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Defining the confidence level in the reference temperature determination for a finite set of toughness

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Abstract

The ASTM E08.08 subcommittee has approved a standard, ASTM E1921-97, to determine the reference temperature, T_0, for ferritic steels in the transition region. This T_0 value positions a master curve in the transition region. The minimum number of valid experimental results in the data set to be analyzed by this standard for a given temperature, T, is six (however, no confidence level is given when using six specimens for the T_0 determination).

In this study a Monte-Carlo simulation was used to find the probability that T_0 will be within a given tolerance for a set of specimens, using randomly generated subsets representing the experimental results. The T_0 dependence on the number of results are presented graphically in two forms: (1) the confidence level when T_0 is obtained from a given set of valid experimental results, and (2) the minimum number of valid experimental results necessary to have T_0 with a given confidence level (from 70% to 98%).

From the results it was found that the confidence level on T_0 determination, when using only six valid results obtained at $(T - T_0) = 0°C$, is less than 90% and that the best range for testing is $-25°C \leqslant (T - T_0) \leqslant +50°C$.

Keywords: Fracture mechanics; Transition; Reference temperature; Master curve; Reliability

1. Introduction

The trend in fracture toughness behavior versus test temperature is well known for ferritic steels. Three regions of fracture are observed. At the lower temperatures there is a brittle region, or lower shelf, where all fracture is caused by a cleavage mechanism that results is a sudden unstable fracture with a low toughness value. At the higher temperatures there is a ductile region or upper shelf, where the fracture occurs by ductile tearing, resulting in slow stable crack growth and resulting in much higher values of toughness. In both these regions the scatter in the results is relatively small if the specimen size requirements in the fracture toughness test standards are satisfied. Between these two well-defined regions there is the transition

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region where the fracture may have a combined fracture mechanism, beginning with some stable crack extension and terminating with an abrupt cleavage. In this transition region there is extensive scatter in the toughness values and a strong influence of size, geometry and constraint level. To deal with these difficulties in determining fracture toughness in the transition region, a statistical treatment is required.

1.1. Statistical treatment

Following the standard ASTM E1921-97 [1] this is done with a three-parameter Weibull statistical distribution (Eq. (1a)) where P (Eq. (1b)) is the probability that fracture occurs at or below K_{J_c}. N is the number of experimental toughness values obtained at the test temperature T.

$$1 - P = \exp\left(- \left[\frac{K_{J_c} - K_{\min}}{K_0 - K_{\min}} \right]^m \right) \tag{1a}$$

$$P = \frac{i - 0.3}{N + 0.4} \tag{1b}$$

The toughness results should be ranked in an increasing order and after that, i is the order of a given toughness value. In principle the three parameters (m, K_0 and $K_{\min}$) should be obtained by a best fit method for each set of toughness results. However, it was shown by a statistical microstructural model [2] and by a large compilation of results [3] that the exponent m approaches 4 as the number of experiments increases. Statistical simulations [2] showed that the $K_{\min}$ value should be somewhere between 10 and 20 MPa $\sqrt{\mathrm{m}}$. In Ref. [4] it is suggested to set this $K_{\min}$ value at 20 MPa $\sqrt{\mathrm{m}}$, based on a warm prestress argument. This argument was discussed in Ref. [5]. Some approaches were suggested in [6–9] to obtain this $K_{\min}$ lower-bound material value from the measured set of toughness values. The ASTM standard E1921-97 [1] assumes that $K_{\min} = 20$ MPa $\sqrt{\mathrm{m}}$ and $m = 4$. So, in Eq. (1a) K_0 is the only parameter to be determined by a best fit procedure given by Eq. (2).

$$K_0 = \left(\frac{\sum_{i=1}^{N} \left(K_{J_{c,i}} - K_{\min} \right)^4}{N - 0.3068} \right)^{1/4} + K_{\min} \tag{2}$$

Once K_0 is known, and from Eq. (1a), considering 50% of failure probability, that is: $P = 0.5$, the median value of the toughness distribution, $K_{J_c,\mathrm{med}}$, can be determined (Eq. (3)).

$$K_{J_c,\mathrm{med}} = (K_0 - K_{\min})0.9124 + K_{\min} \tag{3}$$

1.2. The master curve

The master curve method was originally proposed by Wallin and co-workers [10–12] to deal with the results of fracture toughness of ferritic steels in the transition region for 1 T specimens. The master curve assumes that the median toughness values of fracture ($K_{J_c,\mathrm{med}}$) follow a predictable trend with temperature (T) given by Eq. (4).

$$K_{J_c,\mathrm{med}} = 30 + 70\mathrm{e}^{0.019(T - T_0)} \tag{4}$$

The T_0 value positions the master curve in the transition region range. The units are MPa $\sqrt{\mathrm{m}}$ for toughness and °C for temperature. From the master curve definition, when T equals T_0 the median toughness ($K_{J_c,\mathrm{med}}$) equals 100 MPa $\sqrt{\mathrm{m}}$.

To conduct a master curve evaluation the measured toughness results, usually J_c, J integral at cleavage, are transformed to equivalent stress intensity factor K_{J_c} assuming plane stress conditions –$K_{J_c} = \sqrt{(J_c E)}$, where E is the elastic modulus. Tests conducted at one temperature, T, are required to determine T_0, and from this, the entire median toughness ($K_{J_c,\mathrm{med}}$) versus temperature behavior can be predicted.

When the toughness results are from specimens of the same geometry, but with a different size and thickness, a correction based on the weakest link theory is applied to transform them to equivalent unit thickness (1 T) results. Recent study [13], based on a large database of experimental results for steels in the transition, showed that the master curve shape correctly describes the trends in toughness with $(T - T_0)$ between −75°C and 50°C.

1.3. The reference temperature (T_0) determination

The ASTM E1921-97 standard [1] provides guidelines for the reference temperature (T_0) determination adopting the three parameter Weibull statistical distribution. Initially, the K_0 parameter of the Weibull distribution should be determined from the measured set of toughness using Eq. (2) and supposing there are only valid results. A K_{J_c} value is valid if it is lower or equal than the $K_{J_c,\mathrm{limit}}$ given by Eq. (5) where b_0 is the specimen remaining ligament, σ_{ys} is the material yielding stress and $M = 30$. In the standard there are guidelines to treat those sets with some invalid data but having at least six valid ones.

$$K_{J_c,\mathrm{limit}} = \sqrt{\frac{\sigma_{ys} E b_0}{M}} \tag{5}$$

From the K_0 value, and using Eq. (3), the $K_{J_c,\mathrm{med}}$ value is obtained and using Eq. (4) in the appropriate fashion, the T_0 value can be determined.

1.4. T_0 adjustment for lower bound curve

In its appendix X3 the standard [1] defines lower and upper bounds curves associated, for example, with 5% and 95% of probability, that can be plotted with the master curve. These bounds, determined from the assumption of an infinite sample size, can encompass most of the scatter in the results. A margin adjustment, an upward temperature shift (ΔT_0) of the lower bound curve (Eq. (6)), is added to cover the uncertainties in T_0 obtained with a reduced number of values. The factor β in this equation is given in Table 1 and γ is a standard two-tail normal deviate that should be taken from statistical handbooks.

1.5. Number of specimens

The standard [1] recommends that the test temperature T be close to the reference temperature T_0 and, for this, it gives guidelines to determine T based on Charpy V-notch data. As mentioned, a minimum of $N = 6$ valid results, obtained at one temperature and geometry, is required. This number should be increased depending on the test temperature and geometry of the specimen according to Table 1.

Table 1
Number N of valid K_{J_c} values needed to obtain T_0 and β values to adjust the lower bound curve (ASTM E1921, 1997)

$K_{J_c,\mathrm{med}}$ (1 T) MPa $\sqrt{\mathrm{m}}$	(β) 49 to 52 (N) 50 to 52	53 to 57	58 to 65	66 to 83	>83
β (°C)	22.7	21.4	20.1	18.8	18.0
N	10	9	8	7	6

$$\Delta T_0 = \frac{\beta}{\sqrt{N}}\gamma \tag{6}$$

In the standard (ASTM, 1997) there is no defined confidence level associated with a given number of results and test temperature. A natural question is: how many results are necessary to have a given confidence in the obtained T_0? Or: what is the confidence level in the master curve (T_0) when six valid toughness results are used? And how sensitive is this confidence level to a change in the number of test results?

In some cases, it may not be possible to have this minimum number of experimental toughness results required by the ASTM E1921 standard but it is still necessary to characterize the material in the transition. A technique to obtain the master curve from just one single specimen, at a given temperature, was proposed using a lower bound estimate and order statistics to generate a fictitious distribution [14], to deal with those situations where there is just one specimen to test. However, due to the uncertainties in the obtained results, this technique should be reserved for those extreme cases where insufficient material quantities exist to prepare the required number of specimens.

This paper tries to address the questions above. To do this, some Monte-Carlo simulations were performed: subsets of toughness were randomly sampled from an infinite set, with an associated $(T - T_0)_u$ value. The infinite set fits perfectly to the three-parameter Weibull distribution. The ASTM E1921 procedure [1] was applied to each one of these subsets in order to obtain the associated distributions of the $(T - T_0)_s$ values. From these distributions the analyses were done and the desired results were obtained.

2. Work outline

The general idea was to sample subsets of N toughness values from the infinite set, labeled the 'universe'. Each subset can be viewed as an 'imperfect' set because a $(T - T_0)_s$ value, different from the correct one, $(T - T_0)_u$, will be obtained. Applying the procedure proposed in the ASTM E1921-97 standard the respective $(T - T_0)_s$ value of each subset was calculated.

The $(T - T_0)_s$ values associated with all sampled subsets formed a distribution where the median value is expected to be near its target $(T - T_0)_u$ value if the sampled subsets are a good representation of the 'universe' from which they were taken. It was assumed that the central portion of this distribution, defined as $\pm 10°C$ around $(T - T_0)_u$, defines an acceptable value of $(T - T_0)_s$ when N values are used. Comparing the number of subsets falling within this range versus those falling outside defines the probability to have $|(T - T_0)_s - (T - T_0)_u| \leqslant 10°C$. Therefore, each distribution was divided in three regions:

first region: $(T - T_0)_s < (T - T_0)_u - 10°C$
second region: $(T - T_0)_u - 10°C < (T - T_0)_s < (T - T_0)_u + 10°C$
third region: $(T - T_0)_s > (T - T_0)_u + 10°C$

and the number of subsets falling in each region was counted. For the scope of this work it was assumed that the relative counting obtained in the second region defines the desired reliability or confidence level in the T_0 determination when N valid values are experimentally obtained at a temperature T. The adoption of this $\pm 10°C$ spread value around the expected $(T - T_0)_u$ value was arbitrarily chosen as a reasonable range for T_0 determination. If this spread value is reduced (increased), the number of specimens to be tested to give a certain confidence level will increase (reduce).

2.1. Previous work

A direct approach was tried [15] defining two finite 'universes' with $\mathrm{NTOT} = \{20, 40\}$ toughness values each and fitting perfectly the three-parameter Weibull distribution. All possible subsets, formed by N values taken from $\{3, 4, 5, 6, 8, 10\}$, were sampled simulating the sets of experimental measurements. Also, for each NTOT value three $(T - T_0)_u$ values were defined: −50°C, 0°C and +50°C. So the grid of analyses was formed by $2 \times 3 \times 6$ cases. The result of this approach showed that the confidence in the T_0 determination varied with the number of results in one subset and with the temperature. The results also showed a strong influence of the reliability on the value of NTOT, suggesting that a bigger set should be used in the analysis. Sampling all possible subsets became impractical with this approach because the number of calculations needed were prohibitively large.

2.2. Present work

The Monte-Carlo method was used as a new approach to address the problem of how to determine the confidence level in the reference temperature determination when using a set with N experimental toughness values. Nine $(T - T_0)_u$ values: from ranging from −100°C to 100°C with 25°C increments were used with the N values taken as $\{3, 4, 5, 6, 8, 10, 14, 20, 25, 35, 50, 100\}$.

Results from Ref. [16] determine how many subsets should be sampled to give a reasonable distribution of $(T - T_0)_s$. To do so, three values were tested: 10^3, 10^4 and 10^5. The results showed that for 10^4 or more subsets the $(T - T_0)_s$ distributions were identical. So in the present work the results were obtained using 3×10^4 subsets for each calculation. This value was chosen because the $(T - T_0)_s$ distribution curves become slightly smoother than the ones obtained using 10^4 subsets. The subsequent results do not change significantly. To create the curves showing the $(T - T_0)_s$ distributions the range of the 30 000 $(T - T_0)_s$ values was divided in 50 groups. These groups were taken in equal increments of the $(T - T_0)_s$ range from the minimum to the maximum of the 30 000 values. The three regions defined above were considered in the $(T - T_0)_s$ distributions. From these, the curves "minimum N versus $(T - T_0)_u$" and all other results and conclusion were obtained.

2.3. Analyses

Following the philosophy of the Monte-Carlo Method, a 'universe' with infinite values of probabilities $P(0 \leqslant P \leqslant 1)$ was assumed, associated with a given $(T - T_0)_u$ and, from this, 30 000 sets, each one with N values, were randomly sampled.

The analyses were performed in four steps as described:

Step 1 – calculation of $K_{0,u}$ – this value is associated with the infinite universe of probabilities, for a given $(T - T_0)_u$. To obtain this value, the Eqs. (4) and (3) were used.

Step 2 – obtaining each subset with N toughness values ($K_{J_{c,i}}$*)* – for each sampled random probability P_i the Eq. (1a) was applied to obtain the respective K_{J_c} values, using the previous $K_{0,u}$ value (with $m = 4$ and $K_{min} = 20$ MPa $\sqrt{m}$).

Step 3 – treating each subset – the Eqs. (2)–(4), were applied in the appropriate fashion, to each subset with N toughness values, to obtain their $(T - T_0)_s$ values.

Step 4 – treatment of results – for each value of N there are 3×10^4 values of $(T - T_0)_s$ to work with in this step 4.

No censoring [1] was applied in the toughness values that came from the sampled probabilities. That should be noticed that when using $(T - T_0)_u = -100$°C and $N = 3$ and 4, some few subsets came with

$K_{J_c,\mathrm{med}} < 30$. This fact happened about 1000 times for $N = 3$ and about 100 times for $N = 4$. This would cause a $\log(x < 0)$ in the $(T - T_0)_s$ calculation using the Eq. (4) in the reverse order and can imply that the randomly created subset was in the lower shelf region. These subsets were eliminated and new ones were created.

Steps 2 and 3 were repeated 30 000 times for each N value. Steps 1–4 were repeated for each $(T - T_0)_u$ value.

A double fitting of the curves "minimum N versus $(T - T_0)_u$" was performed. Firstly, an exponential equation was used to fit each confidence level curve (70%, 80%, 90%, 95% and 98%). A second fitting was performed to fit the equation coefficients with a third degree polynomial. So the obtained expression gives the minimum number of specimens (min N) to be tested, as a function of the temperature and the desired confidence in the T_0 value.

3. Results

The basic results that were obtained for each $(T - T_0)_u$ and N value were the $(T - T_0)_s$ values and the corresponding distributions. For each distribution curve the number of toughness values within the central region were obtained. This can be seen in the curves "Relative counting in the Region #2", that give the probability of finding toughness results within this region. From these probability curves the curves with the minimum N to obtain a given confidence level were obtained. These curves were fitted with an analytical expression to give the number of toughness results required to determine the reference temperature with a given confidence level.

3.1. $(T - T_0)_s$ distributions

For a given $(T - T_0)_u$ value the distributions of $(T - T_0)_s$ as function of N are like those presented in Figs. 1–4. For a given N the Figs. 5–8 show, typically, how the distributions of the $(T - T_0)_s$ vary with the assumed temperature $(T - T_0)_u$.

The minimum and maximum $(T - T_0)_s$ values, min Dt and max Dt respectively, for each distribution, as function of N and $(T - T_0)_u$ are shown in Fig. 9. In this figure, for each temperature, the lines are shifted in the horizontal to show their variation with N (for the smallest N we have the biggest range).

3.2. Relative counting in the Region #2

Fig. 10 shows the "Relative (%) counting versus N" curves for each $(T - T_0)_u$ value, in the Region #2. The horizontal axis (N) was limited to 50 because all curves tend to the same upper value as N becomes large.

3.3. Minimum N needed to obtain a given confidence level in the T_0 determination

For the scope of this work it was assumed that the confidence in the T_0 determination is the relative counting (%) obtained in the second region, centered in the assumed $(T - T_0)_u$ and spreading over ±10°C. So, with this assumption and from the results presented in Fig. 10, it is possible to obtain the respective minimum N to obtain T_0 with a specified confidence level, for a given $(T - T_0)_u$. Fig. 11 shows the curves "minimum N versus $(T - T_0)_u$", associated with the confidence levels of 80%, 90%, 95% and 98%. The 70% confidence level curve was also obtained but it is not presented in this Fig. 11 (it was used in the fitting process presented in the next section).

From these curves it can be seen, for example, that to have 95% confidence in the T_0 determination, when the tests were performed at $(T - T_0)_u = -25°C$, one should have at least 11 valid results. (This means that in 95% of the T_0 calculations, with 11 K_{J_c} results obtained at $(T - T_0)_u = -25°C$, will give a T_0 value that is,

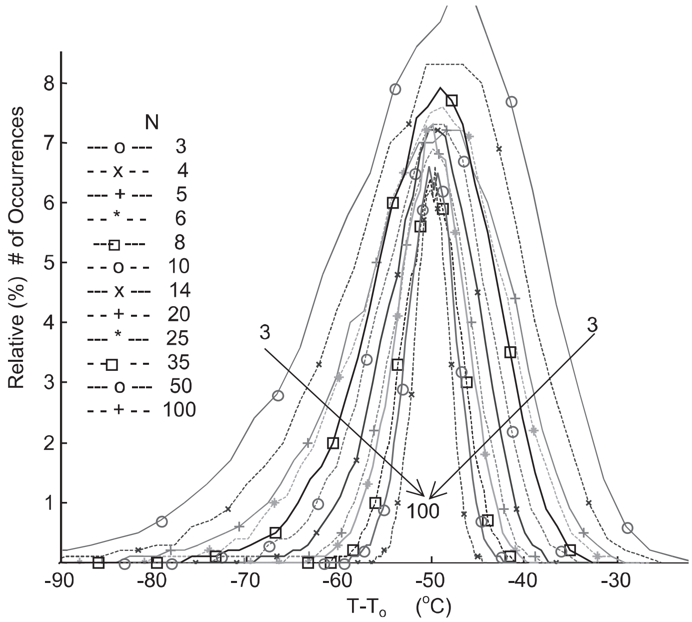

Fig. 1. $T - T_0$ distributions for $(T - T_0)_u = -50°C$.

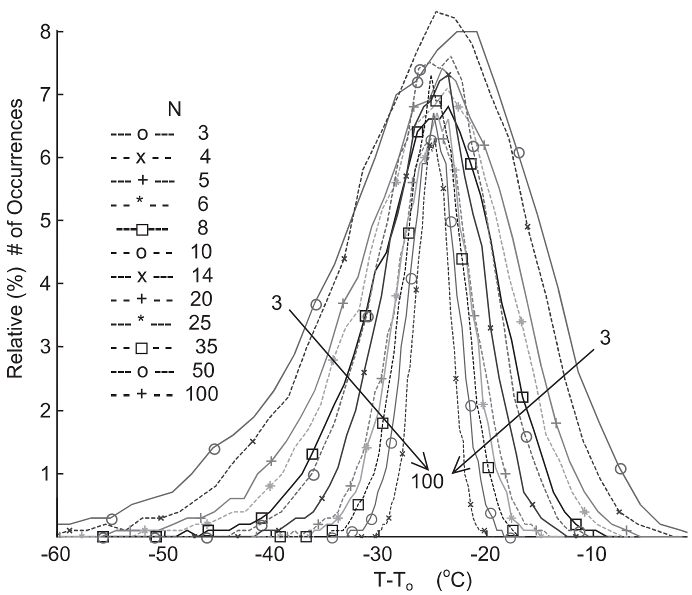

Fig. 2. $T - T_0$ distributions for $(T - T_0)_u = -25°C$.

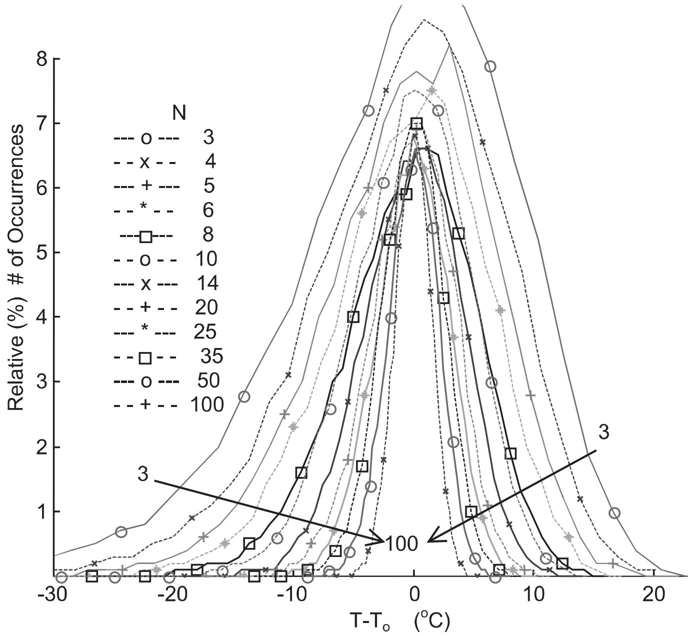

Fig. 3. $T - T_0$ distributions for $(T - T_0)_u = 0°C$.

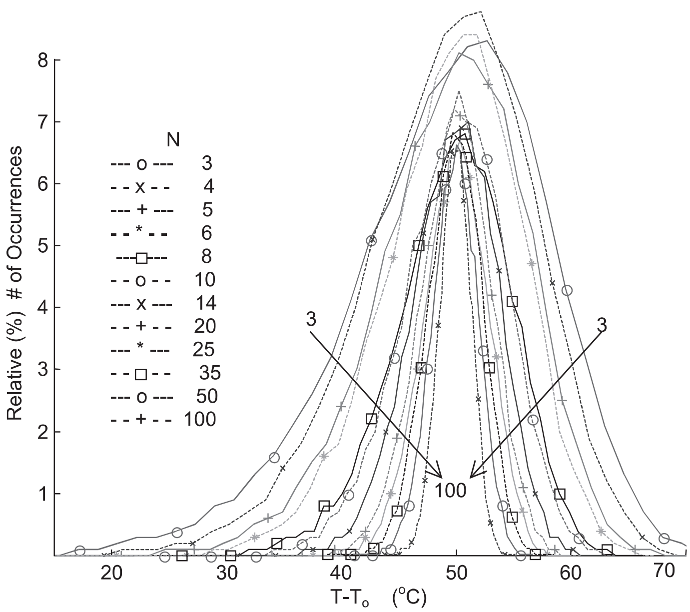

Fig. 4. $T - T_0$ distributions for $(T - T_0)_u = +50°C$.

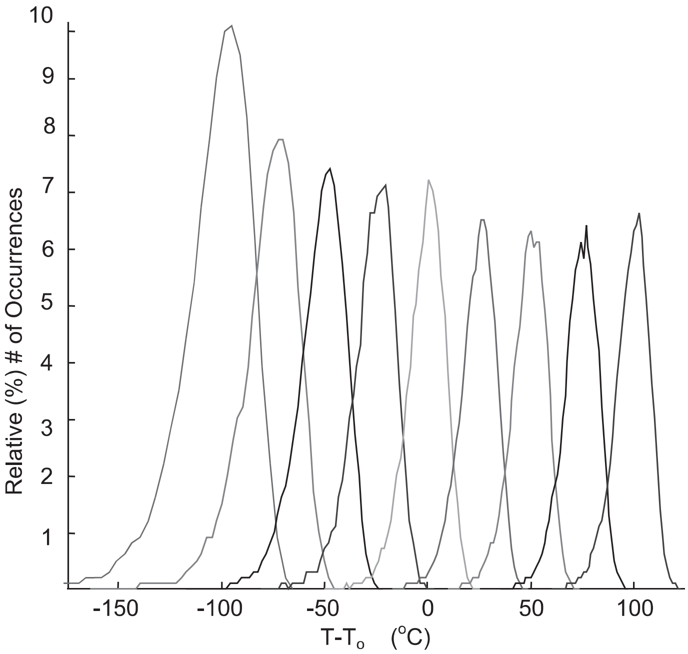

Fig. 5. $T - T_0$ distributions for $N = 3$.

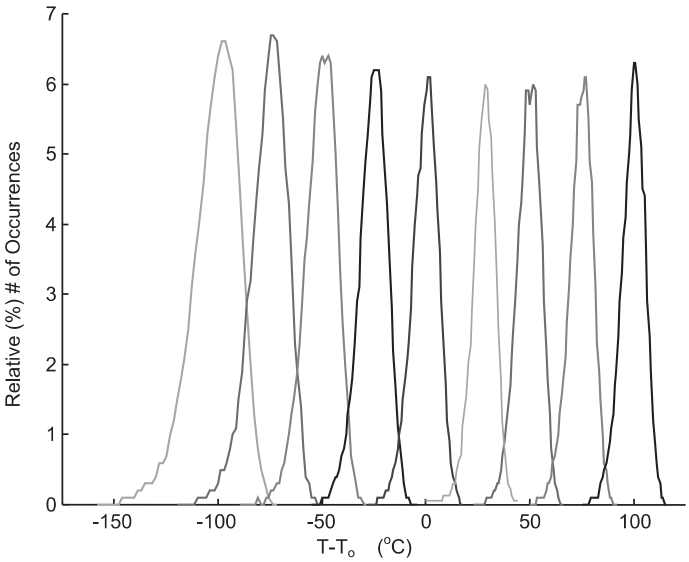

Fig. 6. $T - T_0$ distributions for $N = 6$.

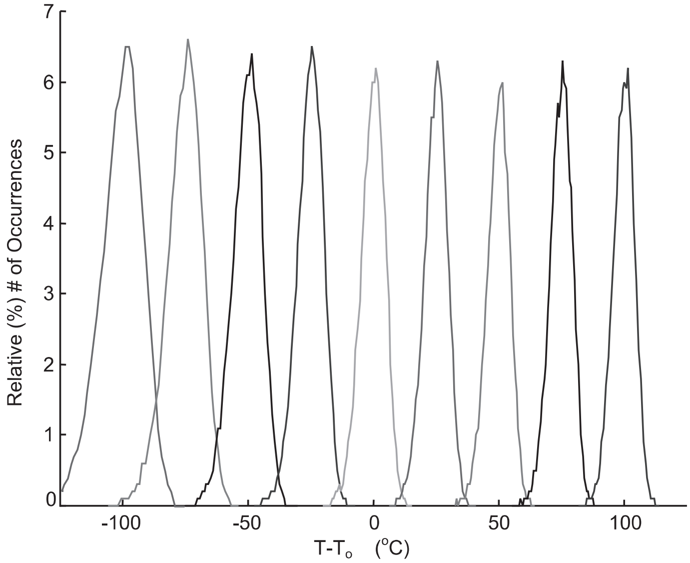

Fig. 7. $T - T_0$ distributions for $N = 10$.

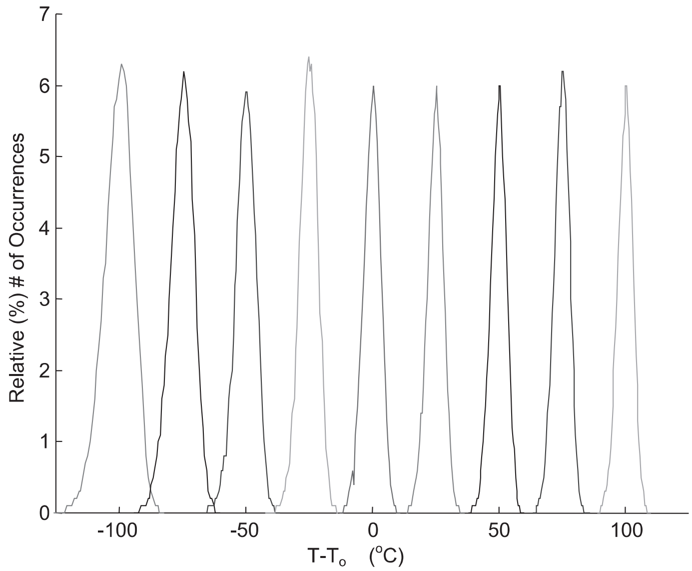

Fig. 8. $T - T_0$ distributions for $N = 20$.

at the maximum, ± 10°C from the actual value.) If the tests were to be performed at $(T - T_0)_u = 0$°C, 10 valid results gives the same 95% confidence level.

3.4. Fitting the curves "minimum N versus $(T - T_0)_u$"

All the curves shown in Fig. 11, plus the one associated with the 70% of confidence level, have the same trend. So, by a process of trial and error a double fitting was performed. Firstly, an exponential equation (7)

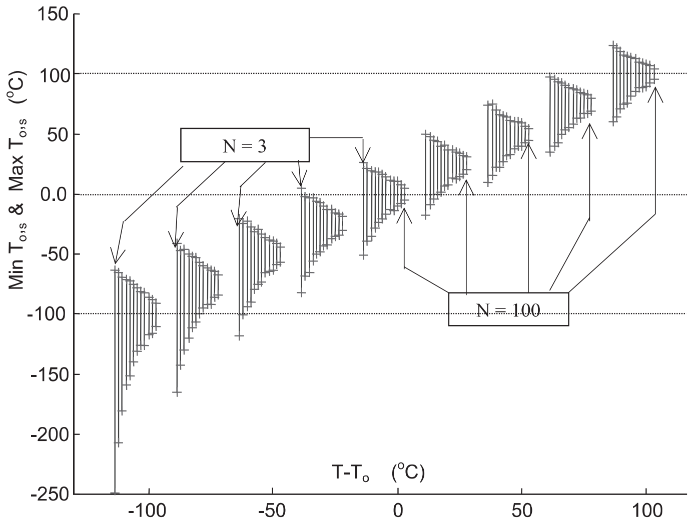

Fig. 9. Minimums and maximums of the $(T - T_0)_s$ distributions as a function of N and $(T - T_0)$.

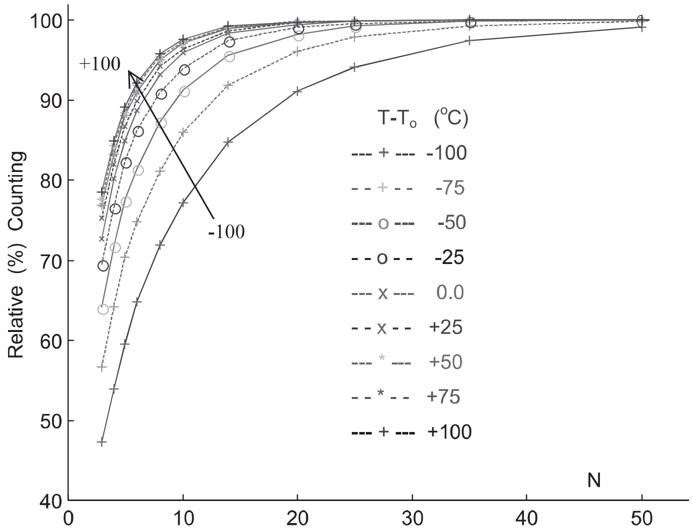

Fig. 10. Relative counting in the Region #2.

was chosen to fit all the curves. The obtained coefficients a_i, b_i and c_i are presented in Table 2. In Eq. (7): $-100°\text{C} \leqslant (T - T_0)_u \leqslant +100°\text{C}$.

As a second step the coefficients a_i, b_i and c_i were fitted using a third degree polynomial (Eq. (8)) to have them as function of the confidence level. In Eq. (8), $0.70 \leqslant x \leqslant 0.98$ is the confidence level. The obtained coefficients d_j, e_j, f_j and g_j are presented in Table 3.

$$\min N = a_i + b_i e^{-(T-T_0)/c_i} \tag{7}$$

$$a_i, \text{ or } b_i, \text{ or } c_i = d_j + e_j x + f_j x^2 + g_j x^3 \tag{8}$$

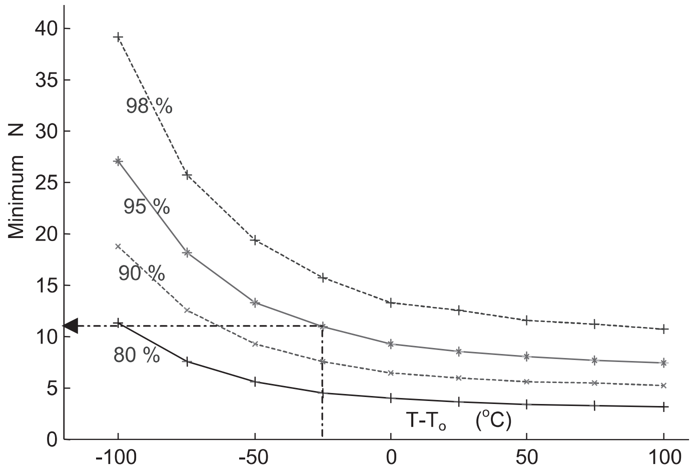

Fig. 11. Minimum N versus $(T - T_0)_u$ for a given confidence level.

Table 2
Coefficients for the fitting of the curves "minimum N versus $(T - T_0)_u$"

	Confidence level	Coefficients		
i	% (x)	a_i	b_i	c_i
1	70% (0.70)	3.00	0.12	28.06
2	80% (0.80)	3.18	0.75	41.99
3	90% (0.90)	5.24	1.22	41.59
4	95% (0.95)	7.40	1.88	42.66
5	98% (0.98)	10.94	2.41	40.76

Table 3
Second fitting coefficients d, e, f and g

		Coefficients			
j		d_j	e_j	f_j	g_j
1	Coeff. a_i	−404.75	1581.7	−2042.8	879.0
2	Coeff. b_i	−116.65	428.6	−525.6	216.5
3	Coeff. c_i	−1389.9	4745.1	−5217.9	1904.1

With this approach the minimum number of specimens to be tested (at a given temperature), min N, to obtain the reference temperature T_0 in the transition, as function of the desired confidence level, is given by Eqs. (7) and (8) with the coefficients defined by Tables 2 and 3. This double fitting procedure can be verified by a direct comparison with the original values. This was done in Fig. 12 were the dotted lines are the fitted curves and the continuous lines are the original curves. The maximum difference between the original and the fitted curves is 2, in the lower temperature region.

3.5. Curves with the confidence level for a given N as function of $(T - T_0)_u$

In the situation post-experiments the number of specimens/results is known, and from this one should be interested to know the level of confidence in the obtained reference temperature. To know the confidence level associated with a given number N of results, the curves "confidence level versus $(T - T_0)_u$" presented in Fig. 13 can be used. To obtain these curves the N values were taken from [3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 18, 20, 22, 24, 26, 30, 40].

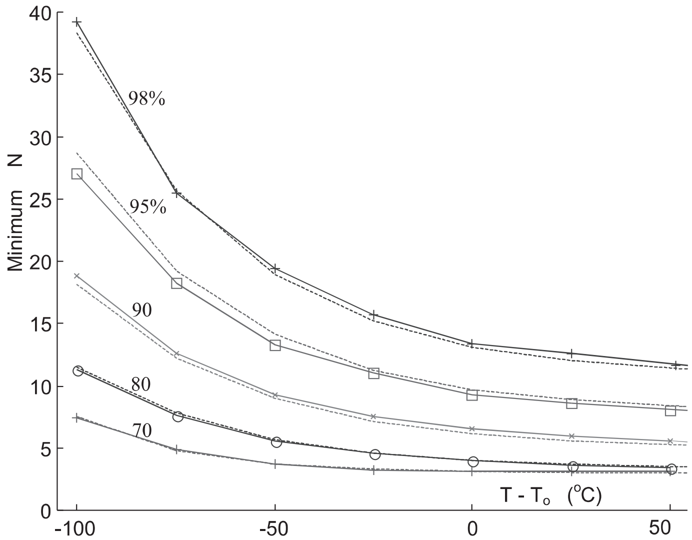

Fig. 12. Comparison between the original (—) and the fitted (- - -) curves.

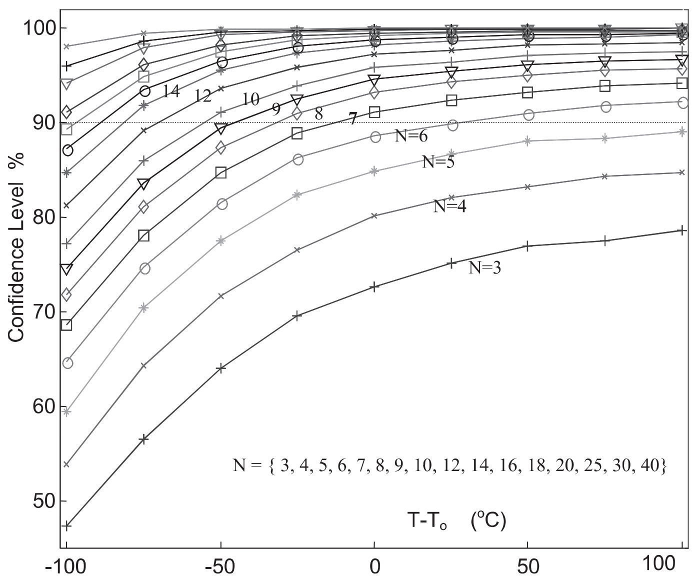

Fig. 13. Confidence level for a given N, as a function of $(T - T_0)$.

4. Discussion

An alternate solution to the results presented here is available in an analytical format [17]. This is presented in an appendix. This solution gives a more direct way to evaluate the confidence level of a T_0 determination. However, it requires the evaluation of an integral and the graphical solutions presented in this paper may be preferred by some engineers.

The numerically obtained distributions of the $(T - T_0)_s$ values became narrower as N and $(T - T_0)_u$ increases. This fact is more evident as function of N (Figs. 1–4) but it occurs also with $(T - T_0)_u$ (Figs. 5–8). Also, as expected, the range of these $(T - T_0)_s$ values shows a reduction as N increases as can be seen, more clearly, in Fig. 9.

The irregular behavior of these distribution curves, typically shown in Figs. 1–4, in some localized regions where, sometimes, one curve overlaps the other, can be explained by the way they were constructed: for each curve, the range of the $(T - T_0)_s$ values was divided in 50 divisions to conduct the counting that defines the curve itself. These ranges can be different from one curve to another, as can be seen in Fig. 9. The area under the curves should be the same (the number of used subsets – 30 000). As consequence each curve has a different weight associated with the horizontal scale.

The overlapping of the extremity regions between two adjacent curves (see Figs. 5–8) is bigger as the $(T - T_0)_u$ is smaller. Usually it is advisable not to conduct tests in the lower temperature regions due to the uncertainties associated. This can be seen in the Fig. 12 or 13 – more specimens are needed to give the same confidence level when tests were performed in the very lower transition region: $(T - T_0)_u < -50°C$. So this overlapping is not a concern for the present study.

The results show that there is no big benefit in testing at the high temperature region $(T - T_0)_u > 50°C$. See Figs. 12 and 13 where the curves are almost flat for higher values of $(T - T_0)_u$.

The results are directly tied to the assumption of the confidence level: that is, a $\pm 10°C$ allowed spread in the central region of the $(T - T_0)_s$ distributions, around the target $(T - T_0)_u$ value. If this spread value is reduced, the number of specimens to be tested to give a certain confidence level will increase.

The third region could be considered in this confidence level definition because it defines a very conservative estimate of the toughness values. If Region #3 were included, the number of specimens to be tested to give a certain confidence level would reduce. Doing this would raise the confidence level for those sets with, in the average, less than seven values. However, this option was not adopted in this work to allow that the obtained confidence level curves have some degree of conservatism.

No censoring [1] was applied to the randomly obtained subsets of toughness values but when a subset presented a $K_{J_c,\mathrm{med}} < 30$ MPa$\sqrt{\mathrm{m}}$ it was eliminated and a new subset was generated.

5. Conclusion

A method was devised to determine how the confidence in the reference temperature, T_0, determination varies with the number of specimens used in the experimental measurements. This dependence was obtained numerically and presented graphically as function of the $(T - T_0)$ temperature and number N of results. The resulting curves were fitted to allow one single expression that gives the minimum number N of results (min N), or specimens to be tested, at a given temperature, as function of the desired confidence level.

From these curves, it can be seen that using six specimens, or six valid results, as stated in the ASTM E1921/97 [1], one can get less than 90% of confidence when the test temperature (T) is the same of the reference temperature T_0 (i.e. $(T - T_0)_u = 0°C$). For the same $(T - T_0)_u$ about 12 results are required to have a 95% of confidence in the T_0 determination. When using six specimens the confidence level in the T_0 determination is greater than 90% only when the test is conducted above $(T - T_0) = 25°C$.

The behavior shown by the curves in Figs. 11–13 is directly related to the master curve definition. Due to the uncertainties no tests should be performed at $(T - T_0) \leqslant -50°C$, the lower region of the transition. The results show that there is no big benefit in testing at the high temperature region $(T - T_0)_u > 50°C$ of the transition because the curves of the confidence level become almost flat in this part of the transition. And, since the master curve does not define the end of the transition, there is a possibility that this temperature level could result in ductile, upper shelf, fracture behavior. Therefore, the best range for testing in the transition is $-25°C \leqslant (T - T_0) \leqslant 50°C$.

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Appendix A

The standard deviation of the K_0 parameter of the Weibull distribution, with a fixed $m = 4$, and obtained with N experimental results (K_{J_c} values), is given approximately by Eq. (A.1)

$$\sigma = \frac{0.254(K_0 - K_{\min})}{\sqrt{N}} \tag{A.1}$$

Combining this equation with the expression for the master curve, and considering ΔT the accuracy, the following error expression (A.2) can be obtained

$$\Lambda = \frac{\sqrt{N}}{0.254}\left\{1 - \frac{\frac{77\exp[0.019(T-T_0)]}{\exp[0.019(\Delta T)]} + 11}{77\exp[0.019(T - T_0)] + 11}\right\} \tag{A.2}$$

Considering this expression as a normally distributed one, with a null median and deviance ± 1, the cumulative probability can be obtained from (A.3)

$$P(\Delta T) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\Lambda} \exp\left(-\frac{x^2}{x}\right)\mathrm{d}x \tag{A.3}$$

The probability to obtain, with N experimental results, a T_0 value within the actual T_0 value $\pm\Delta T$, i. e. the absolute difference between the actual T_0 and the obtained T_0, is lower or equal than ΔT, can be given by Eq. (A.4)

$$P(\pm\Delta T) = P(\Delta T) - P(-\Delta T) \tag{A.4}$$

This analytical formulation might be a more exact solution but the average engineer may prefer a graphical solution that he can read directly, rather than an analytical solution that has to be evaluated each time it is applied.

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