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A THEORETICAL STUDY OF T-VIOLATION IN ^{192}Pt

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Abstract: A theoretical study is made of T-violation in nuclear γ-decay in terms of a phenomenological T-violating internucleon potential. The results are applied to experimental data obtained for γ-decay in ^{192}Pt and an upper limit is derived for the strength of the T-violating potential.

1. Introduction

Various measurements have been made in the last few years of nuclear γ-ray angular correlations and Mössbauer transitions aimed at detecting T-violating effects [1,2]. Such effects manifest themselves as T-odd asymmetries and are proportional to sin η where η is the phase angle between interfering multipoles (e.g. M1-E2) in the γ-transition under test and which takes the values $\eta = 0$ or π if time-reversal-invariance holds [1,2]. Recently, Holmes *et al.* [3,4]) have searched for T-violating asymmetries in two γ-decays (γ and γ' – see fig. 1) in ^{192}Pt and have obtained the following experimental result:

$$|\sin \eta_1' + 0.19 \sin \eta_1| = (4 \pm 5) \times 10^{-3}, \tag{1}$$

where η_1 and η_1' are the corresponding (M1-E2) phase angles in the transitions γ_1 and γ_1' respectively.

The object of this paper is to attempt an interpretation of this result in terms of a simple phenomenological T-violating potential included in the nuclear Hamiltonian. In sect. 2 we introduce some general considerations and then in the following sections go on to deal with the specific decay.

2. General considerations

Including T-violating interactions (from whatever fundamental cause) the total nuclear Hamiltonian relevant to the discussion of nuclear γ-transitions can be written in the form

$$H = H_1 + \mathscr{H}_1(A), \tag{2}$$

where

$$H_1 = H_0 + V_{\mathrm{t.v.}},$$

$$\mathscr{H}_1(A) = \mathscr{H}_0(A) + \mathscr{H}_{\mathrm{t.v.}}(A).$$

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Here H_0 is the time-reversal-invariant nuclear Hamiltonian, $\mathscr{H}_0(A)$ is the usual electromagnetic interaction, $V_{\text{t.v.}}$ is a parity-conserving T-violating internucleon potential and $\mathscr{H}_{\text{t.v.}}(A)$ is a parity-conserving T-violating electromagnetic interaction. Each term is taken to be a function of the nucleon co-ordinates. Because of current conservation it is impossible to construct a T-violating nucleon-photon vertex interaction when the nucleon is on the mass shell [5]) and this means, therefore, that $\mathscr{H}_{\text{t.v.}}(A)$ is invariably a two- (or higher) body operator equivalent in status to the usual exchange operators which contribute to nuclear γ-transitions. The total Hamiltonian should, of course, be gauge invariant and although this condition determines parts of $\mathscr{H}(A)$ and $\mathscr{H}_{\text{t.v.}}(A)$ there is no unique prescription [e.g. Sachs [6])] and full detailed forms for these two terms can only be obtained by reference to the fundamental elementary particle interactions which generate H. Here it should be noted, however, that in the case of *electric* multipole operators stemming from $\mathscr{H}_{\text{t.v.}}(A)$, as will be seen, Siegert's theorem [6, 7]) enables the corresponding matrix element to be related simply to matrix elements of $V_{\text{t.v.}}$ in the lowest order of kR (where k is the photon momentum and R a typical nuclear dimension)†. This is not possible with magnetic multipole operators. The precise forms of such multipole operators have been worked out by Clement and Heller [8]) and Coutinho [9]) for specific elementary particle models of T-violation in the electro-magnetic interaction.

Consider now the electric multipole contribution to the γ-transition $a \rightarrow b+\gamma$ between two eigenstates ϕ_a and ϕ_b of H_1. Using the notations and conventions of Rose and Brink [10]) we have:

$$(\phi_a|T^{e'}_{LM}(k)|\phi_b) = (\phi_a|[H_1, D_{LM}(k)]|\phi_b) = (E_a-E_b)(\phi_a|D_{LM}(k)|\phi_b), \tag{3}$$

where $T^{e'}_{LM}(\text{k})$ is the electric interaction multipole operator, the prime indicating that T-violating effects are included, and $D_{LM}(k)$ is given explicitly by

$$D_{LM}(k) = \frac{m\beta}{\hbar^2 k}\alpha^e_L \sum_i (1+\tau^{(i)}_3) r^L_i C^{(i)}_{LM}, \tag{4}$$

where

$$\alpha^e_L = \frac{(ik)^L}{(2L-1)!!}\left(\frac{L+1}{2L}\right)^{\frac{1}{2}},$$

$$C^{(i)}_{LM} = \left(\frac{4\pi}{2L+1}\right)^{\frac{1}{2}} Y_{LM}(\theta_i, \phi_i).$$

In these expressions $k(= E_a-E_b)$ is the momentum of the emitted photon††, β is the nuclear magneton, m is the nucleon mass, and r_i, θ_i, ϕ_i are the co-ordinates of the ith nucleon and $\tau^{(i)}_3$ its 3-component of isospin. It should be noted that $T^e_{LM}(k) \sim k^L$ and $D_{LM}(k) \sim k^{L-1}$.

† See footnote on pag. 160.

†† Since $V_{\text{t.v.}}$ is odd under time reversal its diagonal matrix elements are identically zero. For this reason E_a and E_b can be taken to be the eigenvalues of H_0 correct to first order in $V_{\text{t.v.}}$.

Treating $V_{\text{t.v.}}$ as a perturbation and expanding $\phi_{a,b}$ in terms of the eigenstates $|a, b\rangle$ of H_0 gives

$$(\phi_a|T^{e'}_{LM}(k)|\phi_b) = (E_a - E_b)\left[\langle a|D_{LM}(k)|b\rangle + \sum_{c\neq b}\frac{\langle a|D_{LM}(k)|c\rangle\langle c|V_{\text{t.v.}}|b\rangle}{E_b - E_c} + \sum_{c\neq a}\frac{\langle a|V_{\text{t.v.}}|c\rangle\langle c|D_{LM}(k)|b\rangle}{E_a - E_c}\right]. \tag{5}$$

This expression does not require specific knowledge of the form of $\mathscr{H}_{\text{t.v.}}(A)$ and T-violation is introduced solely through the T-violating potential $V_{\text{t.v.}}$.

A corresponding relation can be written down for the matrix element of the magnetic interaction multipole operator $T^{m'}_{LM}$. However, in this case there is no simple relation of the type given in eq. (3) and so the final expression takes the form

$$(\phi_a|T^{m'}_{LM}(k)|\phi_b) = \left[\langle a|T^{m}_{LM}(k)|b\rangle + \sum_{c\neq b}\frac{\langle a|T^{m}_{LM}(k)|c\rangle\langle c|V_{\text{t.v.}}|b\rangle}{E_b - E_c} + \sum_{c\neq a}\frac{\langle a|V_{\text{t.v.}}|c\rangle\langle c|T^{m}_{LM}(k)|b\rangle}{E_a - E_c} + \langle a|T^{m(\text{t.v.})}_{LM}(k)|b\rangle\right], \tag{6}$$

where $T^{m}_{LM}(k)$ is the usual magnetic interaction multipole operator [10]) having an energy dependence k^L, and $T^{m(\text{t.v.})}_{LM}(k)$ is a 2-body magnetic multipole operator stemming from $\mathscr{H}_{\text{t.v.}}(A)$.

As mentioned in sect. 1, the size of the T-violating physical effect is proportional to $\sin\eta$ where η is the relative phase between the electric and magnetic multipoles contributing to the transition under investigation and is equal to 0 or π if time-reversal-invariance holds. In the usual case of interfering $\mathrm{E}(L+1)$ and $\mathrm{M}L$ multipoles we can write for the mixing ratio δ in terms of reduced matrix elements [10]):

$$\delta = \frac{(\phi_a||T^{e'}_{L+1}(k)||\phi_b)/(2L+3)^{\frac{1}{2}}}{(\phi_a||T^{m'}_{L}(k)||\phi_b)/(2L+1)^{\frac{1}{2}}} = |\delta|\,e^{i\eta}, \tag{7}$$

where

$$\eta = \eta_0 + \varepsilon^{e}_{L+1} - \varepsilon^{m}_{L}, \tag{8}$$

and $\eta_0 = 0$ or π. To first order in the T-violating terms ε^{e}_{L+1} and ε^{m}_{L} are given by (see eqs. (5) and (6))

$$i\varepsilon^{e}_{L+1} = [\langle a||D_{L+1}(k)||b\rangle]^{-1}\left[\sum_{c\neq b}\frac{\langle a||D_{L+1}(k)||c\rangle\langle c|V_{\text{t.v.}}|b\rangle}{E_b - E_c} + \sum_{c\neq a}\frac{\langle a|V_{\text{t.v.}}|c\rangle\langle c||D_{L+1}(k)||b\rangle}{E_a - E_c}\right], \tag{9}$$

$$i\varepsilon^{m}_{L} = [\langle a||T^{m}_{L}(k)||b\rangle]^{-1}\left[\sum_{c\neq b}\frac{\langle a||T^{m}_{L}(k)||c\rangle\langle c|V_{\text{t.v.}}|b\rangle}{E_b - E_c} + \sum_{c\neq a}\frac{\langle a|V_{\text{t.v.}}|c\rangle\langle c||T^{m}_{L}(k)||b\rangle}{E_a - E_c} + \langle b||T^{m(\text{t.v.})}_{L}(k)||a\rangle\right]. \tag{10}$$

These expressions can now form a basis from which to calculate the magnitude of the T-violating effect in the particular transitions under study in the γ-decay of ^{192}Pt. This is done in sect. 3.

3. The decay of ^{192}Pt

Elements of the ^{192}Pt level scheme are shown in fig. 1 where the two decays under test for T-violation are γ_1 and γ_1'. As noted in sect. 1 it has proved impossible experimentally to disentangle the two cascades γ_1-γ_2 and γ_1'-γ_2' so that a composite experimental result as set out in eq. (1) is obtained. From the theoretical point of view the problem is then, to estimate the sizes of η_1 and η_1' in terms of the strength of some assumed T-violating interaction using the general results of sect. 2.

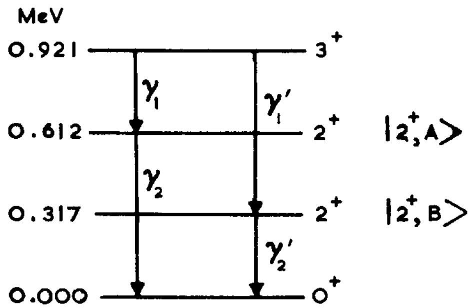

Fig. 1. Elements of the ^{192}Pt level scheme.

For the purposes of this calculation we first of all assume that the perturbation sums in eqs. (9) and (10) can be replaced by a single term resulting from the mutual admixing of the two adjacent 2^+ states ($|2^+, A\rangle$ and $|2^+, B\rangle$ in fig. 1) in ^{192}Pt. This is probably a reasonable assumption to make because of the small energy denominator ($\Delta E \approx 296$ keV) and the fact that there seem to be no other nearby states having the same spin and parity as the other states involved in the γ_1 or γ_1' transitions †. From eqs. (9) and (10) we can, therefore, write for the different phase factors:

γ_1 *transition*:

$$i\varepsilon_2^{\mathrm{e}}(\gamma_1) = \frac{\langle 3^+||D_2(k_1)||2^+, \mathrm{B}\rangle}{\langle 3^+||D_2(k_1)||2^+, \mathrm{A}\rangle} \frac{\langle 2^+, \mathrm{B}|V_{\mathrm{t.v.}}|2^+, \mathrm{A}\rangle}{\Delta},$$

$$i\varepsilon_1^{\mathrm{m}}(\gamma_1) = \frac{\langle 3^+||T_1^{\mathrm{m}}(k_1)||2^+, \mathrm{B}\rangle}{\langle 3^+||T_1^{\mathrm{m}}(k_1)||2^+, \mathrm{A}\rangle} \frac{\langle 2^+, \mathrm{B}|V_{\mathrm{t.v.}}|2^+, \mathrm{A}\rangle}{\Delta} + \xi_1; \tag{11}$$

† Recently Gari and Huffman [11]) have pointed out in connection with *parity violating* potentials that (i) there are uncertainties in making such a truncation in a perturbation expansion in which use has been made of Siegert's theorem and (ii) Siegert's theorem can only be used with complete confidence in the static limit [12]) (which is not the case here since $V_{\mathrm{t.v.}}$ is momentum dependent). No account is taken of either of these uncertainties in this paper since we are only concerned with making a crude estimate of the T-violating effect.

γ_1' *transition*:

$$i\varepsilon_2^{\rm e}(\gamma_1') = \frac{\langle 3^+||D_2(k_1')||2^+, {\rm A}\rangle}{\langle 3^+||D_2(k_1')||2^+, {\rm B}\rangle}\, \frac{\langle 2^+, {\rm A}|V_{\rm t.v.}|2^+, {\rm B}\rangle}{-\Delta},$$

$$i\varepsilon_1^{\rm m}(\gamma_1') = \frac{\langle 3^+||T_1^{\rm m}(k_1')||2^+, {\rm A}\rangle}{\langle 3^+||T_1^{\rm m}(k_1')||2^+, {\rm B}\rangle}\, \frac{\langle 2^+, {\rm A}|V_{\rm t.v.}|2^+, {\rm B}\rangle}{-\Delta} + \xi_1'; \tag{12}$$

where $\Delta = E_{\rm A} - E_{\rm B}$ ($= 0.296$ MeV) and

$$\xi_1 = \frac{\langle 3^+||T_1^{\rm m(t.v.)}(k_1)||2^+, {\rm A}\rangle}{\langle 3^+||T_1^{\rm m}(k_1)||2^+, {\rm A}\rangle}, \qquad \xi_1' = \frac{\langle 3^+||T_1^{\rm m(t.v.)}(k_1')||2^+, {\rm B}\rangle}{\langle 3^+||T_1^{\rm m}(k_1')||2^+, {\rm B}\rangle}. \tag{13}$$

Now the probabilities for the two decays γ_1 and γ_1' are given by [10]

$$P(\gamma_1) = \frac{4}{\hbar} k_1 \frac{|\langle 3^+||T_2^{\rm e}(k_1)||2^+, {\rm A}\rangle|^2}{5} [1 + |\delta(\gamma_1)|^{-2}],$$

$$P(\gamma_1') = \frac{4}{\hbar} k_1' \frac{|\langle 3^+||T_2^{\rm e}(k_1')||2^+, {\rm B}\rangle|^2}{5} [1 + |\delta(\gamma_1')|^{-2}], \tag{14}$$

where $\delta(\gamma_1)$ and $\delta(\gamma_1')$ are the mixing ratios (see eq. (7)) for the two transitions. Using these relations together with eq. (3) and the known energy dependence of the multipole operators $T_L^{\rm e,\,m}(k)$ it is straightforward to obtain the following expressions for the phases η_1 and η_1' in the two transitions:

$$\eta_1 = \eta_0(\gamma_1) + \varepsilon_{L+1}^{\rm e}(\gamma_1) - \varepsilon_L^{\rm m}(\gamma_1)$$
$$= \eta_0(\gamma_1) \mp \frac{i\langle 2^+, {\rm B}|V_{\rm t.v.}|2^+, {\rm A}\rangle}{\Delta} \left[\frac{P(\gamma_1')k_1(1+|\delta(\gamma_1)|^{-2})}{P(\gamma_1)k_1'(1+|\delta(\gamma_1')|^{-2})}\right]^{\frac{1}{2}} \left[\frac{k_1}{k_1'} - \frac{\delta(\gamma_1)}{\delta(\gamma_1')}\right] \frac{k_1}{k_1'} - \xi_1, \tag{15}$$

$$\eta_1' = \eta_0(\gamma_1') + \varepsilon_{L+1}^{\rm e}(\gamma_1') - \varepsilon_L^{\rm m}(\gamma_1)$$
$$= \eta_0(\gamma_1') \mp \frac{i\langle 2^+, {\rm B}|V_{\rm t.v.}|2^+, {\rm A}\rangle}{\Delta} \left[\frac{P(\gamma_1)k_1'(1+|\delta(\gamma_1')|^{-2})}{P(\gamma_1')k_1(1+|\delta(\gamma_1)|^{-2})}\right]^{\frac{1}{2}} \left[\frac{k_1'}{k_1} - \frac{\delta(\gamma_1')}{\delta(\gamma_1)}\right] \frac{k_1'}{k_1} - \xi_1'. \tag{16}$$

In the above expressions the sign ambiguity ($\mp$) derives from the square-rooted term. Use has also been made of the fact that $\langle 2^+, {\rm B}|V_{\rm t.v.}|2^+, {\rm A}\rangle$ is imaginary so that $\langle 2^+, {\rm B}|V_{\rm t.v.}|2^+, {\rm A}\rangle = -\langle 2^+, {\rm A}|V_{\rm t.v.}|2^+, {\rm B}\rangle$.

From experiment [3,4] we have the following input data to the above expressions:

$$k_1 = 0.309 \text{ MeV}, \qquad k_1' = 0.604 \text{ MeV},$$

$$P(\gamma_1)/P(\gamma_1') = \text{branching ratio} = 77/22,$$

$$\delta(\gamma_1) = 5.66^\dagger,\ \delta(\gamma_1') = 1.63^\dagger.$$

† These values, of course, do not include any possible imaginary parts due to T-violating effects and are used solely to evaluate the different square brackets in eqs. (15) and (16). It should be stressed that they are the values appropriate to the Rose-Brink [10]) definition of δ (see eq. (7)).

On substituting these values we obtain:

$$\sin\eta_1 = \mp 0.651 \frac{i\langle 2^+, \mathrm{B}|V_{\mathrm{t.v.}}|2^+, \mathrm{A}\rangle}{\Delta} - \xi_1,$$

$$\sin\eta_1' = \mp 13.39 \frac{i\langle 2^+, \mathrm{B}|V_{\mathrm{t.v.}}|2^+, \mathrm{A}\rangle}{\Delta} - \xi_1'. \tag{17}$$

Taken with the experimental result of Holmes *et al.* [3,4]) given in eq. (1) this leads to

$$\left| \mp 13.41 \frac{i\langle 2^+, \mathrm{B}|V_{\mathrm{t.v.}}|2^+, \mathrm{A}\rangle}{\Delta} - (\xi_1' + 0.19\xi_1) \right| = (4 \pm 5) \times 10^{-3}. \tag{18}$$

Unless there is a virtually complete accidental cancellation between the two terms in the modulus this enables us to impose an approximate limit on the magnitude of the matrix element of $V_{\mathrm{t.v.}}$, namely,

$$\left| \frac{i\langle 2^+, \mathrm{B}|V_{\mathrm{t.v.}}|2^+, \mathrm{A}\rangle}{\Delta} \right| \approx (3 \pm 4) \times 10^{-4}. \tag{19}$$

Or, using the experimental value of $\Delta (= 0.296$ MeV)

$$|i\langle 2^+, B|V_{\mathrm{t.v.}}|2^+, A\rangle| \approx 90 \pm 110 \text{ eV}. \tag{20}$$

The next task, therefore, is to attempt a theoretical evaluation of the above matrix element.

4. Evaluation of $\langle 2^+, \mathrm{B}\,|\,V_{\mathrm{t.v.}}\,|\,2^+, \mathrm{A}\rangle$ and discussion

The possible phenomenological forms of *T*-violating potentials have been set out by Herczeg [13]) and specific forms deriving from a *T*-violating electromagnetic interaction have been obtained by Huffman [14]) and Clement and Heller [8]). It is clear from inspection of these results that $V_{\mathrm{t.v.}}$ must be velocity dependent and, whatever its origin, highly complicated. Further, although collective wave functions are available for the different states in ^{192}Pt [e.g. Kumar [15])] no detailed microscopic forms suitable for calculating the above matrix element have been obtained.

In view of this it is only possible to obtain a very crude estimate of the limits imposed by eqs. (19) and (20) on the strength of a possible *T*-violating potential. In order to do this we note that there is a strong electromagnetic transition between the states $|2^+, \mathrm{A}\rangle$ and $|2^+, \mathrm{B}\rangle$ and therefore it is reasonable to assume that there are sizeable components of each state differing by at most a single particle orbital. Under this circumstance the main contribution to the matrix element $\langle 2^+, \mathrm{B}|V_{\mathrm{t.v.}}|2^+, \mathrm{A}\rangle$ can be represented by using an equivalent *single particle T-violating potential* for $V_{\mathrm{t.v.}}$. This method [e.g. Michel [16])] has also been used to describe parity-violating effects in nuclei but it should be stressed that the approach is only likely to be approximately valid when the above assumption holds. For other states the implicit neglect of terms differing by two single particle orbitals could be serious.

We take the following single particle form for $V_{\text{t.v.}}$:

$$V_{\text{t.v.}} = G_{\text{t.v.}} \sum \tfrac{1}{2}(\boldsymbol{p}_i \cdot \hat{\boldsymbol{r}}_i + \hat{\boldsymbol{r}}_i \cdot \boldsymbol{p}_i), \tag{21}$$

where $G_{\text{t.v.}}$ is a measure of the strength of the T-violating potential and $\hat{\boldsymbol{r}}_i$ is a unit vector. Assuming no velocity dependence of the strong internucleon potential other than a possible spin-orbit term, $V_{\text{t.v.}}$ can be written

$$V_{\text{t.v.}} = imG_{\text{t.v.}}[H_0, \sum_i r_i]. \tag{22}$$

It then follows straightforwardly that

$$\left| \frac{i\langle 2^+, \text{B}|V_{\text{t.v.}}|2^+, \text{A}\rangle}{\Delta} \right| = |-G_{\text{t.v.}} m\langle 2^+, \text{B}| \sum_i r_i |2^+, \text{A}\rangle| \approx |G_{\text{t.v.}} mR|, \tag{23}$$

where R is a typical nuclear dimension and, as mentioned earlier, it is assumed that the states $|2^+, \text{A}\rangle$ and $|2^+, \text{B}\rangle$ have similar structures. More detailed calculations [Coutinho [9])] starting with a spin and isospin dependent two-body potential lead in the end to a similar result.

On this basis the experimental limit given in eq. (19) leads to

$$|G_{\text{t.v.}} mR| \approx (3 \pm 4) \times 10^{-4}. \tag{24}$$

The only theoretical estimate of $G_{\text{t.v.}}$ is that which can be derived (by averaging the two-body potential) from the work of Huffman [14]) which used a specific model of T-violation in the electromagnetic interaction. After much approximation Coutinho [ref. [9])] has concluded that the value of $G_{\text{t.v.}}$ so derived corresponds to a value rather lower than the upper limit implied by the above result. Experiment, therefore, does not rule out the presence of a T-violating potential having a strength of the order of magnitude suggested by Huffman's work.

It is also interesting to make a comparison with the strength of the parity-violating internucleon potential ($G_{\text{p.v.}}\, \boldsymbol{\sigma} \cdot \boldsymbol{p}$) deriving from the weak interaction. Here, Michel [ref. [16])] obtained

$$|G_{\text{p.v.}} mR| \approx 1 \times 10^{-7}, \tag{25}$$

a value which has subsequently been confirmed by experiment [see e.g. Blin-Stoyle [2]) for a review].

Clearly, there are many approximations in the latter stages of the foregoing work. But in view of the many uncertainties surrounding the nuclear structure and the possible form of the T-violating potential it is probably not worth trying to improve on them at present. There is every incentive, however, to try and reduce the experimental upper limit given in eq. (1) and to carry out similar experiments in nuclei having structures more amenable to theoretical treatment.

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