1800\text{ nm}$ ,  $e = 2900\text{ nm}$ ,  $f = 1200\text{ nm}$  and  $g = 2000\text{ nm}$ .

### 3.1. Mean lifetime and energy-transfer rate constant determination

The following non-radiative energy-transfer (ET) processes (dipole–dipole interaction) were considered been involved in the optical cycle of  $\text{Tm}:\text{Ho}:\text{ZBLAN}$  under 797 nm excitation.

- (i)  $\text{Tm}^{3+}({}^3\text{H}_4):\text{Tm}^{3+}({}^3\text{H}_6) \rightarrow \text{Tm}^{3+}({}^3\text{F}_4):\text{Tm}^{3+}({}^3\text{F}_4)$ —cross relaxation (CR);
- (ii)  $\text{Tm}^{3+}({}^3\text{F}_4):\text{Ho}^{3+}({}^5\text{I}_8) \rightarrow \text{Tm}^{3+}({}^3\text{H}_6):\text{Ho}^{3+}({}^5\text{I}_7)$ —energy transfer ( $\text{ET}_1$ );
- (iii)  $\text{Ho}^{3+}({}^5\text{I}_7):\text{Tm}^{3+}({}^3\text{H}_6) \rightarrow \text{Ho}^{3+}({}^5\text{I}_8):\text{Tm}^{3+}({}^3\text{F}_4)$ —back-transfer (BT);
- (iv)  $\text{Tm}^{3+}({}^3\text{F}_4):\text{Ho}^{3+}({}^5\text{I}_7) \rightarrow \text{Tm}^{3+}({}^3\text{H}_6):\text{Ho}^{3+}({}^5\text{I}_6)$ —energy transfer ( $\text{ET}_2$ );
- (v)  $\text{Tm}^{3+}({}^3\text{H}_4):\text{Ho}^{3+}({}^5\text{I}_8) \rightarrow \text{Tm}^{3+}({}^3\text{H}_5):\text{Ho}^{3+}({}^5\text{I}_7)$ —energy transfer ( $\text{ET}_3$ ).

These ET processes have been previously investigated in  $\text{Tm}:\text{Ho}:\text{ZBLAN}$  glasses [3] by means of using a time-resolved luminescence technique and a tunable OPO laser excitation (1100–2000 nm). The luminescence decay of  ${}^3\text{H}_4$  and  ${}^3\text{F}_4$  excited states of  $\text{Tm}^{3+}$  in  $\text{Tm}:\text{Ho}:\text{ZBLAN}$  were best fitted by using a combination of the Inokuti–Hirayama function (transfer parameter  $\gamma$ ) and a proposed localized interaction described by an exponential term with a decay rate  $K$ , according to Ref. [3]. Energy-transfer parameters  $\gamma$  and  $K$  were used to determine the mean lifetime of the donor ( ${}^3\text{H}_4$ ) and acceptor ( ${}^5\text{I}_7$ ) states.

The following equations have been used to fit the luminescence decay curves of  $\text{Tm}^{3+}$  at 1470 nm with short laser pulse of 4 ns (10 Hz) at 797 nm ( $E \sim 10\text{ mJ}$ ) and  $\text{Ho}^{3+}$  luminescence transient (at 2000 nm) excited by  $\text{Tm}^{3+}({}^3\text{F}_4) \rightarrow \text{Ho}^{3+}({}^5\text{I}_7)$  transfer induced by pulsed laser excitation at 1671 nm (4 ns,  $E \sim 10\text{ mJ}$ ) [3].

- (i)  $1.47\ \mu\text{m}$ -luminescence dec

$$I_D(t) = a \exp\left(-\frac{t}{\tau_D} - \gamma\sqrt{t}\right) + b \exp(-Kt) \quad (1)$$

- (ii)  $2\ \mu\text{m}$ -luminescence transient:

$$I_A(t) = (a + b) \exp\left(-\frac{t}{\tau_A}\right) - \left[ a \exp\left(-\frac{t}{\tau_D} - \gamma\sqrt{t}\right) + b \exp\left(-\frac{t}{\tau_D} - Kt\right) \right] \quad (2)$$

where  $\tau_D = \tau_{D_3} = 1.46\text{ ms}$  is the intrinsic lifetime of  ${}^3\text{H}_4$  excited state of  $\text{Tm}^{3+}$  in Eq. (1).  $\tau_D = \tau_{D_2} = 8.9\text{ ms}$  is the intrinsic lifetime of donor  ${}^3\text{F}_4$  state of  $\text{Tm}^{3+}$  and  $\tau_A = \tau_{A_5} = 16.4\text{ ms}$  is the intrinsic lifetime of the acceptor  ${}^5\text{I}_7$  state of  $\text{Ho}^{3+}$  in Eq. (2). The fitting parameters  $\gamma$  and  $K$  are the  $\text{Tm}$ – $\text{Ho}$  transfer parameters.  $a$  and  $b$  are non-dimensional parameters related to the Inokuti–Hirayama [9] and localized transfer contributions [3], respectively. The mean lifetime of  ${}^3\text{H}_4$  and  ${}^3\text{F}_4$  excited levels of  $\text{Tm}^{3+}$  represented by  $\tau_3$  and  $\tau_2$  were obtained by using the integration of normalized luminescence decay curves. For instance,  $\tau_2$  was better determined by integration of the raise-time component of the luminescence transient curve of  ${}^5\text{I}_7$  level of  $\text{Ho}^{3+}$ —the acceptor state. The luminescence decay of  ${}^5\text{I}_7$  level is exponential with a lifetime  $\tau_5$ .  $\tau_3$  and  $\tau_2$  mean lifetimes were obtained using:

- (i)  $1.47\ \mu\text{m}$ -luminescence decay integration:

$$\tau_3 = \frac{1}{(a + b)} \int_0^\infty \left\{ a \exp\left(-\frac{t}{\tau_{D_3}} - \gamma\sqrt{t}\right) + b \exp(-Kt) \right\} dt \quad (3)$$

(ii) 2  $\mu\text{m}$ -luminescence raise-time integration:

$$\begin{aligned} \tau_{\text{risetime}}(^5\text{I}_7) &= \tau_2 \\ &= \frac{1}{(a+b)} \int_0^\infty \left\{ a \exp\left(-\frac{t}{\tau_{D_2}} - \gamma\sqrt{t}\right) + b \exp\left(-\frac{t}{\tau_{D_2}} - Kt\right) \right\} dt \end{aligned} \quad (4)$$

$W_{\text{CR}}$ ,  $K_1$ ,  $K_2$  and  $K_3$  transfer rate constant of CR, ET<sub>1</sub>, ET<sub>2</sub> and ET<sub>3</sub> processes, respectively were obtained using

$$W_{\text{CR}} = \frac{1}{\tau_3} - \frac{1}{\tau_{D_3}} \quad (5)$$

$$K_1 = \frac{1}{\tau_2} - \frac{1}{\tau_{D_2}} \quad (6)$$

$$K_2 = \frac{1}{\tau_5} - \frac{1}{\tau_{D_5}} \quad (7)$$

$$K_3 = \frac{1}{\tau_3} - \frac{1}{\tau_{D_3}} - K_1 \quad (8)$$

Mean lifetimes values were obtained by applying the integration method described by Eqs. (3) and (4) using the transfer rate constants obtained by Eqs. (5)–(8)—mean lifetimes and transfer rate constants were given in Tables 1 and 2. Branching ratio of luminescence, radiative lifetime and intrinsic total lifetime of

**Table 1**  
Experimental values of energy transfer parameters ( $\gamma$  and  $K$ ) obtained from Ref. [3] for 1.47  $\mu\text{m}$  luminescence of  $^3\text{H}_4$  excited state of  $\text{Tm}^{3+}$  for two sets of  $\text{Tm}(y):\text{Ho}(x):\text{ZBLAN}$ .

Tm:Ho (mol%)		Transfer parameter (best fitting)					Transfer rates	
(y)	(x)	$\gamma$ ( $\text{s}^{-1/2}$ )	$K$ ( $10^3 \text{ s}^{-1}$ )	$a$	$b$	$\tau_3$ ( $\mu\text{s}$ )	$W_{\text{CR}}$ ( $10^3 \text{ s}^{-1}$ )	$K_3$ ( $\text{s}^{-1}$ )
0.5	0	12.1	1.8	0.95	0.05	961	0.355	0
0.5	1	11.9	1.9	0.92	0.09	952	0.366	0
1	1	40.2	3.2	0.82	0.20	432	1.602	28
3	1	182	25.0	0.64	0.40	50.6	19.08	0
6	1	357	153.2	0.31	0.73	9.7	102.4	0
9	1	231	395.6	0.09	0.97	5.6	176.9	0
1	0.5	42.3	2.7	0.16	0.16	437	1.602	0
1	2	53.3	2.4	0.11	0.11	368	1.602	429
1	3	66.6	3.4	0.07	0.07	310	1.602	934
1	4	73.3	10.3	0.09	0.09	236	1.602	1941

$a$  represents the fraction of Inokuti–Hirayama contribution and  $b$  a fraction of localized transfer contribution (transfer constant  $K$ ). Mean lifetime  $\tau_3$  was obtained by integration of luminescence decay.  $\tau_D \sim \tau_{R_3} = 1.46 \text{ ms}$ .  $W_{\text{CR}}$  is the cross-relaxation rate between the excited  $^3\text{H}_4$  and the ground  $^3\text{H}_6$  states of  $\text{Tm}^{3+}$  in ZBLAN.

**Table 2**  
Experimental values of energy transfer parameters ( $\gamma$  and  $K$ ) obtained from Ref. [3] for 2  $\mu\text{m}$  luminescence of  $^5\text{I}_7$  excited state of  $\text{Ho}^{3+}$  for two sets of  $\text{Tm}(y):\text{Ho}(x):\text{ZBLAN}$ .

Tm:Ho (mol%)		Transfer parameters (best fitting)				Mean lifetimes (integration)		Transfer rates	
(y)	(x)	$\gamma$ ( $\text{s}^{-1/2}$ )	$K$ ( $10^3 \text{ s}^{-1}$ )	$a$	$b$	$\tau_2$ ( $\mu\text{s}$ )	$\tau_5$ (ms)	$K_1$ ( $10^3 \text{ s}^{-1}$ )	$K_2$ ( $\text{s}^{-1}$ )
0.5	0	0	0	1	0	8900	–	0	0
0.5	1	111.6	412	0.84	0.22	128.9	16.4	7.65	0
1	1	151.4	308.8	0.83	0.18	71.2	16.4	13.93	0
3	1	281.2	162.8	0.77	0.27	21.3	12.4	46.84	19.7
6	1	275.3	221.9	0.34	0.70	12.2	4.8	81.85	147
9	1	223	316.3	0.21	0.85	11.1	2.0	89.97	439
1	0.5	140.9	127	0.82	0.24	82.2	13.8	12.05	0
1	2	215.7	184.3	0.53	0.49	25.4	15.4	39.25	3.9
1	3	280.6	289.7	0.38	0.64	11.5	15.8	86.84	2.3
1	4	281.0	389.9	0.27	0.76	8.8	12.5	113.14	19

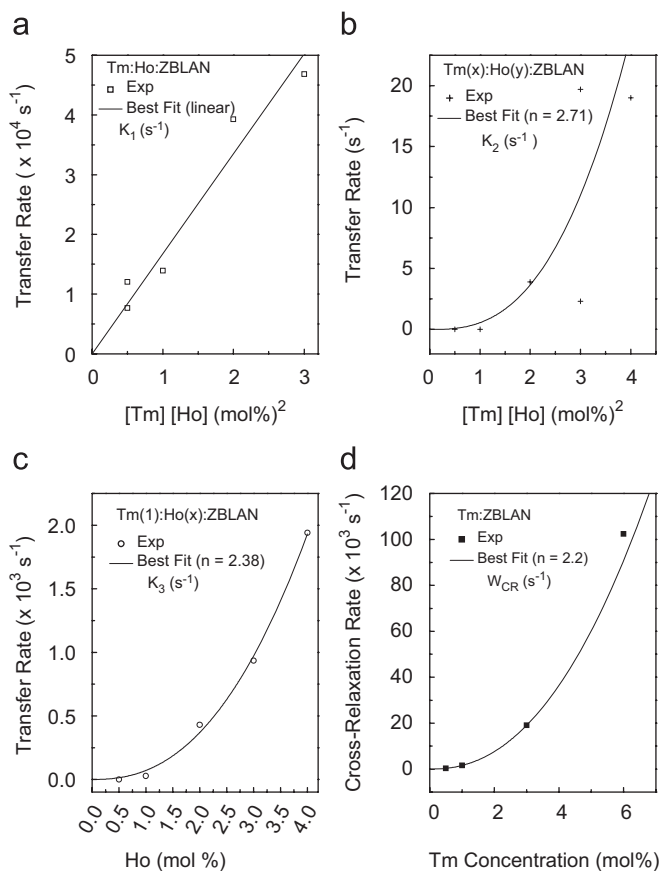
$a$  represents the fraction of Inokuti–Hirayama contribution and  $b$  a fraction of localized transfer contribution (transfer constant  $K$ ). Mean lifetime  $\tau_2$  was obtained by integration of raise-time component of 2  $\mu\text{m}$ -emission of  $^5\text{I}_7$  acceptor state ( $\text{Ho}^{3+}$ ).

the two lowest excited states of  $\text{Ho}^{3+}$  and  $\text{Tm}^{3+}$  ions are given in Table 3. In spite of  $K_2$  rate is expected to be dependent on the excitation density ( $\text{Tm}^{3+}$ ) (because ET<sub>2</sub> process involves two excited states interaction), we observed  $K_2$  rate slightly increasing for  $\text{Tm}^{3+}$  excitation density increase from  $0.5 \times 10^{18}$  to  $2 \times 10^{18} \text{ cm}^{-3}$ . However, it becomes constant when the excitation density is  $\geq 3 \times 10^{18} \text{ cm}^{-3}$ . The excitation density was estimated by measuring the mean energy of the laser pulse at 1671 nm and the excitation volume estimated to be  $1.57 \times 10^{-2} \text{ cm}^{-3}$ . This result is similar to that one reported [10] for the energy transfer up-conversion (ETU) involving two excited  $\text{Ho}^{3+}$  in the  $^5\text{I}_7$  level in Ho-doped ZBLAN [11]. This result justifies the use of  $K_2$  values mentioned in Table 2 as a rate constant of ET<sub>2</sub> process in the rate equations system (class 1) for numerical simulation since higher excited  $\text{Tm}^{3+}$  ion densities ( $N^* \sim 10^{19} \text{ cm}^{-3}$ ) are usually present in the system to have stimulated emission at 1.5  $\mu\text{m}$  under cw operation.

**Table 3**  
Branching ratio of luminescence, radiative lifetime and total lifetime (experimental) of  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  in ZBLAN (lifetime measurements done in low concentrations of  $\sim 0.15 \text{ mol}\%$ ).

	Branching ratio ( $\beta$ )	Radiative lifetime (ms)	Total lifetime (ms)	Non-radiative decay ( $\text{s}^{-1}$ )
<i>Transition (<math>\text{Tm}^{3+}</math>)</i>				
$^3\text{H}_4 \rightarrow ^3\text{H}_6$	0.9	1.32	1.46	$\sim 0$
$^3\text{H}_4 \rightarrow ^3\text{F}_4$	0.1			
$^3\text{F}_4 \rightarrow ^3\text{H}_6$	1	8.9	6.56	$\sim 0$
<i>Transition (<math>\text{Ho}^{3+}</math>)</i>				
$^5\text{I}_6 \rightarrow ^5\text{I}_8$	0.9	5.87	3.5	115
$^5\text{I}_6 \rightarrow ^5\text{I}_7$	0.1			
$^5\text{I}_7 \rightarrow ^5\text{I}_8$	1	12.6	16.4	$\sim 0$
Tm: Ho (mol %)	$K_1$ ( $\text{s}^{-1}$ )	$K_2$ ( $\text{s}^{-1}$ )	$K_3$ ( $\text{s}^{-1}$ )	
(y):(x)				
0.5:0.3	2550	0	4	
0.5:0.4	3355	0	8	
0.5:0.5	4295	0.013	14	
0.5:1	8456	0.09	71	
0.5:1.5	12617	0.26	188	
0.5:2	16778	0.6	371	

Values of  $K_1$ ,  $K_2$ ,  $K_3$  obtained by inspection of best-fit curves of experimental rate constant ( $K_i$ ) as a function of  $[\text{Tm}^{3+}]$  and  $[\text{Ho}^{3+}]$  concentrations measured for the  $\text{Tm}(0.5):\text{Ho}(x):\text{ZBLAN}$  samples. These  $K_i$  values were used in the numerical simulations of the rate equations system.  $\sigma = 2.8 \times 10^{-21} \text{ cm}^2$  at 797 nm (absorption cross-section of  $^3\text{H}_6 \rightarrow ^3\text{H}_4$  transition).

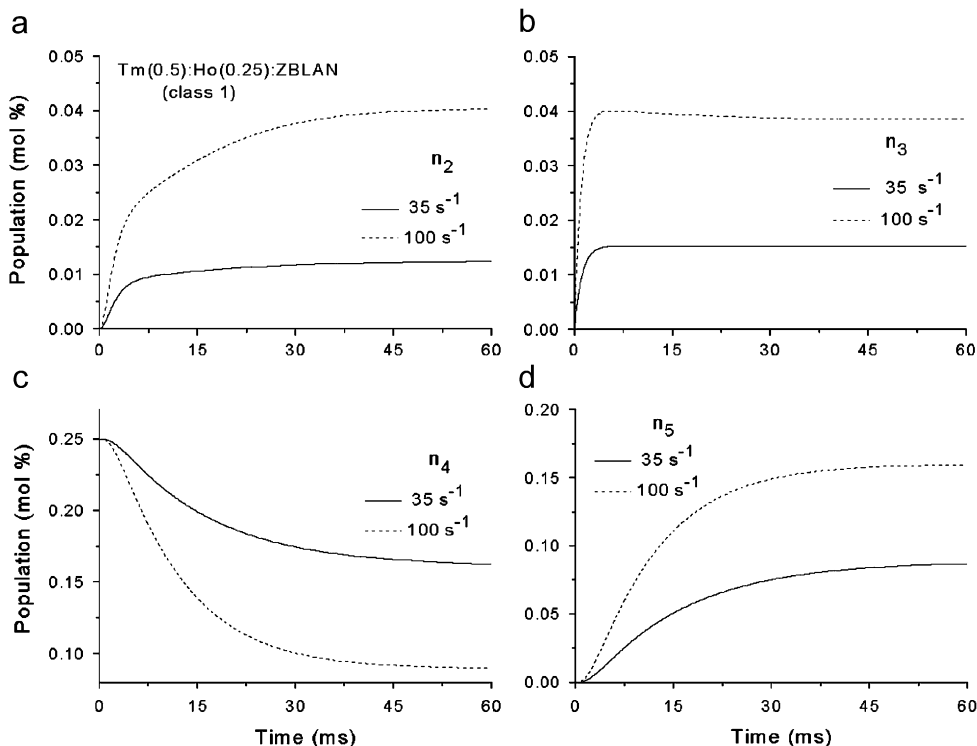


**Fig. 2.** Probability rate constants  $K_1$ ,  $K_2$  and  $K_3$  ( $\text{s}^{-1}$ ) due to Tm–Ho interactions as a function of  $[\text{Tm}^{3+}][\text{Ho}^{3+}]$  concentrations product. (d) Tm–Tm cross-relaxation rate ( $W_{\text{CR}}$ ) as a function of  $\text{Tm}^{3+}$  concentration.

Fig. 2(a) shows that  $K_1$  has a linear dependence on  $([\text{Tm}][\text{Ho}])$  concentration, as is expected for a migration-assisted energy transfer involving excited donor and acceptor ions in the ground state. However,  $K_2$  and  $K_3$  exhibit non-linear concentration dependence, i.e.,  $K_2 \propto ([\text{Tm}][\text{Ho}])^{n=2.71}$  (see Fig. 2(b)) and  $K_3 \propto [\text{Ho}]^{n=2.38}$  (see Fig. 2(c)), respectively. In addition,  $W_{\text{CR}}$  exhibits a quadratic dependence on  $\text{Tm}^{3+}$  concentration,  $W_{\text{CR}} \propto [\text{Tm}]^{n=2.2}$ , as expected for a cross-relaxation process (see Fig. 2(d)). Values of  $K_1$ ,  $K_2$ ,  $K_3$  and  $W_{\text{CR}}$  rate constants were estimated for Tm(0.5%):Ho(x%) systems by inspecting the best-fitted curves shown in Figs. 2(a)–(d) and are listed in Table 3. These rate constants values were used in the numerical simulations performed in Section 3.3.

### 3.2. Rate equations for optical excitation of the $^3\text{H}_4$ level

Previous investigation of  $^3\text{H}_4$  luminescence ( $\sim 1470 \text{ nm}$ ) of  $\text{Tm}^{3+}$  in Tm:Ho:ZBLAN glass, excited by short laser pulses (4 ns) of 780 nm, has shown that a residual  $^3\text{F}_4$  luminescence ( $\sim 1800 \text{ nm}$ ) of  $\sim 20\%$  remains besides the very strong  $\text{Tm}(^3\text{F}_4) \rightarrow \text{Ho}(^5\text{I}_7)$  energy transfer [3]. In addition, this reminiscence luminescence of  $^3\text{F}_4$  state exhibits an exponential decay with a lifetime similar to that one observed for single Tm-doped ZBLAN ( $\sim 7 \text{ ms}$ ). On the other hand, residual  $^3\text{F}_4$  luminescence was not observed in the case of Tm:Ho in tellurite [10], chalcogenide [7] and Tm:Tb in germanate (GLKZ) [4] glasses. This luminescence effect (residual) evidences that the fast energy migration through  $^3\text{F}_4$  excited states leads to two distinct types of  $\text{Tm}^{3+}$  excited ions in ZBLAN glass (or classes): (i) *class 1*—excited  $\text{Tm}^{3+}$  ions close by  $\text{Ho}^{3+}$  that has a  $K_1$  transfer rate resultant from  $\text{Tm}(^3\text{F}_4) \rightarrow \text{Ho}(^5\text{I}_7)$  transfer minus  $\text{Ho}(^5\text{I}_7) \rightarrow \text{Tm}(^3\text{F}_4)$  back-transfer rates and (ii) *class 2*—excited  $\text{Tm}^{3+}$  ions ( $^3\text{F}_4$ ) far from  $\text{Ho}^{3+}$  ions (or isolated) having an intrinsic lifetime of  $\sim 7 \text{ ms}$ . According to that observation, one can assume that 80% of  $\text{Tm}^{3+}$  ions interact with  $\text{Ho}^{3+}$  ions (*class 1*) and 20% behaves as



**Fig. 3.** Time evolution of  $n_2(t)$  and  $n_3(t)$  populations of  $\text{Tm}^{3+}$  and  $n_4(t)$  and  $n_5(t)$  populations of  $\text{Ho}^{3+}$ , numerical solutions of rate equations system (*class 1*) for Tm(0.5%):Ho(0.25%):ZBLAN continuous pumped at 797 nm. Pumping rates of 35 and  $100 \text{ s}^{-1}$  were used.  $\text{Ho}^{3+}$  ground state depopulation ( $n_4$ ) precludes the population inversion ( $n_3 - n_2 < 0$ ) for higher pumping rate ( $100 \text{ s}^{-1}$ ) shown by dashed line of (c).

isolated  $\text{Tm}^{3+}$  ions (*class 2*). Based on this argument, we must have two distinct rate equations systems for  $\text{Tm}:\text{Ho}:\text{ZBLAN}$  glass. One set of rate equations for *class 1* that includes  $ET_1$ ,  $ET_2$ ,  $ET_3$  and  $CR$  processes and another set for *class 2* (see rate equations system presented in Section 3.2).

Based on Fig. 1, one obtains the following rate equations system for the  $\text{Tm}:\text{Ho}:\text{ZBLAN}$  considered for continuous laser pumping the  $^3\text{H}_4$  ( $\text{Tm}^{3+}$ ) level ( $n_3$ ) at 797 nm. The  $n_1$ ,  $n_2$  and  $n_3$  represent the population of the  $^3\text{H}_6$ ,  $^3\text{F}_4$  and  $^3\text{H}_4$  levels of  $\text{Tm}^{3+}$ , and  $n_4$ ,  $n_5$  and  $n_6$  are the populations of the  $^5\text{I}_8$ ,  $^5\text{I}_7$  and  $^5\text{I}_6$  levels of  $\text{Ho}^{3+}$ , respectively.

(i) Rate equations system for *class 1*:

In this case, it was used  $n_1 + n_2 + n_3 = n_{\text{Tm}}$  and  $n_4 + n_5 + n_6 = n_{\text{Ho}}$ , where  $n_{\text{Tm}}$  and  $n_{\text{Ho}}$  are the  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  concentrations given in mol%:

$$\frac{dn_1}{dt} = -R_p n_1 + \frac{B_{21}}{\tau_{R_2}} n_2 + \frac{B_{31}}{\tau_{R_3}} n_3 - W_{\text{CR}} n_1 n_3 + K_1 n_2 n_4 + K_2 n_2 n_5 - K_{\text{BT}} n_1 n_5 \quad (9)$$

$$\frac{dn_2}{dt} = 2W_{\text{CR}} n_1 n_3 + \left( \frac{B_{32}}{\tau_{R_3}} + W_{\text{nr}}(32) \right) n_3 - \frac{\beta_{21}}{\tau_{R_2}} n_2 - K_1 n_2 n_4 - K_2 n_2 n_5 + K_3 n_3 n_4 \quad (10)$$

$$\frac{dn_3}{dt} = R_p n_1 - W_{\text{CR}} n_1 n_3 - \left( \frac{\beta_{32''} + \beta_{32} + \beta_{31}}{\tau_{R_3}} + W_{\text{nr}}(32) \right) n_3 - K_3 n_3 n_4 \quad (11)$$

$$\frac{dn_4}{dt} = -K_1 n_2 n_4 + \frac{\beta_{64}}{\tau_{R_6}} n_6 + \frac{\beta_{54}}{\tau_{R_5}} n_5 - K_3 n_3 n_4 + K_{\text{BT}} n_1 n_5 \quad (12)$$

$$\frac{dn_5}{dt} = K_1 n_2 n_4 - \frac{\beta_{54}}{\tau_{R_5}} n_5 + \left( \frac{\beta_{65}}{\tau_{R_6}} + W_{\text{nr}}(65) \right) n_6 + K_3 n_3 n_4 - K_2 n_2 n_5 - K_{\text{BT}} n_1 n_5 \quad (13)$$

$$\frac{dn_6}{dt} = K_2 n_2 n_5 - \left( \frac{\beta_{65} + \beta_{64}}{\tau_{R_6}} + W_{\text{nr}}(65) \right) n_6 \quad (14)$$

(ii) Rate equations system for *class 2*:

In this case, it was used  $n_1 + n_2 + n_3 = n_{\text{Tm}}$ .

$$\frac{dn_1}{dt} = -R_p n_1 + \frac{B_{21}}{\tau_{R_2}} n_2 + \frac{B_{31}}{\tau_{R_3}} n_3 - W_{\text{CR}} n_1 n_3 \quad (15)$$

$$\frac{dn_2}{dt} = 2W_{\text{CR}} n_1 n_3 + \left( \frac{B_{32}}{\tau_{R_3}} + W_{\text{nr}}(32) \right) n_3 - \frac{\beta_{21}}{\tau_{R_2}} n_2 \quad (16)$$

$$\frac{dn_3}{dt} = R_p n_1 - W_{\text{CR}} n_1 n_3 - \left( \frac{\beta_{32''} + \beta_{32} + \beta_{31}}{\tau_{R_3}} + W_{\text{nr}}(32) \right) n_3 \quad (17)$$

$R_p = \sigma_{13} n_1 (I_p/h\nu)$  is the pumping rate ( $\text{s}^{-1}$ ),  $I_p$  is the intensity of pumping given in ( $\text{W cm}^{-2}$ ) and  $h\nu$  is the photon energy of pumping radiation.  $\beta_{ij}$  represents the luminescence branching ratio and  $\tau_{R_i}$  is the radiative lifetime of  $^5\text{I}_6$ ,  $^5\text{I}_7$  ( $\text{Ho}^{3+}$ ) and  $^3\text{H}_4$ ,  $^3\text{F}_4$  ( $\text{Tm}^{3+}$ ) excited states labeled as level  $i = 6, 5$  for  $\text{Ho}^{3+}$  and  $i = 3, 2$  ( $\text{Tm}^{3+}$ ).

### 3.3. Numerical solution of the rate equations

Rate equations system was solved based on the Runge–Kutta numerical method of fourth order by means of a program

developed in Scilab language.  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  populations were determined for  $\text{Tm}(0.5\%):\text{Ho}(x\%):\text{ZBLAN}$  system using the numerical solutions obtained for simulating a cw laser pumping at 797 nm. The  $\text{Tm}^{3+}$  and  $\text{Ho}^{3+}$  populations were obtained after the populations get the equilibrium (after  $\sim 80$  ms).  $\text{Tm}^{3+}$  populations were obtained by considering the population distribution of *classes 1* and *2*, according to  $n_i = 0.8n_i(1) + 0.2n_i(2)$  where  $i = 1, 2$  and *3*. The  $\text{Tm}^{3+}$  population inversion was obtained using  $\Delta n = n_3 - n_2$ .

Fig. 3 shows the time evolution  $n_2(t)$  and  $n_3(t)$  populations of  $\text{Tm}^{3+}$  and  $n_4(t)$  and  $n_5(t)$  ( $\text{Ho}^{3+}$ ) for a pumping rates of 35 and  $100 \text{ s}^{-1}$  after cw laser pumping at 797 nm. Looking the population values of Figs. 3(a) and (b) at  $t = 60$  ms, one can see that the pumping rate of  $35 \text{ s}^{-1}$  produces a population inversion ( $n_3 - n_2$ ) of  $4.8 \times 10^{17} \text{ cm}^{-3}$  (*class 1*) in  $\text{Tm}(0.5\%):\text{Ho}(0.25\%)$  system. A negative population inversion was observed by increasing the pumping rate to  $100 \text{ s}^{-1}$ , as a consequence of the  $\text{Tm}(^3\text{F}_4) \rightarrow \text{Ho}(^5\text{I}_7)$  transfer saturation induced by severe  $\text{Ho}^{3+}$  ground state depopulation. By comparing Fig. 3(c) and (d) one concludes that  $n_2$  population increases at expenses of  $n_4$  population—ground state of  $\text{Ho}^{3+}$ . The saturation effect of  $\text{Tm} \rightarrow \text{Ho}$  transfer might be observed to happen for some high pumping rate ( $> 100 \text{ s}^{-1}$ ) in more  $[\text{Ho}^{3+}]$  concentrated system ( $> 0.3 \text{ mol}\%$ ) (Fig. 4). Fig. 5 shows the population

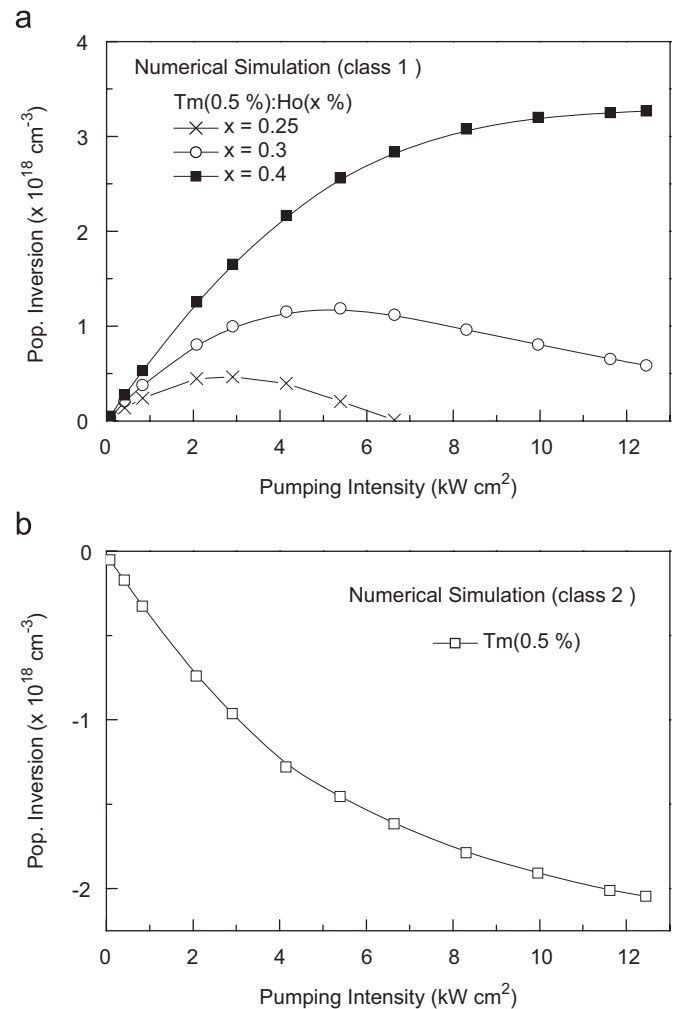
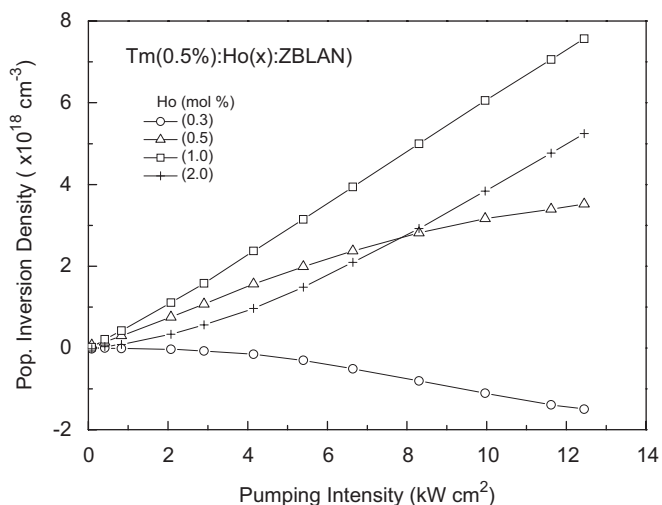
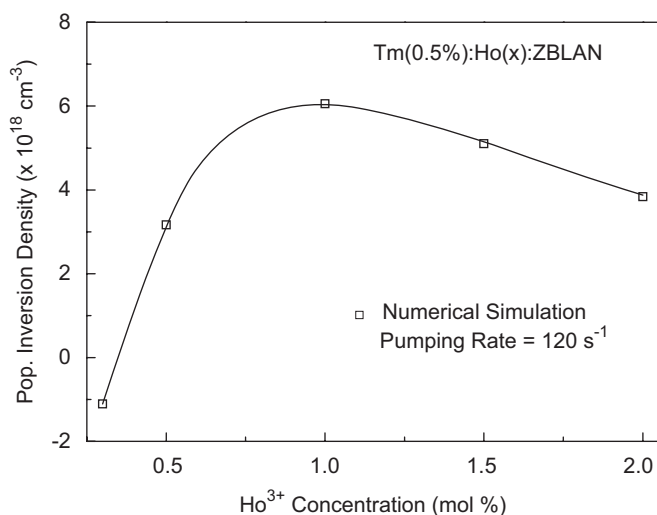


Fig. 4. The population density difference ( $n_3 - n_2$ ) obtained from numerical solutions of the rate equations system applied to  $\text{Tm}(0.5\%):\text{Ho}(x\%)$  system is shown as a function of the pumping intensity at 797 nm. (a) The result of numerical simulation applied to  $\text{Tm}-\text{Ho}$  system of *class 1* for many  $[\text{Ho}^{3+}]$  concentrations, where  $x = 0.25, 0.3$  and  $1 \text{ mol}\%$ . (b) The result applied to  $\text{Tm}(0.5\%)$  system of *class 2*.



**Fig. 5.** It is shown the population density difference ( $n_3-n_2$ ) obtained using the numerical solutions of rate equations system for Tm(0.5%):Ho( $x$ ) system as a function of cw pumping intensity at 797 nm, for many Ho<sup>3+</sup> concentrations, where  $x = 0.3, 0.5, 1$  and 2 mol%.



**Fig. 6.** Results of population density difference ( $n_3-n_2$ ) obtained by numerical simulations applied to Tm(0.5%):Ho( $x$ ) system for a constant pumping intensity of 10 kW cm<sup>-2</sup> (or pumping rate of 120 s<sup>-1</sup>) at 797 nm, for many Ho<sup>3+</sup> concentrations (where  $x = 0.3, 0.5, 1, 1.5$  and 2 mol%).

density difference ( $n_3-n_2$ ) obtained using the numerical solutions of the rate equations system of Tm(0.5%):Ho( $x$ ):ZBLAN as a function of the pumping intensity at 797 nm performed for many Ho<sup>3+</sup> concentrations, where  $x = 0.5, 1$  and 2 mol%. Fig. 5

(open squares) shows a population inversion ( $n_3-n_2$ ) that linearly increases on increasing the pumping intensity for [Ho<sup>3+</sup>] concentration equal to 1 mol%, at least for the highest intensity of 12 kW cm<sup>-2</sup> investigated. Fig. 6 shows that the population inversion ( $n_3-n_2$ ) is maximized for 1 mol% of Ho<sup>3+</sup> in Tm(0.5%):Ho( $x$ ):ZBLAN for a constant pumping intensity of 10 kW cm<sup>-2</sup>.

#### 4. Conclusions

In this work, we investigated the population inversion in Tm(0.5%):Ho( $x$ ):ZBLAN under continuous pumping at 797 nm for using it as optical amplifiers operating near 1.5  $\mu$ m. Highest population inversion of Tm<sup>3+</sup> was obtained using 1 mol% of Ho<sup>3+</sup>. A population inversion density of  $1.6 \times 10^{18}$  Tm<sup>3+</sup> ions cm<sup>-3</sup> was obtained for a pump intensity of 2.7 kW cm<sup>-2</sup>, which is  $\sim 5.3$  times bigger than that one verified in Tm(0.1%):Ho(0.15%):Ge-Ga-As-S-CsBr [7] ( $\sim 3 \times 10^{17}$  cm<sup>-3</sup>). This population inversion density is also higher than that one estimated for Tm(0.1%):Tb(0.2%):GLKZ [4] ( $\sim 2 \times 10^{17}$  cm<sup>-3</sup>) and for Tm(0.1%):Tb(0.15%):Ge-Ga-As-S-CsBr [7] ( $\sim 2.5 \times 10^{17}$  cm<sup>-3</sup>) glasses for similar pumping conditions. This result points that Tm:Ho:ZBLAN doped with 0.5 mol% of Tm<sup>3+</sup> and codoped with 1 mol% of Ho<sup>3+</sup> maximizes its amplification potential gain based on the Tm<sup>3+</sup>-stimulated emission near 1.5  $\mu$ m, so making this material very promising system for light signal amplification in S-band of telecommunication.

#### Acknowledgment

The authors thank the financial support from FAPESP (Grants nos. 1995/4166-0 and 2000/1086-0) and CNPq.

#### References

- [1] E.R.M. Taylor, L.N. Ng, J. Nilsson, R. Caponi, A. Pagano, M. Potenza, S. Sordo, IEEE Photon. Technol. Lett. 16 (3) (2004) 777; E.R. Taylor, L.N. Ng, N.P. Sessions, H. Buerger, J. Appl. Phys. 92 (1) (2002) 112.
- [2] W. Ryba-Romanowski, S. Golab, G. Dominiak-Dzik, M. Zelechower, J. Gabrys-Pisarska, J. Alloys Compds. 325 (2001) 215.
- [3] L.D. da Vila, L. Gomes, L.V.G. Tarelho, S.J.L. Ribeiro, Y. Messaddeq, J. Appl. Phys. 95 (10) (2004) 5451.
- [4] A.F.H. Librantz, L. Gomes, G. Pairier, S.J.L. Ribeiro, Y. Messaddeq, J. Lumin. 128 (2008) 51.
- [5] L.A. Bueno, A.S.L. Gomes, Y. Messaddeq, C.V. Santilli, J. Dexpert-Ghys, S.J.L. Ribeiro, J. Non-Cryst. Solids 351 (2005) 1743.
- [6] L.D. da Vila, L. Gomes, C.R. Eyzaguirre, E. Rodriguez, C.L. César, L.C. Barbosa, Opt. Mater. 27 (2005) 1333.
- [7] J.H. Song, J. Heo, S.H. Park, J. Appl. Phys. 97 (2005) 083542.
- [8] A.F.H. Librantz, L. Gomes, G. Pairier, S.J.L. Ribeiro, Y. Messaddeq, in: Proceedings of the SPIE, USA, 6190 (2006) G-1.
- [9] M. Inokuti, H. Hirayama, J. Chem. Phys. 43 (6) (1965) 1978.
- [10] L.D. da Vila, L. Gomes, C.R. Eyzaguirre, E. Rodriguez, C.L. César, L.C. Barbosa, Opt. Mater. 27 (2005) 1333.
- [11] A.F.H. Librantz, S.D. Jackson, F.H. Jagosich, L. Gomes, G. Poirier, S.J.L. Ribeiro, Y. Messaddeq, J. Appl. Phys. 101 (2007) 123111.