PERGAMON

Available online at www.sciencedirect.com

SCIENCE DIRECT®

Scripta Materialia 48 (2003) 1561–1564

Scripta MATERIALIA

www.actamat-journals.com

Comment on "Microstructural path and temperature dependence of recrystallization in commercial aluminum"

P.R. Rios [a,*], A.F. Padilha [b]

[a] *Escola de Engenharia Industrial Metalúrgica de Volta Redonda, Universidade Federal Fluminense, Av. dos Trabalhadores, 420, Volta Redonda, RJ 27255-125 Brazil*

[b] *Universidade de São Paulo, Departamento de Engenharia Metalúrgica e de Materiais da EPUSP, Av. Prof. Mello Moraes, 2463, São Paulo, SP 05508-900 Brazil*

Received 23 January 2003; received in revised form 28 January 2003; accepted 30 January 2003

Abstract

R.A. Vandermeer and D. Juul Jensen introduced the concept of impingement-compensated chord length as an alternative to Cahn and Hagel's interface-average grain boundary migration rate. In this work a new expression for the impingement-compensated chord length is derived.

Keywords: Microstructure; Kinetics; Nucleation; Growth; Recrystallization

1. Introduction

This paper comments on a recent paper by Vandermeer and Jensen [1] on the microstructural path analysis of the recrystallization of commercial aluminum.

Cahn and Hagel's [2] interface-averaged grain boundary migration rate, $\langle v \rangle_{CH}$, can be estimated from microstructural quantities by:

$$\langle v \rangle_{CH} = \frac{1}{S_V} \frac{dV_V}{dt} \tag{1}$$

where V_V is the volume fraction of transformed or recrystallized region and S_V is the interface area between transformed or recrystallized region and untransformed or unrecrystallized region per unit of volume.

The above expression is normally accepted to be the standard way to estimate growth rate during recrystallization [3]. In recent papers, Vandermeer and Jensen [1,4] proposed a new quantity: the impingement-compensated chord length, $\langle \lambda \rangle_{ex}$, to study the growth of the recrystallized grains. This impingement-compensated chord length is given by:

$$\langle \lambda \rangle_{ex} = \langle \lambda \rangle \frac{-\ln(1 - V_V)}{V_V} \tag{2}$$

where $\langle \lambda \rangle$ is the mean intercept length of the recrystallized regions.

* Corresponding author. Tel.: +55-2433443030; fax: +55-2433443029.

E-mail addresses: prrios@metal.eeimvr.uff.br (P.R. Rios), padilha@usp.br (A.F. Padilha).

1359-6462/03/$ - see front matter
doi:10.1016/S1359-6462(03)00055-1

Their idea is interesting and it has an advantage over $\langle v \rangle_{\mathrm{CH}}$: it does not require the determination of the derivative but it depends only on quantities that can be readily measured from a planar section. The disadvantages are that it implicitly assumes that all regions are randomly distributed and that it is not really the growth rate but it is related to the integral of the growth rate. In any case it looks advantageous to have an alternative way of estimating growth rate, particularly if it is easy to calculate from experimental quantities.

Unfortunately Eq. (2) has some problems. In this work it will be shown why this is so and a new expression will be proposed for the impingement-compensated chord length. Although in what follows frequent allusions are made to recrystallization the treatment is general and the results can be applied to any heterogeneous transformation.

2. A new expression for impingement-compensated chord length

2.1. Problems of Eq. (2)

Eq. (2) will be applied to the simple case in which one has an assembly of equal spheres in extended space so that according to Fullman [3,5]

$$\langle \lambda \rangle_{\mathrm{ex}} = \frac{4R_{\mathrm{ex}}}{3} \tag{3}$$

where R_{ex} represents the radius of the growing spherical grains in extended space.

For equal spheres the following is valid:

$$V_{V\mathrm{ex}} = \frac{4\pi R_{\mathrm{ex}}^3}{3} N_V \tag{4}$$

$$S_{V\mathrm{ex}} = 4\pi R_{\mathrm{ex}}^2 N_V \tag{5}$$

where $V_{V\mathrm{ex}}$ and $S_{V\mathrm{ex}}$ are the extended volume and extended interface area per unit of volume. There is no impingement in extended space and $S_{V\mathrm{ex}}$ always represents the total extended interface area of the growing grains.

Moreover from JMAK theory [6–8] and from DeHoff [9]:

$$V_{V\mathrm{ex}} = \ln\left(\frac{1}{1 - V_V}\right) \tag{6}$$

$$S_{V\mathrm{ex}} = \frac{S_V}{(1 - V_V)} \tag{7}$$

It is worthy of note that as $V_V \to 1$, $V_{V\mathrm{ex}} \to \infty$, $S_{V\mathrm{ex}} \to \infty$ and $R_{\mathrm{ex}} \to \infty$.

One can also recall a standard result from quantitative metallography [10]:

$$\langle \lambda \rangle = \frac{4V_V}{S_{V\mathrm{T}}} \tag{8}$$

where

$$S_{V\mathrm{T}} = S_V + 2S_{V\mathrm{R}} \tag{9}$$

$S_{V\mathrm{T}}$ is the total interface area per unit of volume, $S_{V\mathrm{R}}$ is the interface area between the recrystallized regions per unit of volume. It is clear that as $V_V \to 1$ then $S_V \to 0$ and $S_{V\mathrm{T}} \to S_{V\mathrm{R}} \to$ finite value, $S_{V\mathrm{R}}(V_V = 1)$ and $\langle \lambda \rangle$ tend to the mean intercept length of the fully transformed microstructure.

Combining Eqs. (3)–(8) with Eq. (2) one obtains:

$$S_{V\mathrm{ex}} = S_{V\mathrm{T}} \tag{10}$$

The above equation is clearly incorrect. It predicts that for $V_V \to 1$, $S_{V\mathrm{ex}} \to S_{V\mathrm{T}} \to$ finite value, $S_{V\mathrm{T}}(V_V = 1)$. As a consequence it predicts that $S_{V\mathrm{ex}}$ tends to a finite value whereas it goes to infinity for $V_V \to 1$. Moreover $S_{V\mathrm{ex}}$ can only be approximately equal to $S_{V\mathrm{T}}$ before impingement, afterwards:

$$S_{V\mathrm{ex}} > S_{V\mathrm{T}} \tag{11}$$

So Eq. (2) yields a wrong consequence and therefore it cannot be correct.

2.2. New derivation of Eq. (2)

In order to derive Eq. (2) Vandermeer and Jensen defined two quantities, Z and Z_{ex}, as a function of the intercept lengths λ and λ^{ex} of the growing grains in real and extended space, respectively:

$$Z = V_V = \frac{1}{L}\sum_{i=1}^{N} \lambda_i \tag{12}$$

$$Z_{\mathrm{ex}} = V_{V\mathrm{ex}} = \frac{1}{L}\sum_{i=1}^{N} \lambda_i^{\mathrm{ex}} \tag{13}$$

where L is the length of an arbitrary test line.

Definitions cannot be wrong but in this case they are misleading. The problem lies in the upper value of the summation, N. Eqs. (12) and (13) suggest that they are the same when in fact they are not. Eq. (13) must be rewritten as:

$$Z_{ex} = V_{Vex} = \frac{1}{L}\sum_{i=1}^{N_{ex}} \lambda_i^{ex} \tag{14}$$

N in Eq. (12) represents the number of intercepts of a test line of length L with recrystallized grains in real space. On the other hand N_{ex} in Eq. (14) represents the number of intercepts of a test line of length L with recrystallized grains in extended space. In extended space there is no impingement and grains can grow freely into one another. As a consequence, a test line of length L will intercept *more* recrystallized grains in extended space than in real space. Therefore:

$$N_{ex} > N \tag{15}$$

So the impingement-compensated chord length defined by Vandermeer and Jensen must be redefined as:

$$\langle\lambda\rangle_{ex} = \frac{1}{N_{ex}}\sum_{i=1}^{N_{ex}} \lambda_i^{ex} \tag{16}$$

or:

$$\langle\lambda\rangle_{ex} = \frac{N}{N_{ex}}\left(\frac{\sum_{i=1}^{N}\lambda_i}{N}\right)\left(\frac{L}{\sum_{i=1}^{N}\lambda_i}\right)\left(\frac{1}{L}\sum_{i=1}^{N_{ex}}\lambda_i^{ex}\right) \tag{17}$$

Substituting Eqs. (6), (12) and (14) into Eq. (17):

$$\langle\lambda\rangle_{ex} = \left(\frac{N}{N_{ex}}\right)\langle\lambda\rangle\frac{-\ln(1-V_V)}{V_V} \tag{18}$$

Eq. (18) is the correct expression instead of Eq. (2). Eq. (18) is not very useful as it requires knowledge of N/N_{ex}. It can be shown with the help of Eq. (24), which is derived below, that:

$$\frac{N}{N_{ex}} = \frac{S_{VT}}{S_{Vex}} \tag{19}$$

The right hand side of Eq. (19) can be estimated using an equation proposed by Yamamoto et al. [11] for site saturated transformations:

$$\frac{S_{VT}}{S_{Vex}} = \alpha\left(\frac{V_V}{V_{Vex}}\right)^{2/3} \tag{20}$$

where α is a shape factor, not too far from unity for equiaxed grains. Using Eqs. (6), (19) and (20):

$$\frac{N}{N_{ex}} \approx \left(\frac{V_V}{-\ln(1-V_V)}\right)^{2/3} \tag{21}$$

The ratio N/N_{ex} is the factor missing in Eq. (2). It can be regarded as the magnitude of the error that is made when Eq. (2) is used instead of Eq. (18). It can be seen that the factor N/N_{ex} decreases slowly from $N/N_{ex} = 1$, for $V_V = 0$ to $N/N_{ex} \approx 0.36$ for $V_V = 0.99$.

2.3. Alternative expression for the impingement-compensated chord length

In order to preserve the idea of an impingement-compensated chord length another expression has to be found, preferably relying only on measurements carried out on a planar section. This can be done noticing that:

$$S_{Vex} = \frac{4N_{ex}}{L} \tag{22}$$

Combining Eqs. (14), (16) and (22):

$$\langle\lambda\rangle_{ex} = \frac{4V_{Vex}}{S_{Vex}} \tag{23}$$

Eq. (23) is the application in extended space of a well-known real space quantitative metallography result [10]. Notice that since V_{Vex} varies as $(R_{ex})^3$ and S_{Vex} varies as $(R_{ex})^2$, $\langle\lambda\rangle_{ex}$ varies as (R_{ex}) so that $\langle\lambda\rangle_{ex} \to \infty$ as $R_{ex} \to \infty$ for $t \to \infty$.

Using Eqs. (6) and (7), Eq. (23) can be expressed in terms of S_V and V_V:

$$\langle\lambda\rangle_{ex} = \frac{-4(1-V_V)\ln(1-V_V)}{S_V} \tag{24}$$

This expression is not undefined but $V_V \to 1$ and $S_V \to 0$ as $t \to \infty$ and, as shown above, $\langle\lambda\rangle_{ex} \to \infty$.

Eq. (24) is the expression proposed here for the impingement-compensated chord length that should be used instead of Eq. (2).

3. Conclusions

A new expression was proposed for the impingement-compensated chord length, $\langle\lambda\rangle_{ex}$, a concept introduced by Vandermeer and Jensen [3,4]. It is believed that such an expression can be useful to analyze growth rate during recrystallization in addition to the more usual Cahn and Hagel expression. Its main advantage is that it can be easily determined from quantities measured on a planar section without the need to experimentally measure time derivatives. Its disadvantages are: it is not the growth rate but the integral of the growth rate, so the growth rate has to be inferred from the slope of a $\langle\lambda\rangle_{ex}$ vs. time plot; it is implicitly assumed in the derivation of Eq. (24) that all regions are randomly distributed.

Finally it is important to stress that although the present work was done in the context of recrystallization the final result is general and the impingement-compensated chord length obtained here can be applied to any heterogeneous transformation.

Acknowledgements

The authors are grateful to Conselho Nacional de Desenvolvimento Científico e Tecnológico, CNPq, to Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro, FAPERJ (P.R. Rios), and to Fundação de Amparo à Pesquisa do Estado de São Paulo, FAPESP, for the financial support (Contracts n° 99/10796-8, Thematic Project, and n° 01/10134-7, Visiting Professor Scholarship to P.R. Rios).

References

[1] Vandermeer RA, Juul Jensen D. Acta Mater 2001;49:2083.
[2] Cahn JW, Hagel WC. In: Zackey ZD, Aaronson HI, editors. Decomposition of austenite by diffusional processes. New York: Interscience Publ; 1960. p. 131.
[3] Rios PR. Metall Mater Trans 1997;28A:939.
[4] Vandermeer RA, Juul Jensen D. Interface Sci 1998;6:95.
[5] Fullman RL. Trans AIME 1953;197:447.
[6] Kolmogorov AN. Akad Nauk USSR-Ser Matemat 1937;1:355.
[7] Avrami MJ. Chem Phys 1939;7:1103.
[8] Johnson WA, Mehl RF. Trans AIME 1939;135:416.
[9] DeHoff RT. In: Hansen N, Jensen DJ, editors. Annealing processes—recovery, recrystallization and grain growth. Roskilde, Denmark: Risø National Laboratory; 1986. p. 35.
[10] Underwood EE. Quantitative stereology. Reading, MA: Addison-Wesley Publishing Company; 1970.
[11] Yamamoto T, Sakuma T, Rios PR. Scripta Mater 1998;39:1713.