Nonlocal effects in the ^{8}Be breakup

D. Galetti
Instituto de Física Teórica, Universidade Estadual Paulista, Rua Pamplona 145, 01405-900 - São Paulo, SP – Brazil

Salomon S. Mizrahi
Departamento de Física, Universidade Federal de São Carlos, Rodovia Washington Luiz, km 235, São Carlos, SP – Brazil

L. C. Chamon, D. Pereira, and M. S. Hussein
Instituto de Física, Universidade de São Paulo, C.P. 66318, 05315-970 - São Paulo, SP – Brazil

M. A. Cândido Ribeiro
Department of Physics, University of Wisconsin, 1150 University Ave., Madison, Wisconsin 53706
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A collective Hamiltonian for a two alpha particles aggregate, which describes the ^{8}Be nucleus, encompassing a collective potential and an inertia function of that system, is obtained and analyzed through the use of a technique — derived from an approach of the generator coordinate method (GCM) — which allows for the extraction of collective information. The nucleon-nucleon interaction considered here is the one proposed by Volkov plus the Coulomb repulsion. It is shown that nonlocal effects appear in those collective functions describing the spontaneously occurring breakup process. Furthermore, the result for the inertia function stands for a microscopically generated evidence supporting a double-folding-based model of the real part of the nucleus-nucleus nonlocal interaction recently proposed. [S0556-2813(98)03509-2]

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I. INTRODUCTION

The description of the interaction between two colliding heavy nuclei has long been the subject of study in nuclear physics and the determination of its features a major goal to be attained. In the past years systematic accurate and extensive measurements involving elastic scattering at intermediate energies gave rise to a great improvement in the understanding of the nucleus-nucleus interaction [1]. As a consequence of the analysis of the nuclear rainbow scattering that occurs in those cases the real part of that interaction can be nowadays unambiguously described and its determination, not only at the surface, but also at smaller distances, can be accomplished [2,3]. The resulting phenomenological interactions present significant dependence upon the bombarding energies. To account for this dependence some theoretical models have been developed which make explicit use of density dependent interactions [4–6]. On the other hand, it has been recently shown in a description of elastic scattering, using an integrodifferential equation, that the real mean field (with no coupled channels effects) potential dependence on the bombarding energy comes mostly from nonlocal exchange effects [7,8]. That mean field potential has been proposed to be constructed using the usual double folding model with an energy-independent nonlocal exchange interaction. The results have shown that the obtained nucleus-nucleus interaction can be written in such a form so as to embody the nonlocal exchange effects through a simple form. The nonlocal real part of that potential can be rewritten as a local equivalent one which clearly displays the energy dependence preconized by the phenomenological approach. Furthermore, the double folding inspired inertia function that comes in the treatment reveals the fundamental character of the nonlocal effects in the scattering processes, much in the same spirit as has been previously seen in nuclear fusion processes [9,10].

The aim of the present paper is twofold. First, we intend to show that the exchange nonlocal effects, being primarily of quantum mechanical origin, thus pervading general many-body phenomena, manifest themselves also in still another type of nuclear process, namely the nuclear breakup. Second, we intend to show that the microscopic extraction of the nonlocal effects presented here for that particular type of process do in fact stand for a microscopic support for the ansatz previously proposed for the real part of the nucleus-nucleus interaction, as mentioned above [7,8].

As the ^{8}Be is known to be a spontaneous fissionable nucleus, it constitutes a convenient testing ground for studying the nonlocal effects appearing in this particular process. This convenience comes from the fact that this nucleus is light enough, thus leading to calculations much less involved than those necessary for heavier nuclei where spontaneous fission also occurs, and also because the nonlocal effects are more manifest in this case due to the value of the corresponding nonlocality range that is greater than for heavier systems. Since it is known from the experience that the ^{8}Be nucleus decays into two alpha particles, it seems natural to describe it within the generator coordinate method (GCM) [11–14] by a model consisting of a deformed two alpha particles structure. In fact, the resonating group approach [15] could be used as well, but here we will follow the GCM scheme. Thus, we see that we can describe the ^{8}Be nucleus through a model consisting of a preformed structure of two alpha clusters that can be accomplished by considering two harmonic oscillator (HO) potentials symmetrically located about the origin of a coordinate system, each one describing a single cluster [16–19]. Therefore, because we are interested in studying the breakup of the ^{8}Be, *after it is formed*, in the present treatment it is natural to choose the half distance between the centers of the two HO potentials only along the breakup axis, say the z-axis, as the *generator coordinate* (GC). The same is not true if one wants to study the general alpha-alpha scattering problem within the GCM [17], where the generator coordinate is then **r**, the spatial separation between the centers of the alpha clusters. The full 8×8 Slater determinant wave function of the ^{8}Be nucleus is constructed with the 1s single particle wave function of these two HO's thus yielding the parametrized generating function of the method. The microscopic nuclear potential acting between the nucleons that we consider here is the one proposed by Volkov [20], since it is simple enough to handle and suitable to light nuclei calculations. Furthermore, it has also been shown within the GCM that this kind of interaction gives good results for the partial waves $l=0,2,4,6$ phase shifts of an alpha-alpha scattering [17]. As we concern ourselves with the breakup process, we expect both, this simple Volkov potential and the Coulomb repulsive force, to be sufficient to give origin to that particular resonance level which will characterize the ^{8}Be nucleus. The GCM energy kernel we propose is then composed of two parts, namely, the nuclear Volkov-generated contribution and a full microscopic Coulomb term. In the present treatment of the ^{8}Be nucleus we have not carried ahead the calculation of the projected ground state energy within the GCM, in order to compare it with the experimental value.

In this description, the nonlocal effects come mostly from two sources, namely from the nonorthogonality of the GCM states and from the full 8×8 determinantal character of the associated GCM kernels. In fact, using the Volkov nucleon-nucleon interaction plus the Coulomb repulsion we were able to explicitly obtain the GCM kernels which are manifestly nonlocal.

It has been shown in the past that it is possible to extract collective information from the GCM kernels [21–26]. However, these procedures lack some quantum information because they handle directly with the GC, which is a parameter. Although the GC itself is not a dynamical variable, a pair of genuine collective coordinate-momentum variables can be constructed out of the original GC, and, in fact, it has been shown in the past that a collective Hamiltonian can be written which encompasses a collective potential and an inertia function as the GCM kernels are given. In the present case, the main feature of this new collective Hamiltonian is that it embodies the nonlocal effects present in the initial GCM microscopic description of the breakup process. Therefore, in order to extract the collective potential and the inertia function, which are the constituents of the Hamiltonian, out of the GCM kernels, we took advantage of a procedure based on the GCM and on the Weyl formalism [27] presented many years ago [28]. Using a numerical technique derived from this approach [29], it is possible to extract the collective potential (inertia function) as the discretized version of a zeroth-moment (second moment) of a discretized transformed GCM energy kernel. With these numerical results for those functions we were able to discuss the microscopically generated nonlocal effects present in the breakup process.

This paper is organized as follows. In Sec. II we present the calculations of the GCM kernels. In Sec. III we briefly present the numerical technique and results, while in Sec. IV we discuss the nonlocal effects in our particular breakup process. Finally Sec. V is devoted to the conclusions and final remarks.

II. THE GCM KERNELS

An alpha cluster model, based on the GCM has been developed by Brink [18] many years ago to study the structure of light nuclei. We will follow here that method of Brink in order to calculate the overlap and energy kernels that enters the Griffin-Wheeler (GW) equation,

$$\int [\langle\alpha|H|\alpha'\rangle - E\langle\alpha|\alpha'\rangle] f(\alpha')d\alpha' = 0 \tag{1}$$

for the ^{8}Be nucleus, since they constitute the essential quantities for the derivation of the collective potential, inertia function and energy spectrum, as already discussed [29].

We will choose a coordinate system whose z-axis coincides with the path along which the alpha particles motion occurs, each alpha cluster being described by a HO potential whose center is located at a distance z_0 about the origin. The spatial part of the single particle wave function is written as

$$\varphi(x,y,z) = \frac{1}{\pi^{3/4}b^{3/2}}\exp[-(x^2+y^2)/2b^2] \times \exp[-(z\pm z_0)^2/2b^2], \tag{2}$$

where the sign $\pm$ in front of z_0 is introduced in order to specify to which cluster a particular nucleon belongs. The parameter z_0, the half distance between the fragments, will constitute our generator coordinate and b is the HO parameter whose value allows us to fix the α radius.

Once we have constructed the 8×8 Slater determinant of the ^{8}Be nucleus, $|\Psi(z_0)\rangle$, it is immediate to calculate the normalized overlap kernel

$$N(z_0,z_0') = \langle\Psi(z_0')|\Psi(z_0)\rangle = \frac{[\det\langle\varphi_i(z_0')|\varphi_j(z_0)\rangle]^4}{[\det\langle\varphi_i(z_0')|\varphi_j(z_0')\rangle \det\langle\varphi_i(z_0)|\varphi_j(z_0)\rangle]^2}$$

$$= \frac{\{\exp[-(z_0-z_0')^2/2b^2] - \exp[-(z_0+z_0')^2/2b^2]\}^4}{[1-\exp(-2z_0'^2/b^2)]^2[1-\exp(-2z_0^2/b^2)]^2}, \tag{3}$$

where φ_i corresponds to the occupied orbital states and $\det\langle\varphi_i|\varphi_j\rangle$ stands for the determinant whose elements are the overlap of the single particle wave functions.

In the same fashion we calculate the contribution of the kinetic energy term to the GCM energy kernel,

$$T(z_0,z_0')=\langle\Psi(z_0')|\hat{T}|\Psi(z_0)\rangle$$
$$=4N(z_0,z_0')\sum_{i,j}\langle\varphi_i(z_0')|\hat{t}|\varphi_j(z_0)\rangle(B^{-1})_{ij}, \qquad (4)$$

where the factor 4 stands for the spin-isospin degeneracy, **B** is the matrix whose elements are

$$B_{ij}=\langle\varphi_i(z_0')|\varphi_j(z_0)\rangle, \qquad (5)$$

and $\hat{t}$ is the one-body kinetic energy operator. The analytic expression of $T(z_0,z_0')$ is

$$T(z_0,z_0')=4N(z_0,z_0')\left(\frac{\hbar^2}{mb^2}\right)\left\{\frac{3}{2}-\frac{[(z_0-z_0'/2b)]^2\exp[-(z_0-z_0')^2/2b^2]-[(z_0+z_0'/2b)]^2\exp[-(z_0+z_0')^2/2b^2]}{\exp[-(z_0-z_0')^2/2b^2]-\exp[-(z_0+z_0')^2/2b^2]}\right\}, \quad (6)$$

where m is the nucleon mass. The value of the diagonal expression, $T(z_0,z_0)$, in the limit $z_0\to\infty$, gives $6\hbar^2/mb^2$, which is exactly the kinetic energy value of a system of two *free* alpha particles, each being described by $1s$ HO orbital states. In the other limit, $z_0\to 0$, $T(z_0,z_0)$ goes to $8\hbar^2/mb^2$, which is exactly the kinetic energy of a system of eight nucleons occupying the $1s$ and $1p$ orbital states. Expression (6) contains the spurious center-of-mass motion effect, however this can be corrected by subtracting the contribution of the center-of-mass term,

$$\frac{\hat{P}^2}{2mA}=\frac{1}{A}\sum_i\frac{\hat{p}_i^2}{2m}+\sum_{i\neq j}\frac{\hat{p}_i\cdot\hat{p}_j}{mA}, \qquad (7)$$

which gives

$$\left\langle\Psi(z_0')\left|\frac{\hat{P}^2}{2mA}\right|\Psi(z_0)\right\rangle=\frac{T(z_0,z_0')}{A}+\frac{\hbar^2}{2mAb^2}\frac{N(z_0,z_0')}{\exp[-(z_0-z_0')^2/2b^2]-\exp[-(z_0+z_0')^2/2b^2]}$$
$$\times\left(\frac{(z_0-z_0')^2}{b^2}\left\{\exp\left[-\frac{(z_0-z_0')^2}{2b^2}\right]-\exp\left[-\frac{(z_0-z_0')^2+(z_0+z_0')^2}{2b^2}\right]\right\}\right.$$
$$\left.+\frac{(z_0+z_0')^2}{b^2}\left\{\exp\left[-\frac{(z_0+z_0')^2}{2b^2}\right]-\exp\left[-\frac{(z_0-z_0')^2+(z_0+z_0')^2}{2b^2}\right]\right\}\right), \qquad (8)$$

where A is the total number of nucleons. As $z_0\to\infty$, the diagonal part of the second term in Eq. (8) vanishes, while for $z_0\to 0$ it goes to $-\hbar^2/mAb^2$.

The nuclear two-body potential of Volkov [20], together with the Coulomb repulsion, constitute the full interaction between the nucleons,

$$\hat{V}(|\vec{r}_1-\vec{r}_2|)=(1-M+M\hat{P}_x)\{V_{0a}\exp[-\alpha_a|\vec{r}_1-\vec{r}_2|^2]$$
$$+V_{0r}\exp[-\alpha_r|\vec{r}_1-\vec{r}_2|^2]\}+\frac{e^2}{|\vec{r}_1-\vec{r}_2|}, \qquad (9)$$

where $\hat{P}_x$ is the coordinate exchange operator and the values of the constants are: $V_{0a}=-76.69$ MeV, $V_{0r}=408.27$ MeV, $\alpha_a=0.444$ fm^{-2}, $\alpha_r=4.94$ fm^{-2}, $M=0.60$, $e^2=1.44$ MeV·fm, and $b=1.27$ fm is the value of the oscillator parameter; this value for b fixes the alpha radius as $R_\alpha=1.56$ fm.

For a potential of the form

$$\hat{V}(|\vec{r}_1-\vec{r}_2|)=u(|\vec{r}_1-\vec{r}_2|)[(1-M)+M\hat{P}_x], \qquad (10)$$

the corresponding GCM energy kernel is [18]

$$\langle\Psi(z_0')|\hat{V}|\Psi(z_0)\rangle$$
$$=N(z_0,z_0')\sum_{ijkl}\langle\varphi_i(z_0')\varphi_j(z_0')|u|\varphi_k(z_0)\varphi_l(z_0)\rangle$$
$$\times[X_d(B^{-1})_{ki}(B^{-1})_{lj}+X_e(B^{-1})_{kj}(B^{-1})_{li}], \qquad (11)$$

where $X_d=8-10M$, $X_e=10M-2$ and the matrix **B** is that already introduced in Eq. (5). In this way, for the Coulomb forces, we have $X_d^C=8$ and $X_e^C=-2$.

Finally, the analytic expression of the energy kernel of the full two-body potential is constituted of the following terms, namely: Gaussian terms

$$K_G(z_0,z_0') = \frac{2V_o}{(1+2\alpha_i b^2)^{3/2}} \frac{N(z_0,z_0')}{\{\exp[-(z_0-z_0')^2/2b^2]-\exp[-(z_0+z_0')^2/2b^2]\}^2}$$
$$\times\Bigg((X_d+X_e)\left\{\exp\left[-\frac{(z_0-z_0')^2}{2b^2}\right]+\exp\left[-\frac{(z_0+z_0')^2}{2b^2}\right]\right\} - 2(X_d+X_e)\exp\left[-\frac{(z_0-z_0')^2}{4b^2}-\frac{(z_0+z_0')^2}{4b^2}\right]$$
$$\times\exp\left\{\left[-\frac{1}{4}\left(1+\frac{b^2\alpha_i}{1+2b^2\alpha_i}\right)\right]\left[\frac{(z_0-z_0')^2}{b^2}+\frac{(z_0+z_0')^2}{b^2}\right]\right\}\left\{\exp\left[\frac{b^2\alpha_i}{2(1+2b^2\alpha_i)}\left(\frac{z_0-z_0'}{b}\right)\left(\frac{z_0+z_0'}{b}\right)\right]\right.$$
$$\left.+\exp\left[-\frac{b^2\alpha_i}{2(1+2b^2\alpha_i)}\left(\frac{z_0-z_0'}{b}\right)\left(\frac{z_0+z_0'}{b}\right)\right]\right\} + \exp\left[-\frac{b^2\alpha_i}{1+2b^2\alpha_i}\left(\frac{z_0+z_0'}{b}\right)^2\right]\left\{X_d\exp\left[-\frac{(z_0-z_0')^2}{b^2}\right]\right.$$
$$\left.+X_e\exp\left[\frac{(z_0+z_0')^2}{2b^2}-\frac{(z_0-z_0')^2}{2b^2}\right]\right\} + \exp\left[-\frac{b^2\alpha_i}{1+2b^2\alpha_i}\left(\frac{z_0-z_0'}{b}\right)^2\right]\left\{X_d\exp\left[-\frac{(z_0+z_0')^2}{b^2}\right]\right.$$
$$\left.+X_e\exp\left[\frac{(z_0+z_0')^2}{2b^2}-\frac{(z_0-z_0')^2}{2b^2}\right]\right\}\Bigg), \tag{12}$$

and the Coulomb term

$$K_C(z_0,z_0') = \frac{4e^2}{b(2\pi)^{1/2}} \frac{N(z_0,z_0')}{\{\exp[-(z_0-z_0')^2/2b^2]-\exp[-(z_0+z_0')^2/2b^2]\}^2}\Bigg(\exp\left[-\frac{(z_0-z_0')^2}{b^2}\right]+\exp\left[-\frac{(z_0+z_0')^2}{b^2}\right]$$
$$+\frac{\pi^{1/2}}{2}\frac{\Phi(z_0+z_0'/\sqrt{2}b)}{z_0+z_0'/\sqrt{2}b}\left\{2\exp\left[-\frac{(z_0-z_0')^2}{b^2}\right]-\exp\left[-\frac{(z_0-z_0')^2+(z_0+z_0')^2}{2b^2}\right]\right\}$$
$$+\frac{\pi^{1/2}}{2}\frac{\Phi(z_0-z_0'/\sqrt{2}b)}{z_0-z_0'/\sqrt{2}b}\left\{2\exp\left[-\frac{(z_0+z_0')^2}{b^2}\right]-\exp\left[-\frac{(z_0-z_0')^2+(z_0+z_0')^2}{2b^2}\right]\right\}$$
$$-\pi^{1/2}\exp\left[-\frac{(z_0-z_0')^2+(z_0+z_0')^2}{2b^2}\right]\left[\frac{\Phi(z_0/\sqrt{2}b)}{z_0/\sqrt{2}b}+\frac{\Phi(z_0'/\sqrt{2}b)}{z_0'/\sqrt{2}b}\right]\Bigg), \tag{13}$$

respectively, where $\Phi(x)$ is the probability integral [30]. The complete GCM energy kernel can now be immediately written out of these contributions. It is worth mentioning that these expressions were specifically calculated for the breakup process, where the generator coordinate is z, and must not be directly compared with other energy kernels aiming at scattering problems. Besides, differently from approximated versions of the Coulomb term, as has been proposed [17], our expression for this contribution is fully microscopic.

III. NUMERICAL CALCULATIONS

In order to extract numerically the nuclear collective potential and inertia function for the ^{8}Be, we will follow the procedure presented in [29]. The first step consists in the diagonalization of the overlap kernel (3). However, by simple inspection of that expression we note that the overlap is not translationally invariant (it does not depend only on the difference z_0-z_0') around the origin, but for $z_0,z_0'\gtrsim b$ it goes as $\exp[-2(z_0-z_0')^2/b^2]$, thus exhibiting a narrower width than that around the origin. In what refers to the numerical results, the adoption of the asymptotic expression for the overlap kernel, for all values of z_0 and z_0', induces a small distortion in the exact description of the collective potential and inertia function near the origin [28,29]. Besides, for the exact overlap the numerical techniques used to extract the nuclear collective potential and inertia functions become extremely complicated and the results at present are not reliable. But, in fact, it has been verified in a 50 points mesh calculation that the lowest energy levels of the spectrum, obtained with the exact overlap, do not differ significantly from those calculated with the translationally invariant kernel. So, in spite of the approximated character of the description, we adopt here the asymptotic translationally invariant kernel, instead of the exact one, since the calculations can be performed with a high density of points in the interval of interest, which allows more reliable results. The introduction of that approximation leads to a collective potential and an inertia function that will present a slightly modified behavior near the origin (as compared to the expected exact results), where the exact overlap and the adopted one differ; however, this fact does not constitute a drastic drawback since the physically interesting region $z_0,z_0'\gtrsim b$ will be correctly described.

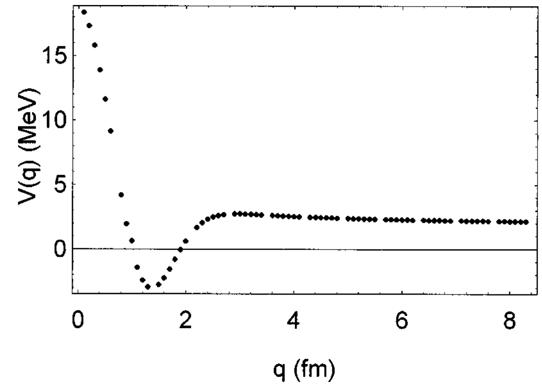

FIG. 1. Collective potential for the ^{8}Be breakup. The constant asymptotic nuclear behavior has been subtracted. The calculations were performed with a 70×70 mesh in the interval $-25 \leqslant z_0 \leqslant 25$ and a step of $\Delta z_0 = 0.7$.

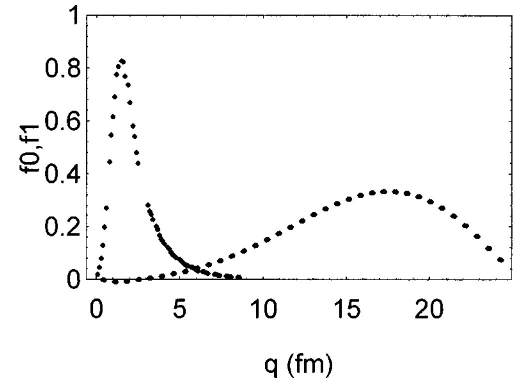

FIG. 2. The intrinsic first and second GCM calculated dimensionless wave functions for the two alpha system. The calculations were performed with a 70×70 mesh in the interval $-25 \leqslant z_0 \leqslant 25$ and a step of $\Delta z_0 = 0.7$.

After the GCM kernels have been calculated, the sums of the generator coordinates $(z_0 + z_0')/2$ are transformed into a genuine collective coordinate and the differences $z_0 - z_0'$ into the canonically conjugated momentum through a Weyl-Wigner mapping [28]. In this way, we end up with an expression for the collective Hamiltonian which has the half-distance between the centers of the densities of the two alpha system as the quantum collective coordinate. Keeping the lowest order terms in its expansion in terms of anticommutators the collective Hamiltonian reads

$$H(q,p) = H^{(0)}(q) - \frac{1}{4\hbar^2}\{p,\{p,H^{(2)}(q)\}\} + \text{higher order terms}, \tag{14}$$

where we identify $H^{(0)}(q) \equiv V(q)$ as the collective potential for the ^{8}Be breakup and $B(q) \equiv \hbar^2/2H^{(2)}(q)$ as the inertia function [29]. The higher order terms will be neglected since they are smaller than the first two. In the numerical scheme, $H^{(2)}(q)$ is the second moment of the matrix representing the transformed mesh associated to the GCM energy kernel and the collective coordinate, q, is now, naturally, the half-distance between the alpha clusters. The numerical calculations have been made with the GC taken in the interval ranging from -25.0 fm to 25.0 fm, with a step of 0.7 fm, corresponding to matrices of order $N = 70$. This particular choice of the interval permits a numerically reliable set of points to be calculated in its central part, namely, up to 15 fm. The collective potential is presented in Fig. 1 (where we have subtracted -43.05 MeV, the constant asymptotic pure nuclear contribution of the potential), while the first two intrinsic wave functions, obtained from a numerical diagonalization of the transformed GW equation as indicated in [28], are depicted in Fig. 2.

Only two energy levels lie below the top of the barrier, namely at 1.14 MeV and 2.12 MeV, respectively, whereas all others occur above. The discretization process and the finiteness of the interval of variation of the GC constitute a constraint in this procedure, leading, obviously, to a discrete spectrum, while the actual one has a continuum.

IV. NONLOCAL EFFECTS

The calculated breakup potential for the ^{8}Be can be seen to be different from the diagonal part of the GCM energy kernel, which corresponds to the variational potential approach for the problem. The difference, which comes from the off-diagonal terms of the GCM energy kernel matrix, reflects the *nonlocal effects* embodied in the formalism and, at the same time, clearly displays a strong repulsion in the inner region. This feature of the potential *is not only due to Coulomb repulsion*; in fact it is strongly marked by the blocking effects of pure kinematical nature. As is well known, the short life of the ^{8}Be nucleus ($\sim 10^{-16}$ s) is basically due to these effects. Thus, at short range, the nuclear part of the nucleon-nucleon interaction and the blocking effects dominate, whereas, asymptotically, the breakup potential tends to the two alphas Coulomb interaction, where no nonlocal effects are expected.

The calculated ratio between the inertia function and the reduced mass of the two alpha system is depicted in Fig. 3 (we remind the reader that the GC adopted is half the distance between the centers of the HO's potentials so that the reduced mass of the system corresponds to eight nucleon masses). The asymptotic behavior of the inertia function goes to the reduced mass of the two alpha system, as expected, since, there, only the direct part of the nucleon-nucleon interaction effects show up, whereas in the alphas'

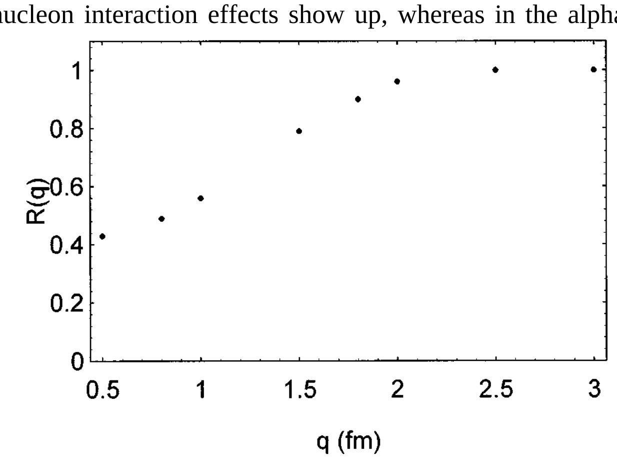

FIG. 3. The microscopically calculated dimensionless ratio $R(q) = \mu^*(q)/\mu$ for the ^{8}Be breakup. The calculations were performed with a 70×70 mesh in the interval $-25 \leqslant z_0 \leqslant 25$ and a step of $\Delta z_0 = 0.7$.

overlapping region, by assuming a peculiar form, the inertia function clearly reveals the nonlocal effects occurring at short distances, due to the exchange effects. At the same time that this result reveals important nonlocal contributions to the breakup process, therefore strongly suggesting the presence of the same characteristics already found in the nuclear fusion processes [9,10], its behavior also confirms a recent proposal for the description of nucleus-nucleus elastic scattering [7,8]. There, it was proposed that the real part of the nucleus-nucleus interaction is described by a nonlocal potential which amounts to having an inertia function that should be given by the expression [31]

$$\mu^*(q)=\frac{\mu}{1+\mu\beta^2/2\hbar^2|V(q)|}, \tag{15}$$

where β is the nonlocality range of the nucleus-nucleus system, μ is its asymptotic reduced mass and $V(q)$ is a double folding potential

$$V_f(q)=\int \rho_1(r_1)v(\vec{q}-\vec{r}_1+\vec{r}_2)\rho_2(r_2)d\vec{r}_1 d\vec{r}_2. \tag{16}$$

Here $v(r)$ is the effective nucleon-nucleon interaction and $\rho_1(r_1)$ and $\rho_2(r_2)$ are the nuclear densities of the colliding partners, respectively. We can directly compare that expression for the inertia function involving the folding potential, used to describe the nuclear scattering, with our previous microscopically derived quantum numerical result, obtained for the breakup using, by its turn, the completely antisymmetrized GCM wave function. For this purpose, we calculate expression (15) using the Wigner part of the Volkov interaction as $v(r)$. This can be analytically accomplished if we use the HO wave functions for the calculations of the alpha densities (as we have done also for the GCM calculations) thus giving

$$V_f(q)=6.4\left[V_{0a}\left(\frac{\alpha_a^2}{\alpha_a^2+2b^2}\right)^{3/2}\exp\left(\frac{-4q^2}{\alpha_a^2+2b^2}\right) + V_{0r}\left(\frac{\alpha_r^2}{\alpha_r^2+2b^2}\right)^{3/2}\exp\left(\frac{-4q^2}{\alpha_r^2+2b^2}\right)\right]. \tag{17}$$

Now, in order to compare the two results for the inertia function, we must first introduce the Jackson and Johnson expression for the nonlocality range [32], namely $\beta=\beta_0 m/\mu$, where μ is the reduced mass of the system, m is the nucleon mass and β_0 is the nucleon-nucleus nonlocality as given by Perey and Buck [33]. The comparison between the microscopically based calculated inertia function and the one calculated from the Volkov double folding potential using expression (17) is shown in Fig. 4. We verify that there is a substantial agreement between the two results in the region $z_0,z_0'\gtrsim b$, which corresponds to the exact description of the overlap kernel, while for $z_0,z_0'\lesssim b$ there appears a slight deviation, since the overlap kernel was approximated in that region. Besides, in the numerical results there are other quantum exchange effects which are not present in Eq. (17). Thus, we can see from the figure that the inertia function for the breakup under study has the same form as that proposed for the description of nucleus-nucleus scattering—this means that it can be quite well described by expression (15)—and, therefore, it is direct to see that it depends on the double folding potential, but not on that describing the breakup, namely the one presented in Fig. 1. In this form, besides being consistent with what is expected for the process, this result also gives full microscopic support for the the ansatz proposed in the nuclear reactions context [7,8].

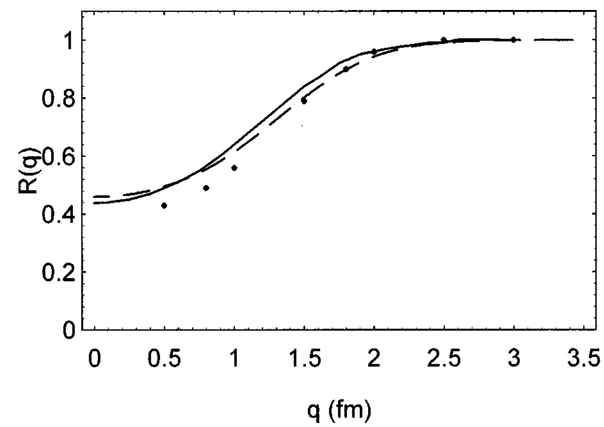

FIG. 4. Comparison between the microscopically calculated dimensionless ratio (points) and the phenomenological one using the folding ansatz, the Volkov interaction, and the Jackson and Johnson nonlocality range (dashed line). Also shown is the same ratio calculated from the nucleon-nucleon M3Y interaction with the parameters given in the text (continuous line).

For the sake of completeness, we have also calculated the inertia function from the double folding potential using the nucleon-nucleon M3Y interaction [34]:

$$v(r)=7999\frac{e^{-4r}}{4r}-2134\frac{e^{-2.5r}}{2.5r}-262\delta(\vec{r}), \tag{18}$$

and HO wave functions for the alpha densities. The pseudopotential, $-262\delta(\vec{r})$ MeV, describes the knock-on exchange collision at 10 MeV/nucleon [35]. The result practically coincides with those of the previous calculation, as can be seen also in Fig. 4.

V. CONCLUSIONS

Recently, we have presented arguments that emphasized the importance of the nonlocal effects arising from the quantum exchange which are present in many-body systems and in particular in nuclear ones. Specifically we have proposed a way to incorporate these effects in the description of the real part of a nucleus-nucleus potential and we have also shown how these nonlocal effects can explain the energy dependence present in the local-equivalent real part of the phenomenological nucleus-nucleus potentials at intermediate energies [7,8]. At the same time, that proposal serves as a test for the nonlocality range for composed nuclear systems as obtained in a simple folding model. In the present paper we have shown that the same nonlocal effects also show up in nuclear breakup processes. Here, we have adopted a full microscopic starting point, namely the generator coordinate method, GCM, for the description of the ^{8}Be nucleus that is known to undergo spontaneous fission into two alpha particles. The nucleons degrees of freedom are washed out in the calculations so that one ends up with the GCM kernels which are then functions only of the parameter associated to the half distance between the clusters. By an appropriate transformation, the GCM energy kernel is converted into a collective Hamiltonian which is written in terms of a collective potential and an inertia function. Therefore, all the underlying nonlocal effects of microscopic origin in the ^{8}Be nucleus are taken into account now in these new functions. The collective potential clearly reveals a shallow pocket that does not allow for a bound state and a strong repulsion at short distances, mainly due to Pauli blocking. In fact, the solution of the GCM equation admits two levels below the top of the barrier but above the asymptotic Coulomb tail; the lowest level is the candidate for representing the ^{8}Be resonance although its energy eigenvalue is above the 100 keV expected for this case. The inertia function, by its turn, bears the hallmark of the presence of nonlocal effects in a marked way. Being a function of the half-distance degree of freedom, the ratio of the effective reduced mass to the reduced mass for the breakup process has a form that clearly resembles the ones already found in other nuclear processes and also that of a nucleon in a self-consistent potential [36]. Furthermore, we have compared the numerical result for that ratio with the one analytically obtained from a double folding inspired expression and the Volkov interaction. The results almost entirely match. We can therefore see that the present microscopic-based treatment of the ^{8}Be exhibits the importance of nonlocal exchange effects in the breakup, which can be of great importance for more accurate descriptions of astrophysical processes, such as the capture of an α particle by a breaking up ^{8}Be in the process that populates the 7.6 MeV Hoyle 0^+ resonance in ^{12}C.

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