Determination of the Fundamental Laser Parameters Using an Acoustooptical Device

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***Abstract*—It is shown that an acoustooptical modulator used inside the laser cavity can emulate a variable transmission mirror. The transmission of the equivalent mirror formed by the modulator and the output coupler is a function of a known applied voltage, in such way that in a single run it is possible to determine the usual behavior of the output power as a function of the mirror transmission. By analyzing this dependence it is possible to obtain laser parameters such as unsaturated gain, pumping rate, and internal losses. Moreover, it is also demonstrated that we can obtain an analytical expression for the internal losses. This technique was used to characterize a CW Nd:YAG laser and it is shown that the rod quality, as well as the depolarization due to the thermal birefringence, are responsible for losses as high as 10% per pass. This is a simple and reliable method for characterizing a laser system in the CW regime.**

INTRODUCTION

THE characterization of the operation conditions of a CW laser is usually performed by changing the output coupler transmission using a set of mirrors and measuring the output power. The optimum output coupling is then easily obtained. In order to determine the internal losses, a measurement of the output power as a function of the pumping power (or energy) is usually done [1]. Both methods present some difficulties; in the former, one must be sure of the mirror quality as well as of the mirror curvatures and to operate the laser in the same spatial mode and in the same region in the gain medium. In the latter, one must infer the laser threshold by a derivative method, therefore assuming a linear dependence on this relation (i.e., assuming constant losses). In both cases the pumping conditions are not discussed. In particular, the pumping rate can provide an inside view of the laser pumping process that, in most cases, is not a straightforward process. By emulating a continuously variable transmission mirror, using an acoustooptical modulator, it is possible, in a single run, to measure the internal losses and the unsaturated gain (that gives the effective pumping rate). This is a direct method that uses one single element, permanently located inside the laser during the measurements, therefore preserving the spatial beam profile.

THEORY

Acoustooptical interaction is generally used in laser intracavity devices like Q switchers, mode lockers, and beam extractors. The usual operation regime is in the Bragg condition, where the impinging beam is partially deflected in the first order by the acoustic wave traveling in the interaction medium. The efficiency of power extraction per pass $\eta(V)$ is given by the known equation [2], [3]:

$$\eta(V) = \frac{I_1(V)}{I} = \sin^2[\beta \cdot V] \tag{1}$$

where I and $I_1(V)$ are the incident and deflected intensities, respectively, V is the peak amplitude of the radio frequency voltage applied to the device, and β is a coefficient, given in V^{-1}, proportional to the material figure of merit, the optical wavelength, the acoustooptical interaction geometry and electrical (piezoelectric) features of the device. The acoustooptical interaction geometry is assumed ideal [4].

For CW operation, the modulator can be considered as a variable transmission element. Its transmission is dependent on the applied voltage according to:

$$T = 1 - \eta(V). \tag{2}$$

Inside the cavity, the modulator is positioned close to the output coupler as shown in Fig. 1. Besides the usual output power by mirror $R(P_2)$ there are also two other deflected beams (P_1 and P_3) due to the action of the acoustooptical device. The total output power (P) is then given by the sum of the three extracted powers: $P = P_1 + P_2 + P_3$. The equivalent reflectivity is given by

$$R_{eq} = R \cdot T^2. \tag{3}$$

The total output power P is related to the outgoing intracavity power P_0 by:

$$P = P_0 \cdot (1 - R \cdot T^2). \tag{4}$$

Each extracted power component is related to P_0 by:

$$P_1 = (1 - T) \cdot P_0; \quad P_2 = (1 - R) \cdot T \cdot P_0;$$
$$P_3 = (1 - T) \cdot R \cdot T \cdot P_0. \tag{5}$$

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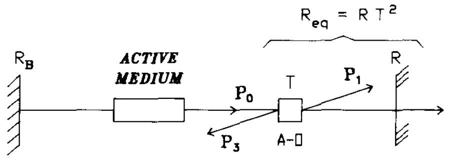

Fig. 1. Acoustooptic modulator inside the laser cavity. The three beams extracted from the cavity are shown: P_1 and P_3 are the diffracted beams, P_0 is the incident beam, and P_2 is the beam transmitted by the output mirror.

We can now express the total output power as a function of the measured power P_2. Using (4) and (5), we have:

$$P = \frac{P_2 \cdot (1 - R \cdot T^2)}{(1 - R) \cdot T}. \tag{6}$$

In order to determine the behavior of the output power as a function of the laser parameters, we assume the simplest model for steady-state oscillation, where the inverted population n_2 is given by [5]:

$$n_2 = \frac{n_{\text{tot}} \cdot W_P}{W_P + \tau_f^{-1} + \sigma_{21} \cdot I/(h\nu)} \tag{7}$$

where W_P is the effective pumping rate parameter, τ_f is the decay time, σ_{21} is the emission cross section, n_{tot} is the total concentration of active centers, and $h\nu$ is the photon energy.

Defining a saturation intensity:

$$I_S = (W_P + \tau_f^{-1}) \frac{h\nu}{\sigma_{21}} \tag{8}$$

the inverted population can be expressed by:

$$n_2 = \frac{n_{\max}}{1 + I/I_S} \tag{9}$$

where the maximum population $n_{\max}$ is the unsaturated population, expressed by:

$$n_{\max} = \frac{n_{\text{tot}} \cdot W_P}{W_P + 1/\tau_f}. \tag{10}$$

In the simplest case, W_P is given by the product of the absorption cross section and the pumping intensity. In the above expression, it is the final **effective** pumping rate, independent of the excitation process.

The gain coefficient γ can be expressed by the product of the inverted population and the emission cross section. Thus,

$$\gamma = \frac{\gamma_{\max}}{1 + I/I_S}. \tag{11}$$

The threshold condition for the laser oscillation is given by

$$R_{eq} \cdot R_B \cdot e^{(\gamma - \alpha) \cdot 2 \cdot l} = 1 \tag{12}$$

where l is the active medium length, α is the loss coefficient (per length unit), R_B is the reflectivity of the back mirror, and R_{eq} is the effective reflectivity of the equivalent mirror.

By combining (11) and (12), we obtain the dependence of the intracavity intensity:

$$I = I_S \cdot \left[\frac{2 \cdot l \cdot \gamma_{\max}}{L - \ln R_{eq}} - 1\right] \tag{13}$$

where L refers to the internal losses, and is given by

$$L = 2 \cdot l \cdot \alpha - \ln R_B. \tag{14}$$

In order to obtain the usual output power, we assume that the intracavity power variation P_E due to the stimulated emission, is given by the product of the intensity, the gain and the interaction volume (the Beer law):

$$P_E = \gamma \cdot I \cdot V_m \tag{15}$$

and it is valid for small signals; V_m is the average interaction volume ($V_m = l \cdot A$). In the equilibrium, this extra power is dissipated either as losses or as useful output power; therefore the output power is given by:

$$P = P_E \cdot \frac{-\ln R_{eq}}{L - \ln R_{eq}}. \tag{16}$$

Finally, the output power is then explicitly given by:

$$P = \frac{A \cdot h \cdot \nu}{2 \cdot \tau_f \cdot \sigma_{21}} \cdot (-\ln R_{eq}) \cdot \left[\frac{2 \cdot l \cdot \gamma_{\max}}{L - \ln R_{\text{eq}}} - 1\right]. \tag{17}$$

The usual optimum output coupling R^{opt} used to extract the maximum output power, is obtained by differentiating (17) with respect to R_{eq} and imposing the maximum condition. Thus, when $2 \cdot l \cdot \alpha >> \ln R_B$, we obtain:

$$R^{\text{opt}} = \frac{\exp\left[2 \cdot l \cdot (\alpha - \sqrt{\alpha \cdot \gamma_{\max}})\right]}{R_B}. \tag{18}$$

It is also possible to obtain the minimum value of the mirror reflectivity $R^{\min}$ in order to sustain the laser oscillation:

$$R^{\min} = \frac{\exp\,[2 \cdot l \cdot (\alpha - \gamma_{\max}]}{R_B}. \tag{19}$$

Now, using (18) and (19), we can have a **closed expression** for the internal losses coefficient α:

$$\alpha = \frac{1}{2 \cdot l} \cdot \frac{[\ln (R^{\text{opt}} \cdot R_B)]^2}{\ln [(R^{\text{opt}})^2 \cdot R_B] - \ln (R^{\min})}. \tag{20}$$

Besides, we can now express the output power for the optimum reflectivity as:

$$P^{\text{opt}} = I_S' \cdot A \cdot l \cdot (\sqrt{\gamma_{\max}} - \sqrt{\alpha})^2 \tag{21}$$

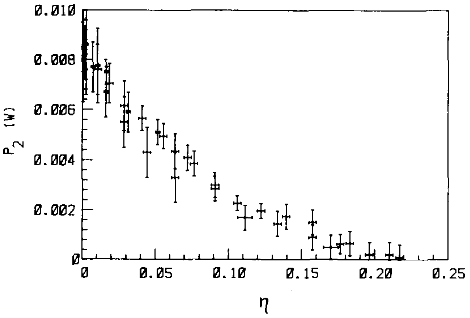

Fig. 2. Output power through the front mirror P_2 as a function of the modulator losses per pass η. The bars indicate the errors in a series of measurements. The lamp pump power is 3.25 kW.

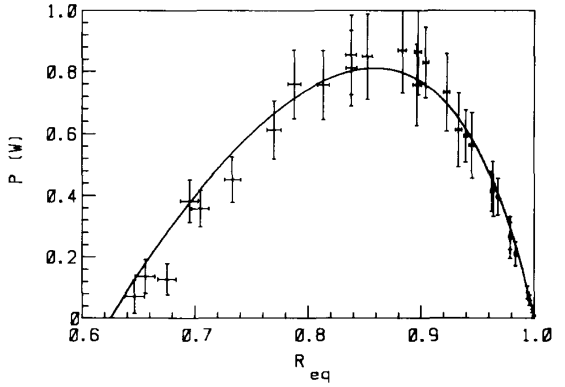

Fig. 3. Total output power (P) as a function of the equivalent reflectivity (R_{eq}). The solid curve corresponds to the best fit of the theoretical function to the data. The lamp pump power is 3.25 kW.

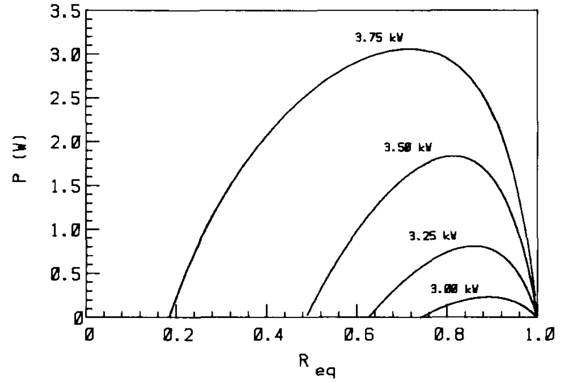

Fig. 4. Total output power (P) dependence on the equivalent reflectivity (R_{eq}), for different pump lamp currents. The curves shown are the best fit to the experimental values (as in Fig. 3).

where $I'_S = h\nu/\sigma\tau$. By using (18), it can be rearranged to:

$$P^{\mathrm{opt}} = I'_S \cdot A \cdot (\ln R^{\mathrm{opt}})^2/(4 \cdot l \cdot \alpha) \tag{22}$$

where we have assumed $R_B \cong 1$. The two above expressions allow us to clearly see that, even in the optimum output coupling, the losses are of fundamental importance.

EXPERIMENTAL RESULTS

The acoustooptical modulator is a commercial one, specially designed to operate as a Q switcher for the 1.064 μm, for unpolarized beam [6]. This can be accomplished by two quarter wave plates located at the ends of the modulator, in such way that the two orthogonal polarization components interact alternatively with the directions of the acoustic propagation ($\vec{E} \parallel \vec{K}$) and its normal ($\vec{E} \perp \vec{K}$) in a way that the final interaction for intracavity operation is polarization independent. The driver and the power supply for 50 MHz operation was home made, limited by a maximum output of 20 W of electrical power. The calibration of the modulator was performed outside the laser, by measuring the ratio of the deflected optical power to the incident one η as a function of the applied RF voltage, thus obtaining the coefficient β by the fit of (1).

The experiments were carried out in a commercial Nd:YAG laser. The output mirror was substituted by a flat one (keeping the original laser beam profile), with a transmission of 0.06%. The laser was operated in the fundamental TEM_{00} mode, that was accomplished by using two internal apertures, and the beam was also polarized by a Brewster plate located close to the back mirror. Fig. 2 shows the behavior of the output power P_2 as a function of $\eta(V)$, at 3 kW of pump lamp power.

The equivalent reflectivity R_{eq} can now be calculated, using (2) and (3) and the values of R and $\eta(V)$. It can be also calculated the equivalent total output power P from the values of P_2 and $\eta(V)$, by using (2) and (6). These new quantities are shown in Fig. 3, where it is also seen the error estimated by a standard analysis of error propagation. In the same figure, it is shown the fit of (17) to these new data, where the fit parameters were the loss coefficient (α), the small-signal gain coefficient ($\gamma_{\max}$) and the constant that is proportional to the average mode area. It was used a standard algorithm for least-squares estimation of nonlinear parameters.

The set of results (fitted curves) for different lamp currents are shown in Fig. 4. The fit of the theoretical expression (17) to each data set, provides us the values of the loss coefficient (α) and the small-signal gain coefficient ($\gamma_{\max}$). By using an average value from the literature for the emission cross section ($6.5 \cdot 10^{-19}$ cm^2), one can then determine $n_{\max} = \gamma_{\max}/\sigma$, the maximum (unsaturated) population. Then, by using (10), one can calculate the effective pumping parameter W_P. The results are shown in Table I. It is assumed a concentration of Nd ions in the rod of 1 mol%, that is equivalent to $n_{\mathrm{tot}} = 1.38 \cdot 10^{20}$ cm^{-3}.

TABLE I

Lamp Pump Power	3.00 kW	3.25 kW	3.50 kW	3.75 kW
gain coefficient $\gamma_{\max}$ (cm^{-1})	$3.3(2) \cdot 10^{-2}$	$4.2(1) \cdot 10^{-2}$	$5.8(3) \cdot 10^{-2}$	$1.2(5) \cdot 10^{-1}$
loss coefficient α (cm^{-1})	$1.2(2) \cdot 10^{-2}$	$1.0(1) \cdot 10^{-2}$	$1.0(1) \cdot 10^{-2}$	$7.0(1) \cdot 10^{-3}$
inverted population ratio $n_{\max}/n_{\mathrm{tot}}$	$3.7(2) \cdot 10^{-4}$	$4.7(1) \cdot 10^{-4}$	$6.5(3) \cdot 10^{-4}$	$1.3(6) \cdot 10^{-3}$
pumping rate W_P (s^{-1})	1.6(1)	2.06(3)	2.8(2)	6.0(2)

CONCLUSION

Internal Losses

From the results in Table I, we can calculate, for a 7.5 cm long rod, the losses per pass inside the laser. They range from ~5 to ~9%, decreasing with the increase of the pump power. They are mostly caused by the combination of the thermally induced birefringence in the Nd : YAG rod and the presence of the intracavity polarizer (Brewster plate). In the Nd : YAG rod, birefringence effect increases linearly with the pump power. However, as the pump power is increased, the thermal lensing effect also increases, reducing the average mode area in the rod, therefore decreasing effects of spatial inhomogeneity. The magnitude of the birefringence also increases as the square of the beam radius in the rod [7]; therefore as the average mode area decreases, the internal losses also decrease. The maximum depolarization effect would correspond to a loss per pass of 19% (due to the Fresnel reflection at the Brewster plate surfaces). For the 9% loss, the average electrical field rotation per pass in the crystal can be calculated and corresponds to 22°. This result is in agreement with previously reported results [7]. We've also measured the polarization of the laser output **without the polarizer** and we observed that **the beam is 99% polarized**. It was also observed that the beam polarization changes, depending on the lamp power and the particular rod position. Therefore, it is very important to carefully align and polarizing element with the thermally induced preferential direction of polarization.

The measured losses are approximately 1.5 times higher than the useful standard mirror transmission (12%); therefore it is possible to increase much more the optimum output transmission if the internal losses are reduced, significantly increasing the useful output power. As an example, for the commercial laser operating at 26 A, one can increase the output power by a factor of 4, by reducing the losses to almost 0.001 cm^{-1}, as can be seen using (21) and the values listed in Table I.

As it is shown in Table I, the losses decrease as the pump power increases, demonstrating that excited state absorptions are not significant for the Nd : YAG laser. The loss values shown in Table I are due to the fitting of expression (17); they agree very closely with the calculated values using (20), as expected.

Gain Per Pass

As it is expected for a system like the CW Nd : YAG laser, the small-signal gain coefficient (shown in first row of Table I) is rather small, ranging from 3.3% cm^{-1} up to 12% cm^{-1}. It is, of course, due to the small emission cross section of the Nd ions in this host, the low concentration of active ions used to circumvent this small σ_{21} and the low pumping rate, as will be discussed further.

Pumping Rate

The pumping rate can be obtained by inverting (10), thus obtaining:

$$W_P = \left[\tau_f \cdot \left(\frac{n_{\mathrm{tot}}}{n_{\max}} - 1\right)\right]^{-1}. \tag{23}$$

The obtained results for $n_{\mathrm{tot}}/n_{\max}$ and the pumping rates are also shown in Table I. The fraction of the available population used as active medium is of the order of one part per thousand, expressing how poor is the net mechanism of population inversion in this laser, due to the fact that the pumping rate is much smaller than the decay time. In order to minimize the rate of spontaneous decay, the population threshold must be minimized by reducing the internal losses. This can be a serious bottleneck for mode-locking operation where the depth of modulation per pass defines the minimum pulsewidth. If we take the standard 1.5 m long resonator (10 ns round-trip time) the maximum gain variation in order to have a steady-state solution is approximately $4 \cdot 10^{-5}$, for a lamp power of 3.75 kW. Also, in the system under analysis, it is easily understood why it is difficult to obtain passive mode locking.

This operation regime was achieved only by increasing the resonator length to 30 m (factor of 20), allowing for a gain variation of approximately 10^{-3}, and using a saturable absorber with an initial transmission of 99.9%. The pulses are twice longer than those obtained in the active mode-locking scheme [8]. Besides, in experiments with synchronous pumping of Nd : YAG lasers, it was reported the generation of pulses longer than the ones obtained with active mode locking [9]; the reported gain variation in this experiment is greater than in the CW pumping but the internal losses are presumably much higher than in the lamp pumping scheme. So, in spite of being capable of developing shorter pulses, the large threshold population limits the maximum intracavity intensity that can be developed in steady-state regime. This is in accordance with the measured values of pumping rate and the above considerations.

It is possible to consider, for more precise calculations, expressions that take into account the Gaussian beam profile, the inhomogeneous pumping, and also the longitudinal hole burning in the active medium [10], [11]. Preliminary calculations show that, by carefully considering these effects, it is possible to obtain the emission cross section of the active medium. By considering an average area for the laser beam inside the medium and homogeneous pumping, the cross section obtained is too small ($1 \cdot 10^{-19}$ cm^2). No particular care was taken to have a single longitudinal mode along the rod, so longitudinal hole burning effect is not as important as the other two corrections.

Summarizing, we have developed as simple method for measuring, in one single setup, the fundamental parameters for lasers under CW pumping. We used a known Nd:YAG lamp-pumped commercial system and a commercial Q switcher to obtain these parameters; in this case, there is a depolarization effect (due to the thermal induced stress in the rod) that, in the presence of the Brewster plate, accounts for the high losses measured; besides, it is clearly shown that the pumping is extremely inefficient for this system. In particular, the method is general enough to be used in any large system, being only limited by the resonator space to insert an acoustooptical device. The determination of the fundamental laser parameters is of particular importance when testing new laser media, new pumping configurations or when evaluating the mode-locking regime.

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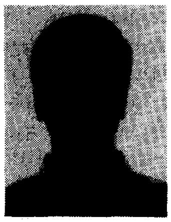

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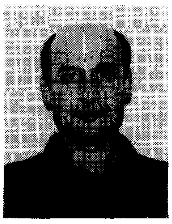

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