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# Kerr-lens mode locking resonator calculation with two nonlinear elements

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## Abstract

The effects of inserting a second nonlinear element in a laser resonator for Kerr-lens mode locking (KLM) are theoretically studied. The use of a highly nonlinear glass (SF 57) in broadband lasers as Cr:LiSAF and Ti:sapphire is analyzed. The role of this auxiliary nonlinear element in the KLM regime is investigated and compared to the regime with the absence of this second nonlinear medium. The analysis of a three-element resonator shows the possibility of enhancing the KLM regime for both short and long cavities as well as presenting a much higher beam sensitivity to the intracavity power. © 2001 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

Since the first work of Spence et al. [1] an extensive study on the Kerr-lens mode locked (KLM) laser regime has been reported [2–9]. Recently, KLM has been shown to be a very powerful technique for ultrashort pulse generation. The presence of a nonlinear (Kerr) medium and a hard aperture in the cavity leads to the mode locking regime. These works describe, in general, two kinds of laser media regularly used in the KLM

regime: the narrow spectral gain ones, like Nd:YLF, Nd:YAG and Nd:YVO<sub>4</sub>, that generate picosecond pulses, and the broad ones, like Ti:Al<sub>2</sub>O<sub>3</sub> and Cr:LiSAF, that generate femtosecond pulses. The narrow-line ones require the insertion of a highly nonlinear medium in the laser resonator to allow the KLM regime. The latter uses the gain medium also as the nonlinear element, in spite of its low nonlinear refractive index.

Several theoretical studies used the nonlinear matrix formalism providing results that analyze and allow the optimization of KLM resonators [10–13]. In this work we used the *ABCD* formalism introduced by Magni et al. [14–17] in order to study the KLM regime in an astigmatic compensated three-element cavity with two nonlinear media, as shown in Fig. 1. The high-reflecting flat mirror (HR) is the plane surface of the gain

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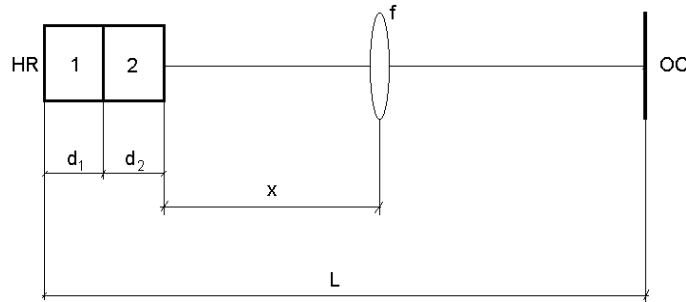


Fig. 1. Schematic of the astigmatic compensated three-element resonator. The medium 1 is the laser gain medium, the medium 2 is the auxiliary highly nonlinear medium, OC is the output coupler, HR is the high-reflecting mirror and  $f$  is the positive lens.

medium properly coated, a concave mirror with radius of curvature  $R$  ( $= 2f$ ) plays the role of the positive lens with focal length  $f$  and the output coupler (OC) is also a flat mirror. The KLM regime is strongly dependent on the distance between the nonlinear medium and the internal lens,  $x$ , that is the adjusting parameter. The cavity is of great interest due to its importance in this regime, as pointed out by references [18–21], due to its compactness, easy adjustment and capacity of operation at high repetition rates. Besides, due to its simplicity, analytical solutions using the  $ABCD$  formalism can be easily obtained.

## 2. Optical path difference introduced by the nonlinearity

We will study the role of a secondary nonlinear element in the cavity, as an auxiliary nonlinear medium. In order to properly evaluate its influence and the optimum length of this auxiliary nonlinear medium in the resonator, it is convenient to consider the optical path introduced by the nonlinearity of this medium for a gaussian beam. The refractive index of a Kerr medium is given by  $n_i = n_{Li} + n_{Ni}|E|^2$ , where  $n_{Li}$  and  $n_{Ni}$  are the linear and nonlinear refractive indexes of medium  $i$  respectively, so that the optical path difference normalized to the intracavity power introduced by the nonlinearity along the propagation direction  $z'$  is given by

$$\Gamma = \int_{z_0}^z \frac{n_N}{A(z')} dz', \quad (1)$$

where  $A(z')$  is the beam spot size;  $z_0$  is the position at the face of the nonlinear medium and  $z$  is the position at the exit of the medium. Two cases will be analyzed.

### 2.1. One Kerr medium only (medium 1)

In this case we consider a gaussian beam reaching this medium so that its beam waist does coincide with the medium input face, so that  $z_0 = 0$ , as shown in Fig. 2a. The spot size  $A(z')$  is given by the well-known equation for the propagation of gaussian beams [14]:

$$A(z') = \pi\omega_0^2 \left( 1 + \left( \frac{z'}{b} \right)^2 \left( 1 - \frac{P}{P_{C1}} \right) \right), \quad (2)$$

where  $\omega_0$  is the radius at the beam waist,  $b = \pi\omega_0^2 n_{L1} / \lambda$ ,  $P$  is the intracavity light power and  $P_{C1}$  is the critical power for self-focusing [14], given by  $P_{C1} = \lambda^2 / 2\pi n_{L1} n_{N1}$ . The normalized optical path  $\Gamma$  due to the nonlinearity is obtained by integrating the Eq. (1) with the help of Eq. (2):

$$\Gamma_1(z) \approx \frac{n_{L1} n_{N1}}{\lambda} \arctan \left( \frac{z}{b} \right). \quad (3)$$

### 2.2. Insertion of an additional Kerr medium

We consider now the insertion of a second nonlinear medium (medium 2) with a critical power of self-focusing  $P_{C2}$  as shown in Fig. 2b.

In order to analyze the influence of the medium 2 exclusively, let us consider the nonlinearity of medium 1 much smaller than that of medium 2,

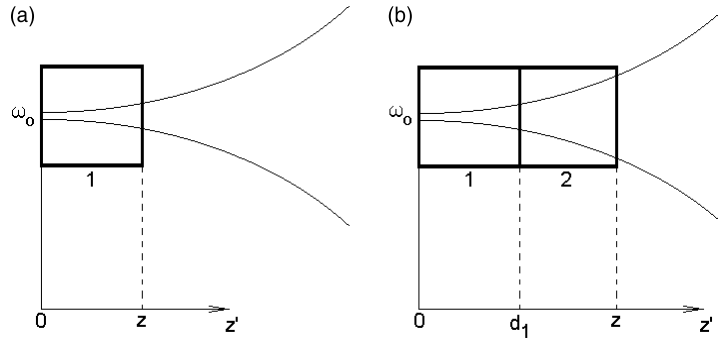


Fig. 2. (a) Gaussian beam propagating along the nonlinear medium 1; (b) along the linear medium 1 and the nonlinear medium 2.

such that medium 1 is considered linear and has now a fixed length  $d_1$ , so that  $z_0 = d_1$  in Eq. (1). In this case, it is more convenient to describe the gaussian beam propagation by using the *ABCD* matrix formalism. The matrix representing the beam propagation, shown in Fig. 2b is given by:

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \sqrt{1-\gamma} \begin{bmatrix} 1 & (z' - d_1)/n_{L2} \\ \frac{\gamma n_{L2}}{(z' - d_1)(1-\gamma)} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & d_1/n_{L1} \\ 0 & 1 \end{bmatrix}, \quad (4)$$

where

$$\gamma = \frac{P}{P_{C2}} \left[ 1 + \frac{1}{4} \left( \frac{2\pi\omega_c^2 n_{L2}}{\lambda(z' - d_1)} - \frac{\lambda(z' - d_1)}{2\pi\omega_0^2 n_{L2}} \right) \right], \quad (5)$$

where  $\omega_c$  is the radius of the beam at the center of medium 2, for  $P = 0$ . The  $z'$ -dependent beam spot size at the nonlinear medium can be obtained from Eq. (4) and is given by

$$\pi\omega^2(z') = \pi\omega_0^2 \frac{n_{L1}}{n_{L2}} \left( A_1^2 + \frac{B_1^2}{b^2} \right). \quad (6)$$

By substituting Eq. (6) in Eq. (1), the optical path normalized to the intracavity power introduced by the nonlinear medium 2 in the gaussian beam is given by:

$$\Gamma_2(z) \approx \frac{n_{L2} n_{N2}}{\lambda} \left[ \arctan\left(\frac{z}{b}\right) - \arctan\left(\frac{d_1}{b}\right) \right]. \quad (7)$$

Notice that, according to Eq. (7),  $\Gamma_2(z)$  only makes sense for  $z \geq d_1$ .

### 2.3. The case of the Cr:LiSAF laser medium and a nonlinear glass

In order to estimate the effect of the presence of the second nonlinear medium, we compare the optical path along the  $z'$  propagation axis introduced exclusively by a Cr:LiSAF crystal (medium 1, Eq. (3)), to the optical path introduced by a SF 57 glass (medium 2, Eq. (7), for  $d_1 = 0.35$  cm) as shown in Fig. 2b. The results are shown in Fig. 3. In this case the following parameters were used:  $\lambda = 835$  nm,  $\omega_0 = 15$   $\mu$ m,  $n_{N2} = 2.6 \times 10^{-15}$  cm<sup>2</sup>/W,  $n_{N1} = 1.5 \times 10^{-16}$  cm<sup>2</sup>/W,  $n_{L2} = 1.8$  and  $n_{L1} = 1.4$ .

Both curves in Fig. 3 were obtained in a conservative approach, i.e. for  $P \ll P_C$ . According to Eqs. (4)–(6), as the power  $P$  increases, the beam divergence decreases, and consequently the beam intensity and the optical path introduced by the nonlinearities of the media are larger, therefore the curves shown in Fig. 3 represent the lower limit case, for  $P \rightarrow 0$ . It can be then noticed that in spite of the greater intensity in the Cr:LiSAF medium than in the SF 57 glass, the optical path due to the nonlinearity of the latter is much larger than the optical path of the first, because of the very high nonlinear refractive index of such glass. As expected, both curves show a saturation behavior, which indicates that for larger values of  $z$  the beam spot size increases, decreasing the intensity as well

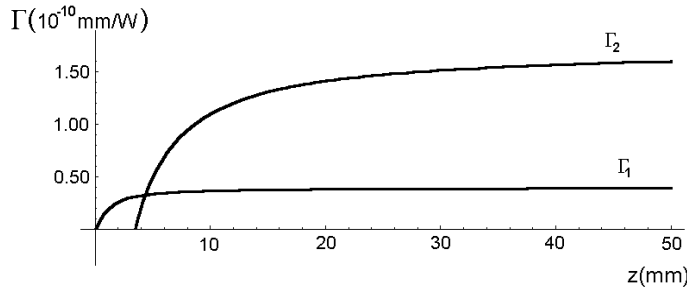


Fig. 3. Optical path introduced by the nonlinearity of the Cr:LiSAF crystal alone (lower curve,  $\Gamma_1$ ). The upper curve,  $\Gamma_2$ , is the optical path introduced by the nonlinearity of the SF 57 glass, positioned at  $d_1 = 0.35$  cm, with a thickness ( $z=3.5$  mm), as shown in Fig. 2.

as the nonlinear contribution for the refractive index. Therefore, Eqs. (3) and (7) are useful for determining the values of  $z$  for which  $\Gamma_1(z_1)$  and  $\Gamma_2(z_2)$  saturate, so  $z_1$  and  $z_2 - d_1$  can be considered the optimum lengths of the media.

### 3. The sensitivity factor calculated by the ABCD matrix formalism with two nonlinear media

For designing KLM resonators Magni et al. [15] introduced a very important parameter that describes the relative intensity dependent variation of the beam spot size, given by:

$$\delta = \left( \frac{1}{\omega} \frac{d\omega}{dp} \right)_{p=0}, \tag{8}$$

where  $p \equiv P/P_C$  is the normalized power. It is well known, from the ABCD matrix formalism for a three-element resonator, that the regions of interest concerning spot size variations occur at the cavity output coupler (for hard aperture KLM) and in the active laser medium (for soft aperture KLM) [20,21]. Consider the  $A$ ,  $B$ ,  $C$  and  $D$  the matrix elements referring to the beam propagation from the output coupler OC to the high-reflecting mirror HR of the resonator (length  $L$ ) shown in Fig. 1. The beam waist at OC is given by:

$$\omega^4 = - \left( \frac{\lambda}{\pi} \right)^2 \frac{AB}{CD}. \tag{9}$$

The nonlinear media 1 and 2 (i.e., the active laser medium and the auxiliary nonlinear medium,

respectively) can be represented by the transfer matrices  $M_1$  and  $M_2$  [15]:

$$M_{1,2} = \sqrt{1 - \gamma_{1,2}} \begin{bmatrix} 1 & d_{e1,2} \\ \frac{\gamma_{1,2}}{d_{e1,2}(1 - \gamma_{1,2})} & 1 \end{bmatrix}, \tag{10}$$

where  $d_{e1,2} = d_{1,2}/n_{L1,L2}$  ( $d_1$  and  $d_2$  are shown in Fig. 1), and  $\gamma_1$  and  $\gamma_2$ , according to Eq. (6), are given by

$$\gamma_1 = \frac{P}{P_{C1} \left( 1 + \frac{b^2(x)}{d_1^2} \right)} \equiv Pk_1$$

and

$$\gamma_2 = \frac{P}{P_{C2} \left[ 1 + \frac{1}{4} \left( \frac{2b(x)}{d_1} \frac{n_{L1}}{n_{L2}} \left( 1 + \frac{y^2}{b^2(x)} \right) - \frac{d_1}{2b(x)} \right)^2 \right]} \equiv Pk_2, \tag{11}$$

where  $y \equiv d_{e2}/2 + d_{e1}$  and  $b(x) = \pi\omega_0^2(x)n_{L1}/\lambda$ , where  $\omega_0(x)$  is the beam waist in the gain medium (shown in Fig. 2) and is given by:

$$\omega_0^4(x) = - \left( \frac{\lambda}{\pi} \right)^2 \frac{B_0 D_0}{A_0 C_0}, \tag{12}$$

where  $A_0$ ,  $B_0$ ,  $C_0$  and  $D_0$  are the matrix elements for a linear resonator. The parameter  $\delta$ , as expressed by Eq. (8) is, however, not suitable for the case of two intracavity nonlinear media, because each of them has its own critical power for self-focusing. A similar parameter to  $\delta$  can be introduced as an alternative to overcome this limitation. This

parameter  $\Delta$  relates the spot size directly to the intracavity power  $P$ , so that

$$\Delta = \left( \frac{1}{\omega} \frac{d\omega}{dP} \right)_{P=0}. \quad (13)$$

Qualitatively not much was modified from Eq. (8) to Eq. (13), except by the fact that  $\Delta$  depends on more than one nonlinear element in the laser cavity.

From Eqs. (9) and (13),  $\Delta$  can be obtained at the OC position as a function of the matrix elements  $A$ ,  $B$ ,  $C$  and  $D$ :

$$\Delta = \frac{1}{4} \left( \frac{1}{A} \frac{dA}{dP} + \frac{1}{B} \frac{dB}{dP} - \frac{1}{C} \frac{dC}{dP} - \frac{1}{D} \frac{dD}{dP} \right)_{P=0}. \quad (14)$$

The product of the matrices representing all the optical components in the resonator, shown in Fig. 1, gives the  $ABCD$  matrix from mirror OC to mirror HR. Taking  $\Delta_{12}$  for two intracavity elements and considering  $l = x + d_{e1} + d_{e2}$ , we obtain from Eq. (14):

$$\Delta_{12} = - \left( \frac{k_1}{4d_{e1}} + \frac{k_2}{4d_{e2}} \right) \left( \frac{f^2}{f+l-L} \right) - \frac{k_2}{4} \times \frac{d_{e1}}{d_{e2}} \left[ \frac{f^2(x-l)}{(f-l)[fL-l(L-l)]} \right]. \quad (15)$$

By displacing the folding concave mirror, the value of the adjusting parameter  $x$  can be properly selected in order to increase the KLM sensitivity. The factors  $k_1$  and  $k_2$  were defined in Eqs. (11). The KLM regime is obtained by inserting a hard aperture close to the output coupling plane in the resonator where  $\Delta$  has high negative values [15]. If the nonlinearity of medium 2 is negligible, as it is usually seen in Cr:LiSAF and the Ti:Al<sub>2</sub>O<sub>3</sub> KLM lasers,  $k_2 = 0$ ,  $\Delta_{12}(k_2 = 0) = \Delta_1$  and Eq. (15) becomes:

$$\Delta_1 = - \frac{k_1}{4d_1} \left( \frac{f^2}{f+l-L} \right). \quad (16)$$

Notice that, except by the multiplicative factor  $k_1$  (as  $\Delta_1 = k_1 \delta_1$ ),  $\Delta_1$  is essentially the same parameter  $\delta$  obtained in Eq. (7) of Ref. [20], that studies KLM regime for compact three-element resonators with only one nonlinear medium. In ref. [20], by using the stability condition  $0 < A_0 D_0 < 1$  of the linear cavity, it was shown that the KLM regime can be only achieved for very compact cavities, where  $L \leq 2f - d_{e1} - d_{e2}$ .

On the other hand, if the nonlinearity of the medium 2 is significantly higher than the linearity of the medium 1, then  $k_2 \gg k_1$  and the terms

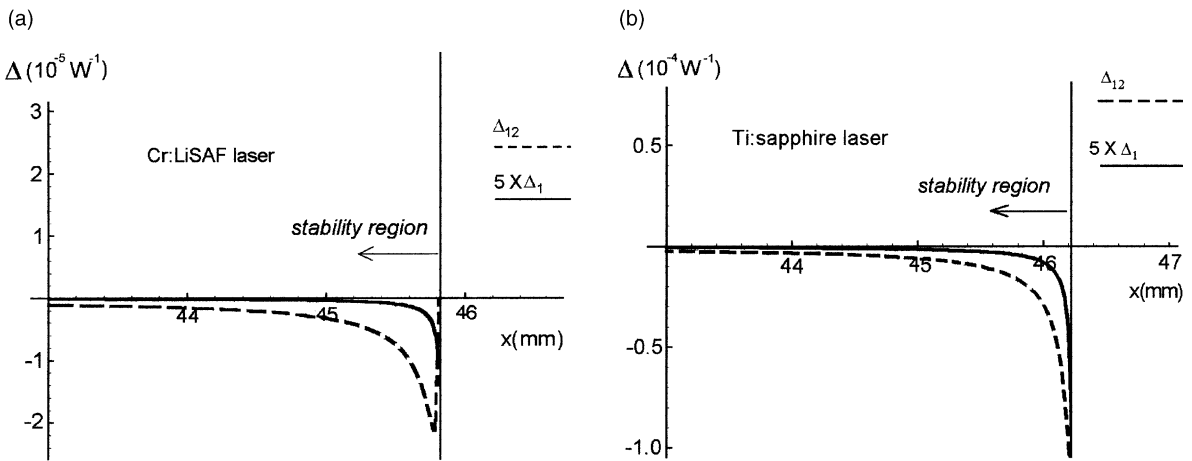


Fig. 4. (a)  $\Delta_{12}$  (dashed curve, with the SF 57 glass) and  $\Delta_1$  (multiplied by 5, solid curve, without auxiliary nonlinear medium) for a Cr:LiSAF compact ( $L = 9.5$  cm,  $d_1 = 0.35$  cm,  $d_2 = 0.3$  cm,  $f = 5$  cm) laser. The linear and the nonlinear refractive indexes of Cr:LiSAF are 1.4 and  $1.5 \times 10^{-16}$  cm<sup>2</sup>/W respectively; the linear and the nonlinear refractive indexes of the SF 57 glass are 1.8 and  $2.6 \times 10^{-15}$  cm<sup>2</sup>/W; (b)  $\Delta_{12}$  (dashed curve, with a SF 57 glass) and  $\Delta_1$  (solid curve, without auxiliary nonlinear medium) for a Ti:Al<sub>2</sub>O<sub>3</sub> compact ( $L = 9.5$  cm,  $d_1 = 0.35$  cm and  $d_2 = 0.3$  cm,  $f = 5$  cm) laser. The linear and the nonlinear refractive indexes of Ti:Al<sub>2</sub>O<sub>3</sub> are 1.76 and  $3.2 \times 10^{-16}$  cm<sup>2</sup>/W respectively.

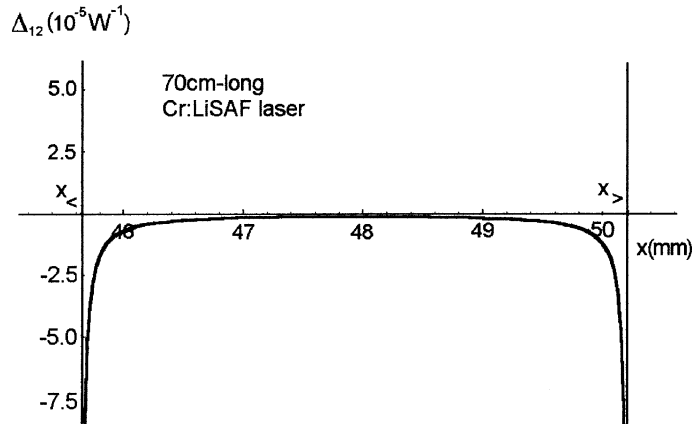


Fig. 5. The parameter  $\Delta_{12}$  for a long-cavity (70 cm) Cr:LiSAF laser with a SF 57 glass as an auxiliary nonlinear medium. The other resonator parameters are the same as those shown in Fig. 4.

containing the factor  $k_2$  in Eq. (15) predominate. Fig. 4a and b compare the behavior of  $\Delta_1$  and  $\Delta_{12}$  for a Cr:LiSAF and a Ti:Al<sub>2</sub>O<sub>3</sub> laser respectively. In both cases, the highly nonlinear medium 2 is a SF 57 glass and the cavity is compact ( $L = 9.5$  cm,  $f = 5$  cm,  $d_1 = 0.35$  cm and  $d_2 = 0.3$  cm).

The curves in Fig. 4 also show the great advantage of using an auxiliary highly nonlinear medium for KLM operation: for a given value of the adjusting parameter  $x$ , the absolute value of  $\Delta_{12}$  is more than one order of magnitude greater than  $\Delta_1$ , which allows a much easier alignment of the cavity and makes the KLM regime much more stable with respect to the thermal lensing and environmental conditions like thermal drift and mechanical instabilities. The highly negative values of  $\Delta_{12}$  also points to a better region for initializing the regime, as pointed out by Cerullo et al. [4,16].

Another remarkable advantage of using two intracavity nonlinear media is the possibility of working with larger cavities. This is due to the second term on the right-hand of Eq. (15). For long cavities ( $L > 4f$ ) the stability requirement  $0 < A_0 D_0 < 1$  for CW regime of the linear resonator leads to the following condition for the adjusting parameter  $x$ :

$$f - d_{e1} - d_{e2} \leq x \leq \frac{L}{2} - d_{e1} - d_{e2} - \left[ \left( \frac{L}{2} \right)^2 - Lf \right]^{1/2}. \quad (17)$$

Notice that as the denominator of the second term of the right hand of Eq. (15) tends to zero, the factor  $\Delta_{12}$  tends to very large negative values, which is highly desirable for KLM action. This condition is satisfied if

$$x \rightarrow \frac{L}{2} - d_{e1} - d_{e2} - \left[ \left( \frac{L}{2} \right)^2 - Lf \right]^{1/2} \equiv x_>, \quad (18)$$

or if

$$x \rightarrow f - d_{e1} - d_{e2} \equiv x_<.$$

Notice that  $x_>$  and  $x_<$  correspond to the greater and the lower limit of  $x$ , respectively, in a long resonator (Eq. (17)). Expressions (17) and (18) clearly show that the values of  $x$  that allow very large negative values of  $\Delta_{12}$  are within the stability region for long cavities. This is only possible by inserting a second intracavity nonlinear medium. Fig. 5 shows the behavior of  $\Delta_{12}$  in a 70-cm-long cavity Cr:LiSAF laser with the same parameters as in Fig. 4.

#### 4. Conclusions

The theoretical results of this work have shown the significant advantage of using an auxiliary highly nonlinear medium in order to achieve a very stable and low threshold KLM regime for the usual broad band, small nonlinearity gain media.

These results are not only applicable for simple three-element resonators, but can be also extended in a qualitative way to more complicated ones, such as “V” and “Z” resonators.

The parameter  $\Delta$ , defined in this work associated to the modulation depth, has much higher negative values when a highly nonlinear element is used as a second intracavity nonlinear medium. With the insertion of this material we have also shown the possibility of achieving KLM regime even in longer cavities, for which  $L > 4f$ , what is not possible if only the broadband laser medium is used as a nonlinear medium.

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