The Pennsylvania State University
The Graduate School
Department of Nuclear Engineering

A Study of Load Change Control in PWRs Using the Methods of Linear Optimal Control

A Thesis in

Nuclear Engineering

by

Ting Yang

Doctor of Philosophy

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Orientador Edward S Kenney

Submitted in Partial Fulfillment of the Requirements for the Degree of

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ABSTRACT

This thesis investigates the application of modern control theory to the problem of controlling load changes in PWR power plants. A linear optimal state feedback scheme resulting from linear optimal control theory with a quadratic cost function is reduced to a partially decentralized control system using "mode preservation" techniques. Minimum information transfer among major components of the plant is investigated to provide an adequate coordination, simple implementation, and a reliable control system.

Two control approaches are proposed: servo and model following. Each design considers several information structures for performance comparison. Integrated output error has been included in the control systems to accommodate external and plant parameter disturbances. In addition, the "cross limit" feature, specific to certain modern reactor control systems, is considered in the study to prevent low pressure reactor trip conditions.

An 11^{th} order nonlinear model for the reactor and boiler is derived based on theoretical principles, and simulation tests are performed for 10% load change as an illustration of system performance.

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CHAPTER I

INTRODUCTION

I.1 - Control Needs and Requirements

The primary control objective of a nuclear power plant follows from two basic conflicting requirements. The power industry demands that the unit be able to handle substantial power transients in order to prevent contingent network instability caused by load variation during normal network operating conditions. On the other hand, a nuclear power plant must operate within severe constraints on the range of safe operating conditions.

A power plant can operate safely if the operating conditions are maintained within the limitations of its components. In the reactor, the cladding is the most susceptible component to damage in the event of departure from nucleate boiling (DNB) caused by a localized high heat flux. Such an uneven power distribution is more likely to occur during power transients, since the control rods are moved from their steady state positions to accommodate the load change. The operating condition of a steam generator is usually limited by thermal constraints such as rapid temperature change or sharp temperature deviations along its structure. Thermal stress is also the most common cause of turbine damage. Such stresses can be reduced by providing slower changes in steam temperature or by keeping the temperature changes small.

With the increasing cost of alternative energy sources, nuclear energy has been shown to be one of the most competitive energy sources. Therefore, because they are usually the cheapest source of electricity, it is usual to operate nuclear power plants at full power all the time. Such use is referred to as "base loaded operation;" a mode of operation which leaves the other more expensive sources of power to handle the load balance in the network during daily load variations.

Although it is reasonable that most of the nuclear power plants operate as base load units, it is still strongly recommended that vendors incorporate into a nuclear power plant's design a load following or maneuvering capability to provide aid in restoring network stability caused by unscheduled outages. The problem is more severe if the nuclear generated power contributes a significant fraction of the overall power in the network.

Some areas of the world may have large hydroelectric supplies. This results in an inversion of the cost picture and reinforces the reason for introducing load following capability. In such cases, hydroelectric is usually the cheapest source of power and, of course, requires power operation maneuverability from reactor plants.

Typically, the nominal response rate of a current nuclear power plant is about 10% of rated output per minute. However, the reactor system suffers gradual changes in the plant parameters with time such as fuel burnup, temperature coefficient, rod worth, residue buildup in ducts and pipes, fission poisoning products,etc.. Some of these effects can be compensated for by using controllable boron concentration in the reactor coolant. These variations generally degrade the plant performance, requiring periodic tuning processes. However, in practice, these situations are often tolerated rather than made subject to complex and costly tuning procedures [1], and sometimes the operator is not sure whether the reactor will be capable of following a specific load schedule [2].

Plant performance deterioration may lead to safety-related problems because diversified transients for nearly identical load changes are more likely to cause operator confusion than are standardized transients during normal operating conditions. There may also be external effects because during a sizeable load variation in the network, poor reactor behavior would require other plants to compensate for this poor response in order to restore the network stability.

Another area that needs improvement is related to the methodologies used to design the control system. A power plant's dynamics consist of interactive processes among several components in the plant; therefore, an appropriate coordination action is necessary. Although some modern control system designs currently used in existing nuclear power plants incorporate coordination features, they represent the applied state-of-the-art evolved from classical control techniques based on single input, single output methods [3]. Therefore, a systematic and multivariable design approach is highly desirable.

On the other hand, safety-related issues suggest decentralization of the control system. The concept is in contrast to the common practice on "Candu" Canadian Power Plants [4]. Such a decentralized control concept reduces the interdependence of the localized control systems and the consequence of the occurrence of a sensor failure. There is also an expected reduction in the overall system implementation cost. However, due to the interactive process among system components, some information exchanges are necessary in order to coordinate the components to provide a smooth and rapid transient response during load changes.

In view of the last two conflicting issues, a minimization of the information exchange necessary for a satisfactory plant operation is required.

In summary, one can write the primary control objectives of a nuclear power plant control system in the form of the following needs:

1. Adequate load following capability;
2. Ability to control transients so that components operate only within safe operating limits;
3. Provide for "self tuning" of the control system;
4. Achieve "robustness" in the sense of accommodating external disturbances, parameter variations, and localized sensor and controller failures;
5. Multivariable and systematic approach for optimal coordination among the components;
6. Use of a partially decentralized control system with minimum required information exchange;
7. Reduce cost consistent with the above objectives.

I.2 - Objective

In view of the increasing requirement for improved control systems for nuclear power plants, it is the objective in this thesis to investigate the applicability of modern control theory to the design of such control systems. This work is directed towards problems of load changes in PWR plants using linear optimal control quadratic functions.

Current control systems are designed using single input, single output classical control methods. However, due to the multi-input, multi-output characteristics of a nuclear plant, the overall control system is designed separately for each major component of the plant. The interactions among these components are either ignored or are incorporated into the system based on the experience of the designer, making the process essentially a "state-of-the-art" design approach. The advantage of using multivariable design methods is that adequate coordination schemes are generated as a direct result of the method.

The overall design problem considers the following objectives: the ability of the control system to maintain adequate performance when the plant is subjected to parameter variations and external disturbances; and, the retention of the simplicity of the resulting control system in its final implementation.

For the simple control systems considered in this thesis, a requirement for the minimum information transfer among major components of the plant is considered and the usual required observers are avoided by using mode preservation techniques. Two control systems are proposed: the first approach is a servo control scheme and the second is based upon the idea of model reference control. The model reference approach is intended to increase robustness of the control system when it is subjected to disturbances. Moreover, such a system will attempt to standardize all transients despite the change of parameters occurring during the core life.

I.3 - PWR Nuclear Power Plant

I.3.1 - Plant Dynamics and Modeling

A pressurized water reactor power plant consists, essentially, of two loops as shown in Figure I.1. The pressurized water coolant in the primary loop carries the heat energy produced in the reactor to the steam generator where the heat is transferred to the secondary loop by generating an adequate amount of steam to the turbine which in turn, produces shaft power from the expansion of the steam. Finally, the shaft drives the generator to produce the required electric power.

The heat source in a nuclear power plant, as is well known, originates as the energy released in the nuclear fission chain reaction. Since the nuclear power is proportional to the neutron density, a specific power level is maintained when the total absorption and leakage rate is equal to the neutron production rate from the fission process. A measurement of such a balance or unbalance, can be expressed in terms of a quantity called reactivity. There are several factors which can affect the reactivity value such as the choice of structural components, fuel

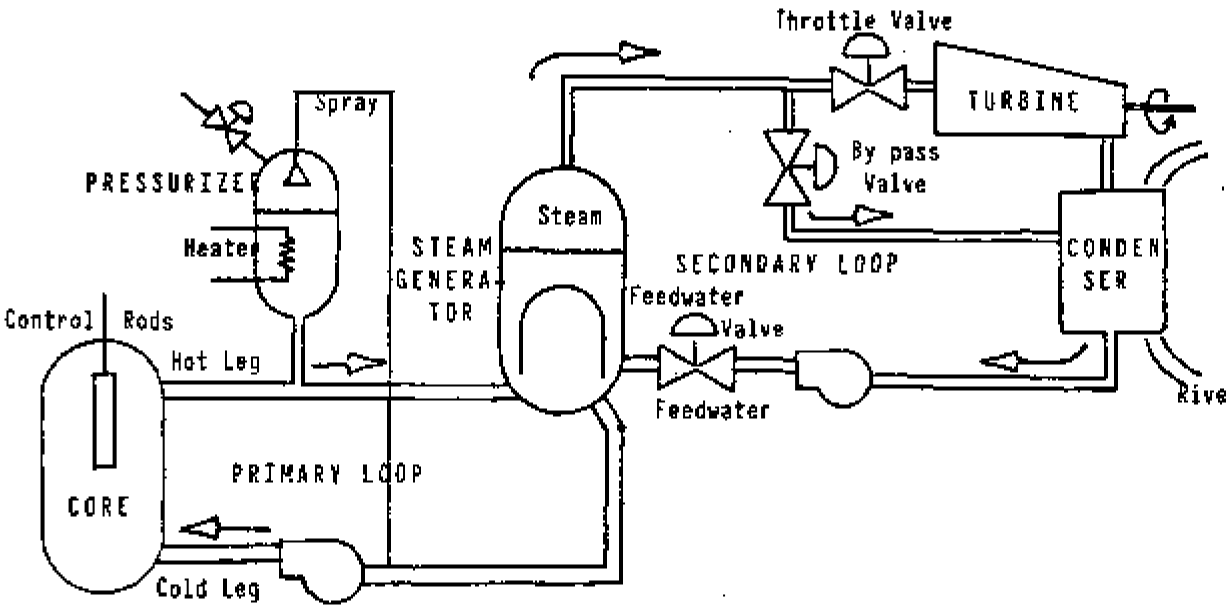

Figure I.1 - PWR Diagram

composition, absorbing control rods, fission products, inherent temperature feedback, etc.. Among these items, for control purposes, it is important to distinguish between those which are uncontrollable and those which can be voluntarily manipulated. Current practice uses control rod positioning in conjuction with the variable concentration of dissolved boric acid in the coolant to provide the desired reactivity control. Because varying boric acid concentration is a very slow dynamic process, it is used to compensate for slowly varying parameters like fission product buildup, xenon poisoning, fuel depletion, and others, leaving the control rods to compensate those varying parameters which cause fast reactivity changes like sudden changes in temperature during load variations.

The energy of fission is mostly dissipated within the fuel, and a small fraction is deposited in the coolant and other structural materials. In a PWR, the primary coolant is maintained at approximately 2200 psia by a pressurizer, in order to insure safe operating conditions. Pumped from the cold leg, the coolant is heated primarily by convection during its passage through the core.

A common boiler currently used in PWR plants is the recirculation type of steam generator. Here, the hot primary coolant enters the steam generator at the bottom and flows through a bundle of inverted U-type tubes. On the secondary loop side, the feedwater mixes with the recirculation flow at a lower pressure, and passes through the riser section where part of it is heated and part is evaporated. The steam-water mixture is then taken to the moisture separators and only dried steam is sent to the turbine.

The amount of steam sent to the turbine is controlled by a throttle valve which also provides control of the pressure in the steam generator. Because of reactor response time limitations, a by-pass valve is used to compensate for excessive production of steam during load reductions.

A simplified model of a PWR plant will now be considered and its linearized form will be used for subsequent control design examples. Note that, for control design in state variable formulation, it is convenient and practical to utilize a simple, but accurate model, rather than a detailed one, since the complexity of the resulting controller is proportional to that of the model.

I.3.1.1 - The Primary Loop Model. Assuming a point reactor kinetic model and an averaged group of delayed neutrons, the neutron density and the precursor concentration can be expressed as the following [5]:

$$\frac{dP}{dt} = \frac{\rho - \beta}{l} P(t) + \lambda C(t) \qquad \text{(I.1.a)}$$

$$\frac{dC}{dt} = \frac{\beta}{l} P(t) - \lambda C(t) \qquad \text{(I.1.b)}$$

where P is the reactor power, and for "lumped" reactor it is proportional to the neutron density [MW];

C is the spatial average precursor concentration expressed in energy units [MW];

ρ is the reactivity;

β is the delayed neutron fraction;

λ is the effective precursor decay constant [sec^{-1}];

and l is the mean generation time [sec].

A typical value for l ranges from 10^{-4} to 10^{-6} seconds.

Such a small time constant leads to numerical difficulties in control simulation on digital computer. In general, the resulting controllers from a direct application of these equations are impracticable. Experience has shown that the fast mode can be neglected by setting $\frac{dP}{dt} = 0$, a procedure called "the prompt jump approximation." The reduced equations become

$$\frac{dC}{dt} = \frac{\rho}{\beta - \rho} \lambda C$$

and

$$P = \frac{1 \lambda}{\beta - \rho} C .$$

If reactivity ρ is expressed in dollar \$ units, and P

·, the equations can be rewritten

$$\frac{\rho}{1 - \rho} C \qquad (I.2.a)$$

$$\frac{1}{3} \frac{C}{(1 - \rho)} . \qquad (I.2.b)$$

nal C divided by full power P_0.

erent temperature feedback

external control in the reactivity,

$$= \rho_c + \frac{\alpha_f}{\beta} (T_f - T_{fo}) + \frac{\alpha_w}{\beta} (T_w - T_{wo}) \tag{I.3}$$

where ρ_c is the reactivity externally controlled using control rods;

α_f is the doppler reactivity coefficient [°F^{-1}];

α_w is the coolant reactivity coefficient [°F^{-1}];

T_f is the core average fuel temperature [°F];

T_w is the core average coolant temperature [°F];

T_{fo} and T_{wo} are fixed values and the term $(\alpha_f T_{fo} + \alpha_w T_{wo})$ is referred to as "power defect" reactivity.

Due to severe limitations on the speed of rod movement, it is more convenient to control the rod speed rather than its position, i.e.,

$$\frac{d \rho_c}{dt} = K_r u_r \tag{I.4}$$

where K_r is the differential rod reactivity worth [$/m];
and u_r is the rod speed [m/sec].

The incorporation of a control delay mechanism can be approximated by a first order time lag:

$$\frac{d u_r}{dt} = \frac{1}{\tau_r} (u_1 - u_r) \tag{I.5}$$

where τ_r is the control mechanism time constant [sec];
and u_1 is the input signal.

For one node fuel model, the fuel temperature variation can be expressed using the energy balance:

$$\frac{1}{M_{fu}\,C_f}\;\frac{d\,T_f}{dt} = F_f\,N_o\,P - A_{fw}\,K_{fw}\,(T_f - T_w)$$

or

$$\frac{d\,T_f}{dt} = \frac{1}{M_{fu}\,C_f}\;(F_f\,N_o\,\frac{1}{\beta}\;\frac{\lambda\,C}{(1-\rho)} - A_{fw}\,K_{fw}\,(T_f - T_w))\,. \quad (I.6)$$

where T_f is the fuel temperature [°F];
M_{fu} is the mass of the fuel [Kg];
C_f is the fuel specific heat [Joules/Kg °F];
F_f is the fraction of power generated within the fuel;
N_o is the full power generation [MW];
A_{fw} is the heat transfer area between the fuel and coolant [m^2];
K_{fw} is the effective heat transfer coefficient between the fuel and coolant [Joules/m^2 °F];
T_w is the core average coolant temperature [°F].

The primary coolant temperature is divided into two nodes: average temperature and outlet temperature.

$$\frac{1}{M_{w1}\,C_w}\;\frac{d\,T_w}{dt} = N_o\,F_{w1}\;\frac{1\,\lambda}{\beta}\;\frac{C}{1-\rho} + A_{fw1}\,K_{fw}\,(T_f - T_w) - W_w\,C_w\,(T_w - T_{w1}) \quad (I.7.a)$$

$$\frac{1}{M_{w2}\,C_w}\ \frac{d\,T_{w2}}{dt} = N_o\,F_{w2}\ \frac{1\,\lambda}{\beta}\ \frac{C}{1-\rho} + A_{fw2}\,K_{fw}\,(T_f - T_w) - W_w\,C_w\,(T_{w2} - T_w)\ . \quad \text{(I.7.b)}$$

where M_{w1} and M_{w2} are the total coolant mass in the lower and upper node, respectively in the core [Kg];

C_w is the coolant specific heat [Joules/Kg °F];

F_{w1} and F_{w2} are the fractions of power generated within the correspondent node;

A_{fw1} and A_{fw2} are the heat transfer area for correspondent node [m^2];

W_w is the coolant flow rate [Kg/sec];

T_{w1} is the inlet coolant temperature [°F].

I.3.1.2 - The Secondary Loop Model. Consider the following diagram, shown in Figure I.2, for the steam generator:

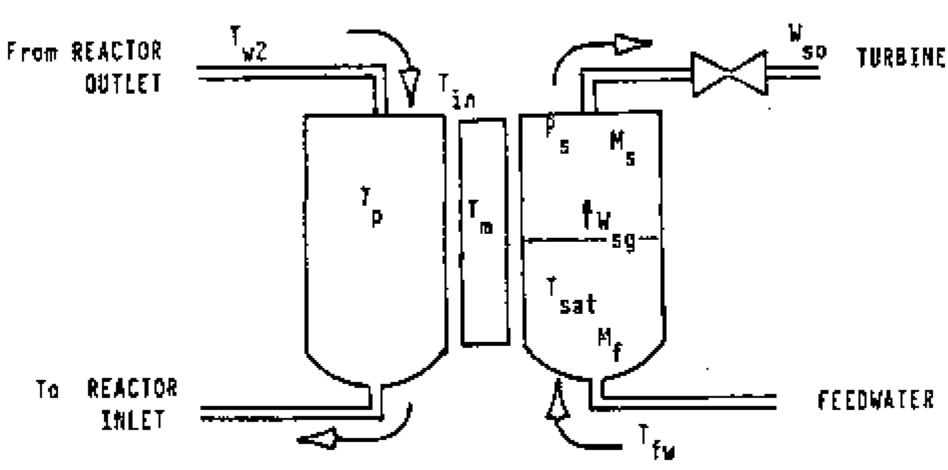

Figure I.2 - Recirculation Type of Steam Generator.

Based on energy balance, one can write:

$$M_p C_p \frac{d T_p}{dt} = W_p C_p (T_{in} - T_p) - A_{pm} K_{pm} (T_p - T_m) \quad \text{(I.8.a)}$$

and

$$M_m C_m \frac{d T_m}{dt} = A_{pm} K_{pm} (T_p - T_m) - A_{ms} K_{ms} (T_m - T_s^*) \quad \text{(I.8.b)}$$

where T_p is the effective coolant temperature in the steam generator, corresponding to reactor inlet temperature [°F];

T_{in} is the inlet temperature of the steam generator corresponding to reactor outlet temperature [°F];

T_m is the temperature of the metal tubes [°F];

W_p is the flow rate [Kg/sec];

A_{pm} is the total heat transfer area between the coolant and the metal tubes [m^2];

K_{pm} is the effective heat transfer coefficient between the coolant and the metal tubes [Joules/m^2 °F];

M_m is the mass of tubes [Kg];

C_m is the specific heat of the metal tubes [Joules/Kg °F];

A_{ms} is the total heat transfer area between the metal tubes and secondary coolant [m^2];

K_{ms} is the effective heat transfer coefficient between the metal tubes and secondary coolant [Joules/m^2 °F];

T_s^* is the saturated temperature correspondent to the pressure in steam generator [°F];

M_p is the total coolant mass in the steam generator [Kg].

The mass balance gives:

$$\frac{d\ M_f}{dt} = W_{fw} - W_{sg} \qquad (I.9.a)$$

$$\frac{d\ M_s}{dt} = W_{sg} - W_{so} \qquad (I.9.b)$$

where M_f is the mass of the fluid in the steam generator [Kg];

M_s is the mass of the steam in the steam generator [Kg];

W_{fw} is the feedwater flow [Kg/sec];

W_{sg} is the steam production rate [Kg/sec];

W_{so} is the steam flowing out rate [Kg/sec].

The volume balance gives:

$$\frac{d\ V_f}{dt} + \frac{d\ V_s}{dt} = \frac{d\ V_{TOT}}{dt} = 0 \qquad (I.10)$$

where V_f is the volume occupied by fluid in the steam generator;

V_s is the volume occupied by steam in the steam generator;

V_{TOT} is the total volume of the steam generator.

For $V_f = M_f\ v_f$ and $V_s = M_s\ v_s$, using Equations I.9.a, I.9.b, and I.10, and assuming

$$\frac{d\ v_t}{dt} = 0 \quad \text{and} \quad \frac{d\ v_s}{dt} = \frac{\partial v_s}{\partial P_s}\ \frac{d\ P_s}{dt}\ ,$$

we have the results:

$$v_{sf}\ W_{sg} + v_f\ W_{fw} - v_s\ W_{so} + M_s\ (\frac{\partial v_s}{\partial P_s})\ \frac{d\ P_s}{dt} = 0. \quad (I.11)$$

where v_f is the specific volume of fluid [m^3/Kg];

v_s is the specific volume of steam [m^3/Kg];

and P_s is the steam pressure [psia].

The energy balance for the steam generator can be expressed as follows:

$$h_{fw}\ W_{fw} - W_{so}\ h_{so} + A_{ms}\ K_{ms}\ (T_m - T_s^*) = \frac{d}{dt}\ (M_f\ u_f + M_s\ u_s)$$

$$= M_f\ \frac{d\ u_f}{dt} + u_f\ \frac{M_f}{dt} + M_s\ \frac{d\ u_s}{dt} + u_s\ \frac{d\ M_s}{dt}\ . \quad (I.12)$$

where u_f and u_s express internal energy of fluid and steam, respectively;

h_{fw} is the feedwater enthalpy [Joules/Kg];

h_{so} is the steam enthalpy [Joules/Kg].

But $u = h - \frac{P_s\ v}{J}$

and, using the approximation

$$\frac{dh}{dt} = \frac{\partial h}{\partial P} \frac{d\,P_s}{dt} ,$$

there results:

$$\frac{d\,u_f}{dt} = ((\frac{\partial h_f}{\partial P}) - \frac{v_f}{J} \frac{d\,P_s}{dt})$$

and

$$\frac{d\,u_s}{dt} = ((\frac{\partial h_s}{\partial P_s} - \frac{v_s}{J}) \frac{d\,P_s}{dt} - \frac{P_s}{J} (\frac{\partial v_s}{\partial P})) \frac{d\,P_s}{dt} .$$

Substituting into I.12, W_{sg} can be expressed as:

$$W_{sg} = \frac{(h_{fw} - h_f + P_s \frac{v_f}{J}) W_{fw} - (P_s \frac{v_s}{J}) W_{so} + A_{ms} K_{ms} (T_m - T_s^*) - B_o \frac{d\,P_s}{dt}}{(h_{sf} - v_{sf} \frac{P}{J})} \quad (I.13)$$

where $B_o = M_f ((\frac{\partial h_f}{\partial P_s}) - \frac{v_f}{J}) + M_s (\frac{\partial h_s}{\partial P_s} - \frac{v_s}{J} - \frac{P_s}{J} (\frac{\partial v_s}{\partial P_s})$

Substituting W_{sg} into Equation I.11, one obtains the following:

$$B_1 \frac{d\,P_s}{dt} = (h_{fw} - h_f + h_{sf} \frac{v_s}{v_{sf}}) W_{fw} - \frac{h_{sf}}{v_{sf}} v_s W_{so} +$$

$$+ A_{ms} K_{ms} (T_m - T_s^*)$$

where $B_1 = [M_f ((\frac{\partial h_f}{\partial P_s} - \frac{v_f}{J}) + M_s ((\frac{\partial h_s}{\partial P_s} - \frac{v_s}{J} - \frac{h_{sf}}{v_{sf}} (\frac{\partial v_s}{\partial P_s}))]$

or

$$\frac{dP}{dt} = \frac{1}{B_1} \left(A_{ms} K_{ms} T_m - A_{ms} K_{ms} T_s^*(P_s) + (h_{fw} - h_f + h_{sf} \frac{v_f}{v_{sf}}) W_{fw} - \frac{h_{sf}}{v_{sf}} v_s W_{so}\right) \tag{I.14}$$

The steam flow W_{so} can be expressed approximately in terms of throttle valve opening C_v, by [6]:

$$W_{so} = K_c P_s C_v .$$

where K_c is a constant.

The saturation temperature T_s^* can be approximated as a linear function of pressure P_s:

$$T_s^* = T_s^o + \left(\frac{\partial T_s}{\partial P_s}\right) P_s . \tag{I.15}$$

where T_s^o is a suitable constant.

Furthermore, one may assume:

$$K_{ms} = K_{ms}^o + \alpha_k P_s . \tag{I.16}$$

where K_{ms}^o and α_k are also suitable constants.

The water mass variation can be obtained by substituting Equations I.13 and I.14 into the fluid mass balance of Equation I.9.a.

$$\frac{d\, M_f}{dt} = (1 - \frac{v_f}{v_{sf}})\, W_{fw} - \frac{v_s}{v_{sf}}\, W_{so} \qquad (I.17)$$

The turbine valve opening dynamic can be approximated by:

$$\frac{d\, C_v}{dt} = \frac{1}{\tau_v}\, (u_v - C_v) \qquad (I.18)$$

where τ_v is the valve positioning time constant [sec]; and u_v is the valve opening input signal.

It is also assumed that the valve positioning control provides an adequate turbine response, and it is, in fact, realizable by imposing certain values on the elements of the weighting matrices.

The differential Equations I.2.a, I.4, I.5, I.6, I.7.a, I.7.b, I.8.a, I.8.b, I.14, I.17 and I.18, and the algebraic Equations I.2.b, I.3, I.15 and I.16, describe the nonlinear model used in this work.

The numerical values for the parameters and constants used in the model, are listed in Table I.1.

Consider the following state variables:

$$\underline{x}^T = [\rho_c, C, T_f, T_w, T_{w2}, T_p, T_m, P_s, M_f, C_v, u_1]$$

and the input variables:

$$\underline{u}^T = [u_R, W_{fw}, u_V].$$

The resulting coefficients of the linearized equation $\dot{\underline{x}} = A\underline{x} + B\underline{u}$ are given in Table I.2.

Table I.1

Numerical Values of Plant Parameters and Constants

Symbol Used	Value	Units	Parameter
β	0.0064	-	Delayed Neutron Fraction
l	1.6E-5	sec	Mean Generation Time
K_r	1.0	\$/m	Rod Reactivity Worth
λ	0.077	sec^{-1}	Precursor Decay Constant
α_f	-1.3E-5	$°F^{-1}$	Doppler Reactivity Coefficient
α_w	-2.0E-4	$°F^{-1}$	Coolant Reactivity Coefficient
P_o	2,200.	MW	Full Rated Power
F_f	0.95	-	Power Fraction in the Fuel
M_{fu}	8.709E4	Kg	Mass of the Fuel
C_f	1.373E-4	MW sec/Kg °F	Fuel Specific Heat
A_{fw}	3.945E3	m^2	Area Between Fuel-Coolant
K_{fw}	5.55E-4	MN/m^2 °F	Heat Transfer Coefficient
$F_{w1,2}$	0.025	-	Power Fraction of the Node
$M_{w1,2}$	6.26E3	Kg	Coolant Mass of the Node
C_w	3.094E-3	MW sec/Kg °F	Coolant Specific Heat
W_w	1.270E4	Kg/sec	Coolant Flow Rate
τ_r	2.0	sec	Rod Speed Time Constant
M_p	3.629E4	Kg	Total Coolant Mass in the S.G.

Table I.1

Continuation ...

Symbol Used	Value	Units	Parameter
A_{pm}	8.383E3	m^2	Heat Transfer Area in the S.G.
K_{pm}	1.312E-2	MW/m^2 °F	Heat Transfer Coef. Primary Loop
K_{ms}	5.247E-3	MW/m^2 °F	Heat Transfer Coef. Second. Loop
M_m	9.027E4	Kg	Mass of Tubes
C_m	2.536E-4	MN sec/Kg °F	Specific Heat of Tubes
K_c	1.363	Kg/sec Psia	Steam Flow Coefficient
τ_v	2.0	sec	Valve Time Constant
h_{fw}	0.784	MW sec/Kg	Feedwater Enthalpy
h_f	1.221	MW sec/Kg	Fluid Enthalpy at Steam Generator
h_s	2.782	MW sec/Kg	Saturated Steam Enthalpy at S.G.
v_f	1.249E-3	m^3/Kg	Specific Volume of Fluid
v_s	3.185E-2	m^3/Kg	Specific Volume of Steam
$\partial h_f/\partial p_s$	3.947E-4	MW sec/KgPsia	Fluid Enthalpy Variation with Pressure
$\partial h_s/\partial p_s$	-6.282E-5	MW sec/Kg Psia	Steam Enthalpy Variation
$\partial v_s/\partial p_s$	-4.246E-5	m^3/Kg/Psia	Variation of Specific Volume with Pressure
M_s^o	4.536E3	Kg	Steam Mass at Steam Generator
$\partial T_s/\partial p_s$	0.1375	°F/Psia	Saturated Temperature Variation with Pressure
J	145.06	m^3Psia/MW sec	Energy Equivalence Constant
K_{ms}^o	2.081E-2	MW/m^2 °F	Constant in Equation I.16
α_k	-1.75E-5	MW/m^2°F Psia	Coefficient in Equation I.16

Table I.2

Numerical Values of the Linear System Coefficients*

A(1,11) = 1.0	
	A(7,5) = 0.2391E1
A(2,1) = 0.2E3	
	A(7,6) = 0.2391E1
A(2,3) = -0.4063	A(7,7) = -0.6694E1
A(2,4) = -0.625E1	A(7,8) = 0.3425
A(3,1) = 0.8743E2	A(8,7) = 0.3353E1
A(3,2) = 0.3366E-1	A(8,9) = =0.5625
A(3,3) = -0.3608	A(8,10) = -0.6819E2
A(3,4) = -0.2549E1	
	A(9,8) = -0.7095
A(4,1) = 0.142E1	A(9,10) = -0.1263E4
A(4,2) = 0.5468E-3	
A(4,3) = 0.5365E-1	A(10,10) = -0.5
A(4,4) = -0.213E1	A(11,11) = -0.2E1
A(4,6) = 0.2029E1	
A(5,1) = 0.142E1	B(11,1) = 0.2E1
A(5,2) = 0.5468E-3	B(8,2) = -0.129E-1
A(5,3) = 0.5365E-1	B(9,2) = 0.104E1
A(5,4) = 0.1928E1	B(10,3) = 0.5
A(5,5) = -0.2029E1	
A(6,5) = -0.3150	The remaining elements
A(6,6) = -0.665	are zero.
A(6,7) = 0.98	

* 0.2E3 is to be read 0.2×10^3

I.3.2 - Current Control System Design

During the early years of power plant operations, each major subsystem was independently and manually controlled [7]. One operator adjusted the turbine throttle valve to match the demanded load, and another operator then controlled heat production in the reactor to maintain the desired throttle pressure. Today, automatic control systems are universally used. Basically, two control strategies have been adopted: first, there is the turbine following mode and secondly, there is the reactor-boiler following mode. In the turbine following mode, the operator adjusts the control rod positions and the feedwater flow to a new desired power operating level and the turbine will follow automatically by adjusting all needed parameters. Similarly, in the reactor-boiler following mode, the operator changes the turbine valve position to supply the required amount of steam for the turbine-generator to generate demanded load, and the reactor-boiler will follow automatically, with all parameters being subsequently adjusted.

Currently, Westinghouse and Combustion Engineering operate their PWR plants using automatic control system based on the reactor-boiler following mode. Babcock and Wilcox have combined both control modes and developed the so called "integrated control system" [8]. Within the framework of this approach, the desired load is set on the unit load demand, and control signals are generated to control the reactor, boiler and turbine simultaneously. Such a control system can be viewed as a state-of-the-art design, evolved from single input, single output methods. In view of this, it is desirable to provide a systematic design basis using analytical techniques, which provide the means to solve the control problem of the overall plant dynamics in a more general fashion, optimally, in light of which, similar or better control performance can be achieved.

The brief description of a Westinghouse PWR control system, presented here, is based on references [9,10].

<u>I.3.2.1 - Reactor Control System</u>. According to a commonly used program, the automatic control system of a reactor regulates the average temperature of the coolant to follow a preprogrammed set point. The set point varies linearly with the load from a minimum T_{ref} at zero power to a maximum T_{ref} at full power. Reactor control is achieved by varying dissolved boron and control rods. Although boron concentration control is slow, it provides a uniform change in the core power distribution and a necessary reserve for control rod shutdown margin. Because of its long delay times, boron concentration is manually

controlled and not included in the automatic reactor system.

The rod control signal is generated by taking the sum of two error signals: the first channel provides the deviation of the average coolant temperature from the preprogrammed temperature, the other channel provides the mismatch power rate between the turbine and the reactor. This is shown in Figure I.3.

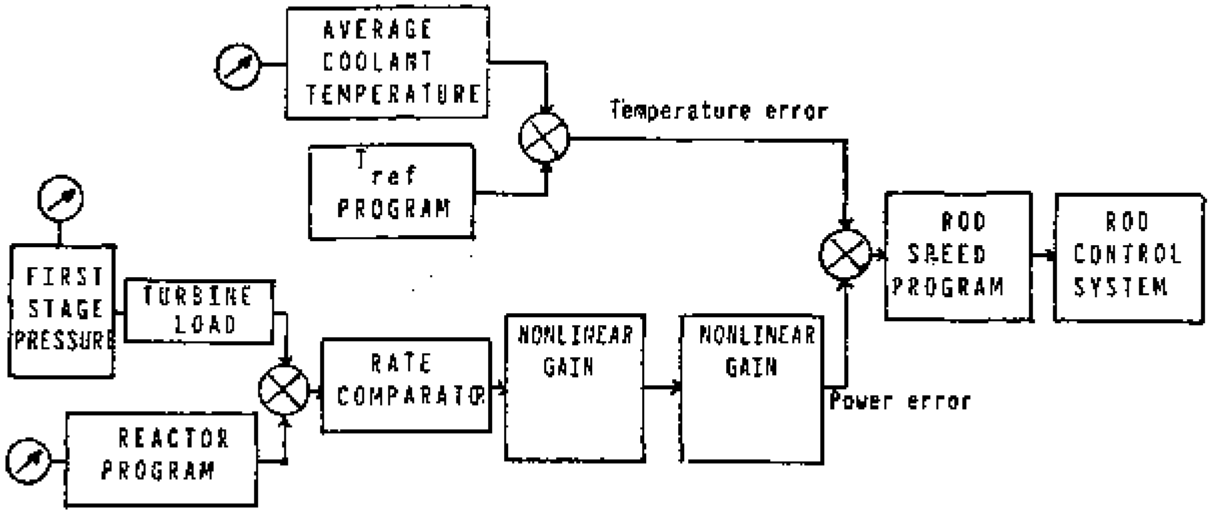

Figure I.3 - Conventional Reactor Control System.

The power rate mismatch signal is designed to speed up the reactor response and it does not produce a steady state error signal during steady state operations, although the reactor and turbine power may not match exactly. Two additional gain units are included in the power channel; the nonlinear gain converts the power rate mismatch signal to a temperature error equivalent signal, and in addition, it compensates for any load change magnitude effects. The variable gain unit compensates for the nonlinear effects of the rod reactivity for different operating levels. Through the other channel, the temperature error provides the fine control desired during near steady state operations. The rod speed program, which converts the total error signal to the desired rod motion, provides dead-band and lock-up characteristics in order to eliminate continuous rod stepping and bistable conditions.

I.3.2.2 - Steam Generator Control System. By positioning the feedwater valve, the feedwater flow control system provides an adequate production of steam to the turbine. The control of feedwater valve positioning is determined by water level error, feedwater flow, and steam flow. The system which accomplishes this is called a three-element control system, as shown in Figure I.4.

Because of expansion and contraction conditions of the coolant during the initial period of a load change, a large weight on the integral water level error in PI_2 is used. This approach delays the control contribution from such an initial misleading indication until the normal condition is restablished using the PI_1 flow error signal.

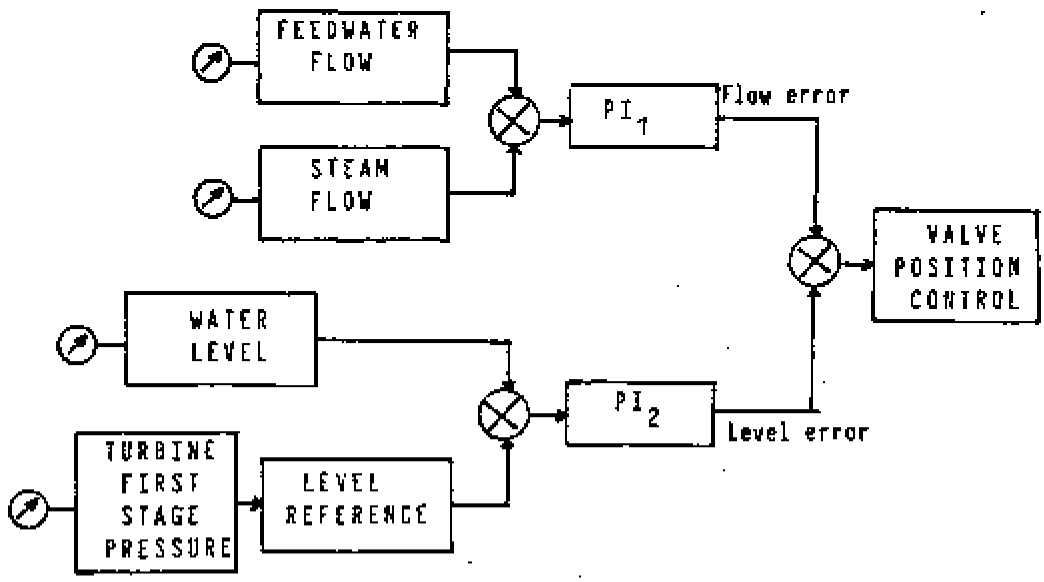

Figure I.4 - Conventional Steam Generator Water Level Control System.

It is important to point out here that Babcock and Wilcox [8] have incorporated an additional feature, so called "cross limits," in their control system which limits a further increase on the feedwater flow demand, if certain mismatches between feedwater flow and reactor power are detected. Thus, when the reactor demand signal is greater than measured power by a specific amount, the feedwater flow will increase in a tracking mode. This action prevents a reactor trip from low reactor coolant pressure due to the subsequent coolant temperature decrease. As shown later, a condition like this can be incorporated into a control system design using the approach proposed in this thesis by placing adequate weight on certain elements of the weighting matrices.

I.4 - Literature Review

Classical control theory has played a central role in the design of all current nuclear power plant control systems, and today's sophisticated control systems tend to represent the end result of a state-of-the-art design evolved from such an approach.

However, early research adopting different strategies based on optimal control, began in the decade of 70's [11, 12, 13]. In [12], Sinha and Bereznai have proposed an approach which included an adaptive feature where a second order reactor model was updated using a least square criteria from which the updated optimal gains were obtained. Because of numerical calculation difficulties inherent in the power plant complexity, most of the published papers have studied the control problem of the reactor core alone, or oversimplified models of the system were assumed.

Bjorlo et al. [14] applied linear quadratic theory to design a control system for a BWR plant. A Kalman filter was used to estimate the inaccessible states, and a single loop controller core was illustrated. Duncombe and

Rathbone [15] and Moore et al. [16] have also included additional plant components in the design. In [16], the maximum likelihood criterion was used to identify the plant, and in [15], researchers used an analog computer for the necessary computation of the Riccati equation. However, results obtained present difficulties for on-line implementation for the control of a real plant. Oguri and Ebizuka [17] suggested closed loop control of power level using dynamic programming, but this approach also required excessive memory and computing time.

Later, in 1976, a feedback control system for a BWR using a 9^{th} order plant was proposed by Shankar et al. [18]. However, only steady state control was considered, and the resulting control system required access to the full state vector. Frogner and Grossman [19], and Bjorn and Espefalt [20] proposed a similar control system and added a Kalman filter which was to be used to estimate certain vector components which represented inaccessible states.

For the variable set point problem, Frogner developed a control system which included a feedfoward signal to speed up the load change response. However, it suffered from a few drawbacks: it could not accommodate disturbances and it required a previous knowledge of the steady state input for each new load. This was achieved by solving the algebraic plant equation which introduced an open loop feature.

Feeley and Tylee developed an advanced protection system and a steam generator control system for the LOFT reactor [21] by using Linear Quadratic Regulator (LQR) theory and an integral control feature. In their approach, the steam generator control system provides zero steady state error and on-line state estimates of some non-measurable variables. The feedfoward signal used the same procedure as that of the Frogner design, therefore, it suffered similar drawbacks. Later Tye [22] proposed a robust controller featuring integral and proportional error feedback for power level change control. However, he considered only a single input and only the primary loop in the system design.

Harvey and Wall [23] presented an on-going study directed toward developing a methodology for designing power generating plant control systems. However, nuclear reactor issues and their constraints were not directly addressed in their work.

To handle variations in the plant parameters, Sinha and Law [24] have proposed an adaptive control approach for power level changes using the so-called "model reference adaptive control" technique. The design, as suggested by these researchers, considered only the reactor and, moreover, an oversimplified model was assumed.

In general, satisfactory results using such a technique are achieved when excitation input signals to the plant are sufficiently rich (broad frequency spectrum) [25], otherwise, the technique may experience convergence problems particularly if a multivariable system is considered. A modified "series-parallel" model reference adaptive control approach was proposed by Irving and Van Mien [26] to control the steam generator of a fast breeder reactor.

Allidina, Hughes, and Tye [27] have developed a self-tuning control system based on a generalized minimum variance strategy. The feedfoward control was adjusted, appropriately compensating the disturbances and insuring minimal variation of the output variables. It was applied to a single input, single output problem in which the primary coolant temperature was maintained at desired levels by adjusting rod positions.

Descriptions of several available methods of nuclear power plant control can be found in McMorron's survey report [28], and an application to steam generator control was presented in [29] using the modal control technique. Ebert [2] reviewed optimal control theory and discussed its generalized applicability in nuclear power reactors. It was shown in his review that a better reactor operation performance could be obtained through application of optimal control methods. However, a more sophisticated plant computer system and better core monitoring equipment is required to support such an implementation.

CHAPTER II

CONTROL SYSTEM DESIGN METHODS

II.1 - General Considerations

When designing a control system, the engineer is first faced with selecting an appropriate design method based on his only information, the plant and its expected performance. Classical control theory provides several effective techniques for designing a control system for single input, single output type plants. However, control of a nuclear power plant consists of regulating several outputs by manipulating several other inputs. Such a system is usually characterized by a coupled multivariable dynamic plant consisting of interactive components such as the reactor core, the primary heat transport system, the boiler, and the turbine-generator. A system this complex would make classical technique excessively complicated or, possibly, sometimes unusable.

On the other hand, modern control theory can be formulated in terms of state-space, in which the interaction among the components can be expressed in an elegant analytical structure and solved optimally. Moreover, such an approach allows the development of systematic design procedures which can be adequately standardized in a computer-aided design package. The control system design presented here was chosen from a variety of available design techniques. The approach is based on the solution of a linear dynamic system with a quadratic cost function. This approach was selected for the following reasons:

- It is achieved with relatively simple computational algorithms and the results are simply realizable and in closed-loop forms;
- The theory is well known and developed;
- It can be easily implemented on any digital computer.

In this Chapter, Section II.2 presents the solution to the standard (conventional) LQR problem and pertinent extensions. These control solutions require information of all states of the plant. However, if some states are unavailable, observers are incorporated in the controller to estimate those inaccessible states. Section II.3 presents some issues on decentralized control and hierarchical control relevant to the design. Finally, Section II.4 presents output feedback control schemes using "preserved modes" technique based on the full state feedback optimal solutions presented previously.

II.2 - Optimal Linear Quadratic Problem (deterministic case)

II.2.1 - Conventional Regulator Problem

Consider the model of the plant to be controlled, described by the following set of coupled nonlinear differential equations:

$$\dot{\underline{x}}_n = f(\underline{x}_n, \underline{u}_n)$$

where $\underline{x}_n$ is the state vector of dimension N;

$\underline{u}_n$ is the input control vector of dimension M.

The optimal regulator problem requires that the plant be formulated on a linear basis. Therefore, one can approximate the model by linearizing the equations around some predertemined values, in particular, the set of steady state operating points $\underline{x}_s$, i.e., $\underline{x}_n + \underline{x}$, then the perturbed plant.

$$\dot{\underline{x}}_n = \dot{\underline{x}}_s + \dot{\underline{x}} = \underline{f}(\underline{x}_s + \underline{x}, \underline{u}_s + \underline{u})$$

which can be approximately expressed as:

$$\dot{\underline{x}}_s + \dot{\underline{x}} = \underline{f}(\underline{x}_s + \underline{u}_s) + \left(\frac{\partial \underline{f}}{\partial \underline{x}}\right)_{\underline{x}_s} \underline{x} + \left(\frac{\partial \underline{f}}{\partial \underline{u}}\right)_{\underline{u}_s} \underline{u} ,$$

which results in the linear plant:

$$\dot{\underline{x}} = A(t)\ \underline{x} + B(t)\ \underline{u} .$$

where $\underline{x}$ is the state vector deviation from steady state of dimension N;

$\underline{u}$ is the input control vector deviation from steady state of dimension M.

thus, A is (N x N) plant matrix;

B is (N x M) input matrix.

The regulator problem is concerned with returning the process from any deviated initial condition to the origin in $\underline{x}$ space in an optimal manner. Thus, the invariant deterministic optimal regulator problem can be stated as follows:

Find a control $\underline{u}(t)$ which minimizes a given cost function

$$J = \tfrac{1}{2}\int_0^T (\underline{x}^T Q\ \underline{x} + \underline{u}^T R\ \underline{u})\ dt + x^T(T)\ S\ x(T),$$

subject to:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

and with

$$\underline{x}(0) = \underline{x}_0 \text{ given,}$$

where matrices A, B, Q, R, and S are time invariant.

Derivation and solution to this problem can be found in several optimal control text books [30, 31, 32]. The well known result is the following:

The optimal solution is given by full state linear feedback control

$$\underline{u}^*(t) = K(t)\, \underline{x}(t) \qquad 0 \leq t \leq T$$

where

$$K(t) = -R^{-1}\, B^T\, P(t)$$

and P(t) satisfies the nonlinear matrix Riccati equation:

$$-\dot{P}(t) = P(t)\, A + A^T\, P(t) - P(t)\, B\, R^{-1}\, B^T\, P(t) + Q$$

with P(T) = S.

The following conditions have to be satisfied in order to give uniqueness and existence of such a solution:

Q is any positive semi-definite symmetric matrix;

R is any positive definite symmetric matrix.

The system is stabilizable if the pair [A, B] is controllable or uncontrollable with unstable subspace contained in the controllable subspace.

From practical considerations, it is helpful if the control period is extended to infinity ($T \rightarrow \infty$), because in this special case, the resulting feedback gain K(t) is time invariant and the Riccati nonlinear differential equation in P is transformed into a set of nonlinear algebraic equations, that is:

$$\underline{u}^*(t) = K\, \underline{x}(t)$$

where

$$K = -R^{-1}\, B^T\, P$$

and P satisfies the algebraic matrix Riccati equation:

$$P\,A + A^T\,P - P\,B\,R^{-1}\,B\,P + Q = 0\ .$$

Usually a direct application of the previous results is unrealizable and sometimes unacceptable because the set points are not at the origin and, in general, some of the states are not available for control. Moreover, the plant is subject to external or parameter variations. Subsequent sections in this Chapter extend the method in attempting to solve basically these problems.

II.2.2 - External Disturbances

The state feedback control procedures presented in the previous section can only accomodate a nonzero initial condition which mathematically represents an impulse type of disturbance. In most practical situations, the system is subject to a variety of types of external disturbances; therefore, an additional control effort is required.

Solution for disturbances takes a variety of forms depending upon the type of disturbance assumed. Consider a linear plant subject to an external disturbance given by:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} + D\underline{w}$$

where $\underline{w}(t)$ is a $(L_w \times 1)$ vector representing the disturbance.

The simplest case occurs when the disturbance is known a priori, then one can easily compute in advance the necessary input signal attempting to eliminate the effect of the disturbance. However, such a situation is rarely practical.

In some cases, the designer assumes a non-deterministic type of disturbance; the existent stochastic control methods require a priori information about the statistical description of the disturbance and the effectiveness of the solution depends upon the accuracy of these data. This approach will not be presented in this paper; it can be found in [30].

Johnson [33] showed that inclusion of certain linearly combined time integrated state variables in the control design, can accommodate any unknown, but constant type of disturbances by regulating all the state variables to zero. In the scheme, the following condition was assumed:

$$\text{range } (D) \subset \text{range } (B) \;. \qquad \text{(II.1)}$$

In other words, there exists a matrix H such that $D = BH$.

Equivalent results to Johnson's are obtained by augmenting the system with an input variable $\underline{u}$ as additional state variables and using a regulator solution with $\dot{\underline{u}}$ (rate of input) as the new input variable. Anderson and Moore [34] referred to this procedure as a regulator problem with input derivative constraints. This feature provides a direct benefit to the reactor design problem because the positioning control of the rods in the reactor are definitely limited by their speed of movement; therefore, an appropriate weight in R can be established to satisfy such a constraint.

The assumption in Equation II.1 can be eliminated if only certain states are important for regulation, providing the following augmented system is controllable, that is:

$$\dot{\underline{x}} = \bar{A}\,\bar{\underline{x}} + \bar{B}\,\underline{u} + \bar{D}\,\underline{w} \quad \text{is controllable.}$$

where $\bar{\underline{x}}^T = [\underline{x}\ \underline{z}]$, with elements $\dot{z}_j = x_j$ being regulating variables;

and $\bar{A}$, $\bar{B}$, and $\bar{D}$ are adequate augmented matrices.

This fact was established in the Davison and Smith paper [35] by the following theorem:

Consider the linear system

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} + \underline{w}$$

$$\underline{y} = C\underline{x}$$

with x(t) a N-dimensional vector of unknown impulse or step type of disturbance.

The necessary and sufficient condition that there exists a realizable state feedback control system such that the eigenvalues of the closed loop system take on pre-assigned values and that the output $\underline{y}(t) \rightarrow 0$ as

$t \rightarrow \infty$, is that the following conditions hold:

(1) [A, B] be controllable;

(2) Rank $\begin{vmatrix} A & B \\ C & 0 \end{vmatrix} = N + L$

where L is the dimension of output vector $\underline{y}$.

Also, the feedback control is given by:

$$\underline{u} = F_1 \underline{x} + F_2 \int_0^t \underline{y}(\tau)\, d\tau$$

with suitable F_1 and F_2 satisfying the pre-assigned eigenvalues locations.

This theorem can also be used in step change type of varying set point problems.

Johnson [36] extended the method to accommodate a wider class of disturbances which can be modeled by the following:

$$\dot{v} = V\, v$$

$$w(t) = H(t)\, v$$

where H and V are known matrices. However, the required information may restrict some applications.

II.2.3 - Set Point Changes and Servo Problems

As mentioned before, the objective in designing a control system for a nuclear power plant is to provide a desired load following response. In this Section, the state space regulator method is extended into a broader class of control problems, referred to as Servo problems, to handle the varying set point situations by means of an external command input.

Kreindler [37] classified output following problems in terms of the type of command input. For the class which uses a polynomial type of command input, the task is called a "Servo Problem," but if the command inputs are some particular given functions of time, it is called a "Tracking Problem," and if it is desirable to follow the outputs of another external plant subject to its own command input, it is called a "Model Following Problem." Here, the task will be referred to as a "Servo Problem" whenever the outputs of the plant are required to follow any external input command. One can note the similarity, in a mathematical sense, between the servomechanism problem and the problem with the plant subject to external disturbances, but they differ as physical identities. Tracking problems can be conceived of which allow us to solve servomechanism systems; however, the solutions require a priori knowledge of the command input [38].

<u>II.2.3.1 - Step Command Input Problems</u>. For "Type One" of multivariable servomechanism problem, in which the output vector is required to follow some arbitrary constant reference output $\underline{y}_r$, consider the following linear system:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

$$\underline{y} = C\underline{x} \ .$$

It is desirable that $\lim_{t \to \infty} \underline{y}(t) = \underline{y}_r$.

The system can be augmented by including the integral error term

$$\dot{\underline{e}} = \underline{y} - \underline{y}_r \ ,$$

which results in the following system:

$$\begin{bmatrix} \dot{\underline{x}} \\ \dot{\underline{e}} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} \underline{x} \\ \underline{e} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \underline{u} + \begin{bmatrix} 0 \\ -I \end{bmatrix} \underline{y}_r . \quad (II.2)$$

For the system in Equation II.2 to be controllable, it must satisfy the controllability condition:

$$\text{rank} \begin{vmatrix} B & AB & \cdots & A^{n+r-1}B \\ 0 & CB & & CA^{n+r-2}B \end{vmatrix} = N + L . \qquad (II.3)$$

Young and Willems [38] showed that the condition II.3 is equivalent to those stated by Davison and Smith in the previous section. They are repeated here:

(1) rank $[B \quad AB \ldots A^{n-1}B] = N$ or $[A \; B]$ controllable

(2) $\text{rank} \begin{vmatrix} A & B \\ C & 0 \end{vmatrix} = N + L$

Notice that the set point $\underline{y}_r$ in Equation II.2 acts like a known disturbance, and if the above conditions are satisfied, the problem that remains is the one of finding an appropriate set of gains in F and F_I such that, when $\underline{u} = F\,\underline{x} + F_I\,e$ is applied, the resulting closed loop system has the desired responses.

Alternative approaches can be used to find the feedback gains F and F_I:

1. Pole assignment methods;
2. Optimal regulator;
3. Extended LQR with model following.

The approaches 2. and 3. are studied for the present design investigation.

In reference [38], Young and Willems used a pole assignment technique for a model following approach and then imbedded it in the cost function in order to find the optimal gains. But, the implementation of the solution required a priori knowledge of the new steady state values in order to feedback the input signal. This requirement makes the realization difficult.

Next, a simple realizable controller of the step command type is derived based on the regulator problem. Consider initially the nonlinear plant:

$$\dot{\underline{x}}_n(t) = f(\underline{x}_n(t), \underline{u}_n(t), \underline{y}_r)$$

where $\underline{y}_r$ expresses the set point.

The linearized plant around steady states $\underline{x}_{so}$, $\underline{u}_{so}$, given by the $\underline{y}_{ro}$ operating set point is:

$$\dot{\underline{x}}(t) = A\,\underline{x}(t) + B\,\underline{u}(t)$$

where $\underline{x}(t) = \underline{x}_n(t) - \underline{x}_{so}$

$\underline{u}(t) = \underline{u}_n(t) - \underline{u}_{so}$

and assuming the controlled states as:

$$\underline{y}_n = C\,\underline{x}_n \quad \text{or} \quad \underline{y} = C\,\underline{x}.$$

Consider a variable $\underline{e}$ such that

$$\dot{\underline{e}} = C\,\underline{x}_n - \underline{y}_{r1}$$

where $\underline{y}_{r1}$ is a new set point.

Then, the resulting system can be written as:

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u}$$
$$\dot{\underline{e}} = C\,\underline{x}_n - \underline{y}_{r1} \tag{II.4}$$

In order to eliminate the independent term $\underline{y}_{r1}$, one can take the time derivatives in the Equations II.4.

$$\frac{d\,\dot{\underline{x}}}{dt} = A\,\dot{\underline{x}} + B\,\dot{\underline{u}}$$

and

$$\frac{d\,\dot{\underline{e}}}{dt} = C\,\dot{\underline{x}}_n\,.$$

But $\dot{\underline{x}} = \dot{\underline{x}}_n$, hence the system is expressed in the following regulator problem:

$$\frac{d}{dt}\begin{vmatrix} \dot{\underline{x}} \\ \dot{\underline{e}} \end{vmatrix} = \begin{vmatrix} A & 0 \\ C & 0 \end{vmatrix} \begin{vmatrix} \dot{\underline{x}} \\ \dot{\underline{e}} \end{vmatrix} + \begin{vmatrix} B \\ 0 \end{vmatrix} \dot{\underline{u}}\,.$$

If the system is controllable, then one can find an optimal solution:

$$\dot{\underline{u}}(t) = K\,\dot{\underline{x}}(t) + K_I\,\dot{\underline{e}}\,(t)$$

such that $\dot{x}(t) \to 0$, $\dot{u}(t) \to 0$, and $\dot{e}(t) \to 0$ as $t \to \infty$.

Note that $\dot{e}(t)$ expresses the error itself, and the new steady state is given approximately by:

$$\begin{vmatrix} \underline{x}_{s1} \\ \underline{e}_{s1} \end{vmatrix} = \begin{vmatrix} A + BF & BF_I \\ C & 0 \end{vmatrix}^{-1} \begin{vmatrix} 0 \\ I \end{vmatrix} \underline{y}_{r1} .$$

Integrating $\dot{u}(t)$, one obtains:

$$\underline{u}(t) = \underline{u}(0) + K\,\underline{x}(t) + K_I\,\underline{e}(0) + K_I\,\underline{e}(t),$$

expressed in terms of the original state variables:

$$\underline{u}_n(t) = \underline{u}_{so} + \underline{u}_n(0) - \underline{u}_{so} + K(\underline{x}_n(0) - \underline{x}_{so}) + K(\underline{x}_n(t) - \underline{x}_{so}) \\ K_I(\underline{e}(0) + \underline{e}(t)).$$

Assuming initial condition at steady state $\underline{x}_n(0) = \underline{x}_{so}$, and since $\underline{e}(0)$ can be arbitrary set to zero, one achieves the following control expression:

$$\underline{u}_n(t) = \underline{u}_n(0) + K(\underline{x}_n(t) - \underline{x}_{so}) + K_I \int_0^t (C\,\underline{x}_n(\tau) - \underline{y}_{r1})\,d\tau . \quad \text{(II.5.a)}$$

One may establish the Q and R matrices of the quadratic cost function as follows:

$$Q = \begin{vmatrix} Q_1 & 0 \\ 0 & Q_e \end{vmatrix} \text{ and } R = R_1$$

then, the partition Q_1 penalizes the rate of changes in states and Q_2 penalizes the error between outputs and set points.

Figure II.1 illustrates the control system diagram. The resulting feedback system is realizable unless the control structure constraint is considered. This problem is treated in Section II.4.

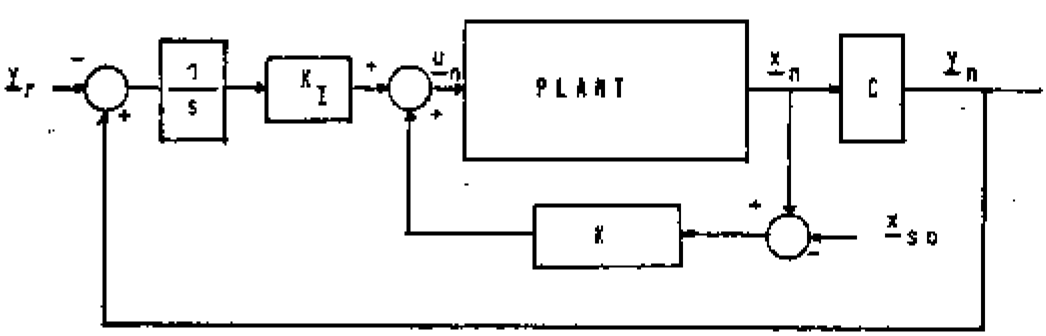

Figure II.1 - Servo Control System Diagram.

Wolfe and Meditch [39] suggested an additional transmission term K_e e(t) in the feedback signal which could improve the tracking response of step inputs without affecting previously established closed loop poles.

<u>II.2.3.2 - A Non Step Command Input Problem</u>. Kreindler [37] developed a servo system that can track a variety of command inputs provided by a command generator of type $\dot{z} = Z\,z$. The method is based similarly on the model following approach in which the command generator is incorporated in the system and the optimal solution is obtained by minimizing a cost function with an additional quadratic term in tracking error, as the following:

$$J = \tfrac{1}{2}\int_0^{\infty} \left((\underline{z}_1 - \underline{x}_1)^T Q_1 (\underline{z}_1 - \underline{x}_1) + \underline{x}^T Q_2 \underline{x} + \underline{u} R \underline{u} \right) dt \;.$$

Consider the augmented system:

$$\dot{\bar{\underline{x}}} = \bar{A}\,\underline{x} + \bar{B}\,\underline{u}$$

where $\bar{\underline{x}}^T = [\underline{x} \;\; \underline{z}]$ of dimension $N + N_2$

$$\bar{A} = \begin{vmatrix} A & 0 \\ 0 & Z \end{vmatrix} , \qquad \bar{B} = \begin{vmatrix} B \\ 0 \end{vmatrix}$$

and $\underline{x}_1$ is the partitioned state in $\underline{x}^T = [\underline{x}_1 \; \underline{x}_2]$ assumed to follow the command input $\underline{z}_1$ in $\underline{z}^T = [\underline{z}_1 \; \underline{z}_2]$.

The cost function can be rewritten as:

$$J = \frac{1}{2}\int_0^{\infty} (\underline{\bar{x}}^T \, \overline{Q} \, \underline{\bar{x}} + \underline{u}^T R \, \underline{u}) \, dt$$

$$\text{with } \overline{Q} = \begin{vmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{vmatrix}$$

$$\text{where } Q_{11} = \begin{vmatrix} Q_1 & 0 \\ 0 & 0 \end{vmatrix} + Q_2$$

$$\text{and } Q_{12} = \begin{vmatrix} -Q_1 & 0 \\ 0 & 0 \end{vmatrix} \quad Q_{22} = \begin{vmatrix} Q_1 & 0 \\ 0 & 0 \end{vmatrix}$$

As expected, the solution is expressed as:

$$\underline{u} = -R^{-1} \quad [B^T \quad 0] \begin{vmatrix} P & P_{12} \\ P_{12}^T & P_{22} \end{vmatrix} \begin{vmatrix} \underline{x} \\ \underline{z} \end{vmatrix}$$

or

$$\underline{u} = F \, \underline{x} + F_Z \, \underline{z}$$

$$\text{where } \overline{P} = \begin{vmatrix} P & P_{12} \\ P_{12}^T & P_{22} \end{vmatrix}$$

satisfies the Riccati equation of the augmented system.

It is interesting to point out that the state feedback gain F (corresponding to P) can be obtained independently from the model generator and the feedforward gain F_Z (corresponding to P_{12}) can be obtained by the partitioned linear equation in P_{12}, in:

$$P_{12} Z + A^T P_{12} - P B R^{-1} B^T P_{12} + Q_{12} = 0 .$$

It was shown in [37] that the necessary and sufficient condition for the last equation in P_{12} to be stable is that:

$$R_e (\lambda_j + \nu_i) \quad 0 \quad \text{for } j = 1, 2 \ldots N \quad \text{and } i = 1, 2 \ldots N_Z$$

where λ_j and ν_i are eigenvalues of (A - BF) and Z, respectively.

The main drawback to this method is that it may require generation of the command input derivatives to feedforward the input signal because the command generator was only used as a fictitious analytical device. However, in the model following problem, the model is formulated as part of the controller which makes all states available.

II.2.4 - Model Following Control System

II.2.4.1 - A Command Generator Type. As mentioned previously, the servo problem can be solved using a model following approach, and again, the LQR technique provides a valuable tool for finding an optimal solution to the problem.

Basically, there are two approaches to formulate a model following problem:

1. Implicit model following;
2. Real model following.

The problem of implicit model following, also called "model in the cost," was studied by Tyler [40]. It consists in finding a set of feedback gains such that the resulting closed loop plant coefficients approach the coefficients of the model.

By penalizing the error between the model and the plant derivatives, the feedback gains solution is obtained in terms of difference between the system and model coefficients. However, this method is rarely used in practical applications because the model-matching ability of this method is highly dependent on whether each coefficient of the system can be independently made to match each of the model coefficients [40].

On the other hand, the real model following, also called "model in the system" approach, is able to handle plant disturbances, unlike the servo problem proposed in [37] by Kreindler. It includes the simulation of the model and, unlike implicit model following, it generates a feedfoward control signal using the model states, in addition to internal state feedback.

Considering the previously defined plant and model equations, the augmented system can be written as:

$$\begin{vmatrix} \dot{\underline{x}} \\ \dot{\underline{x}}_m \end{vmatrix} = \begin{vmatrix} A & 0 \\ 0 & A_m \end{vmatrix} \begin{vmatrix} \underline{x} \\ \underline{x}_m \end{vmatrix} + \begin{vmatrix} B \\ 0 \end{vmatrix} \underline{u} \quad \text{or} \quad \dot{\bar{\underline{x}}} = \bar{A}\,\bar{\underline{x}} + \bar{B}\,\underline{u}$$

with the following cost function:

$$J = \int_0^{\infty} (\underline{y} - \underline{x}_m)^T\, Q\, (\underline{y} - \underline{x}_m) + \underline{u}^T R\, \underline{u})\, dt$$

The problem is expressed in the LQR form and the solution can be expressed as [40, 41]:

$$\underline{u} = K_p\, \underline{x} + K_m\, \underline{x}_m \, .$$

An advantage in using this approach is that it can force the plant response to follow a previously defined standard pattern by feeding foward a necessary input signal regardless the disturbances in the plant. Such a standardized response makes the plant-operator interaction easier, as well as possibly providing additional information for plant diagnostics.

<u>II.2.4.2 - Model Following with External Step Command Input</u>. Consider now that the model system is driven by a step command input:

$$\underline{x}_m = A_m \underline{x}_m + B_m \underline{u}_m$$

$$\underline{y}_m = C_m \underline{x}_m \, .$$

where $\underline{x}_m$ is model state vector of dimension N_m;

$\underline{u}_m$ is model step input vector of dimension M_m;

and $\underline{y}_m$ is output model vector of dimension $L_m = L$.

The augmented system for $\bar{\underline{x}}^T = [\underline{x} \quad \underline{x}_m]$ can be written as:

$$\dot{\bar{\underline{x}}} = \begin{vmatrix} A & 0 \\ 0 & A_m \end{vmatrix} \bar{\underline{x}} + \begin{vmatrix} B \\ 0 \end{vmatrix} \underline{u} + \begin{vmatrix} 0 \\ B_m \end{vmatrix} \underline{u}_m$$

where $\bar{\underline{y}}$ is defined as the output error vector $\bar{\underline{y}} = \underline{e} = \underline{y} - y_m$.

It is desirable that $\underline{e}$ approaches zero and $\bar{\underline{y}} = [C \quad -C_m] \, \bar{\underline{x}} = \bar{C} \, \bar{\underline{x}}$.

Therefore, one can define the following cost function:

$$J = \tfrac{1}{2} \int_0^{\infty} (\underline{\bar{x}}^T \, \bar{Q} \, \underline{\bar{x}} + \underline{u}^T \, R \, \underline{u}) \, dt$$

where $\bar{Q} = \begin{vmatrix} Q & -QC^T C_m \\ -C_m^T CQ & C_m^T CQ^T C_m \end{vmatrix}$ is a square matrix of dimension $(N + N_m)$;

and Q and R are weighting matrices corresponding to the original plant.

For a step type of unknown command input to the model, one can apply Davison's theorem to the problem formulated above, assuming the term $\underline{u}_m$ as a constant disturbance.

Therefore, if the conditions:

$[\bar{A} \, \bar{B}]$ is controllable and the rank of $\begin{vmatrix} \bar{A} & \bar{B} \\ \bar{C} & 0 \end{vmatrix} = N + N_m + L$

are satisfied, then a control input of type

$$\underline{u} = K_1 \, \underline{\bar{x}} + K_2 \int_0^t \bar{y}(\tau) \, d\tau \, ,$$

can assign the eigenvalues of the closed loop system into a proper location in the left half of the complex plane, and the output $\underline{\bar{y}} \rightarrow 0$ as $t \rightarrow \infty$.

Alternatively, the optimal gains K_1 and K_2 can be obtained using a similar approach as derived in Section II.2.3 for step command input problems. It is repeated here:

The modified plant equation is model-augmented

$$\dot{\bar{\underline{x}}} = \begin{vmatrix} A & 0 \\ 0 & A_m \end{vmatrix} \bar{\underline{x}} + \begin{vmatrix} B \\ 0 \end{vmatrix} \underline{u} + \begin{vmatrix} 0 \\ B_m \end{vmatrix} \underline{u}_m$$

and $\bar{\underline{y}} = [C \quad -C_m] \ \bar{\underline{x}} = \underline{e}$

where $\bar{\underline{x}}^T = [\underline{x} \quad \underline{x}_m]$.

Defining variable $\underline{z}$ such that:

$$\dot{\underline{z}} = [C \quad -C_m] \ \bar{\underline{x}} \ .$$

and taking time derivative of the augmented plant equation, one obtains:

$$\frac{d}{dt} \dot{\bar{\bar{\underline{x}}}} = \left| \begin{array}{cc:c} A & 0 & 0 \\ 0 & A_m & \\ \hdashline \multicolumn{2}{c:}{\bar{C}} & 0 \end{array} \right| \dot{\bar{\bar{\underline{x}}}} + \begin{vmatrix} B \\ 0 \end{vmatrix} \dot{\underline{u}}$$

where $\dot{\bar{\bar{\underline{x}}}}^T = [\dot{\bar{\underline{x}}} \quad \dot{\underline{z}}]$

For a controllable system, an optimal solution is obtained for the above quadratic regulator problem as:

$$\underline{\dot{u}}(t) = K\,\dot{\bar{\underline{x}}} = K_1\,\dot{\bar{\underline{x}}} + K_2\,\dot{\underline{z}}$$

such that $\dot{\underline{x}} \to 0$, $\dot{\underline{x}}_m \to 0$, and $\dot{\underline{z}} \to 0$ $(\underline{e} \to 0)$ as $t \to 0$.

Integrating the last equation in $\dot{\underline{u}}$, assuming initial condition at steady state and setting $\underline{z}(0) = 0$, one gets:

$$\underline{u}_n(t) = \underline{u}_n(0) + K_{11}\,(\underline{x}_n(t) - \underline{x}_{so}) + K_{12}\,\underline{x}_m + K_2 \int_0^t (C\underline{x} - C_m\,\underline{x}_m)\,d\ .$$

where $[K_{11} \quad K_{12}] = K_1$

and, assuming the quadratic cost function given by:

$$\bar{Q} = \left[\begin{array}{cc:c} Q & -QC^T C_m & 0 \\ -C_m^T CQ & C_m^T CQ\,C^T C_m & \\ \hdashline & 0 & Q_e \end{array}\right]$$

Note that the upper left partitioned matrix penalizes the rate of change in state and in tracking error, while the Q_e matrix penalizes directly the tracking error. An alternative optimal control solution can be obtained based on integral states for the augmented system in Davison's theorem, but those weighting matrices are established in a different manner, by penalizing state, model state and integrated tracking error.

II.3 - Related Issues on Hierarchical and Decentralized Control

The development of large-scale system theory in the control field has established new control approaches and computational methods due essentially to two reasons:

1. High dimensionality;
2. Nonclassical information structure.

Systems affected primarily by high dimensionality problems, in which an overall plant optimization process is prohibitive, have led to the development of so called "decomposition coordination" methods [42]. Here, the optimization of the overall system is achieved by solving iteractively a set of independent sub-problems (decomposition) using certain coordination variables to take the interconnections into account and to provide a means for applying overall optimal solution convergence criteria (coordination).

Based on this philosophy, a variety of methods have been developed [43, 44, 45, 46]. Since these methods involve an interactive approach, they are faced with convergence and on-line application difficulties as well. Moreover, most of these methods yield open loop solutions; therefore, they are restricted in their use in most practical problems, mainly if process control applications are concerned.

Nevertheless, Singh, Hassen and Titli [47], and Gopalsami and Sanathanan [48], have developed schemes which calculate a complete (localized and coordinated) feedback control scheme based on linear quadratic problems employing "interaction prediction" principle in a decentralized computation structure.

On the other hand, decentralized and hierarchical control system structures have been used to control systems characterized by both, high dimensionality and information structure constraint problems. Information structure constraints for large-scale systems is usually characterized by geographical separation of systems components. For such a problem, Wang and Davison [49] have derived the necessary and sufficient condition for the existence of a local control law to stabilize the system.

Siljak [50] suggested a multilevel control scheme in which the local optimal feedback controllers are computed ignoring the interactions among decomposed subsystems, and an additional control is generated by a global controller to neutralize the effect of interconnections. His objective was emphasized in the design of a reliable control system for a plant subject to structural perturbations. Alternative hierarchical control schemes can be found in the literature, e.g., [51, 52].

Since one of the primary design objectives in this thesis is to avoid excessive information transfer from the subsystems while still providing satisfactory response and coordination, the problem can be solved by expressing it in terms of information structure constraints. It should be noted that a complete decentralization of the control system for the nuclear power plant is unnecessary, and with today's computer technologies, only an unusually large-scale system would require a decentralized computation structure. Indeed, no better coordination can be achieved than coordination schemes obtained by centralized methods. Therefore, a direct application of these schemes is unsuitable for the problem considered in the thesis. However, the hierarchical concept has been considered in order to increase the robustness of the control system in the event of an ocurrence of a central controller failure. Several design methods are available to handle information structure constraint problems using centralized computation structure. They are presented in the next section.

II.4 - Control Structure Constraints

One difficulty found with LQR methods in most practical applications is due to control structure constraints. The optimal solution obtained in the previous section, requires information about all state variables of the plant in order to generate the control signal. Since some states of the plant are inaccessible for measurements due to technical or economical reasons, implementation of the method becomes impracticable. Another type of control constraint, referred to as the "multiple structure" constraint, occurs when each control input has access to a different set of states. Such a characteristic is a common problem found in large-scale systems, in which the information on different sets of state variables is available only at different physical locations. Therefore, it makes the implementation of a centralized controller prohibitive due to the high cost or reduced reliability of the control system. In such a situation, one has to either use a decentralized control system or to develop a control scheme with minimum information transfer from the system components. Figure II.2 illustrates various practical situations of control structure constraints:

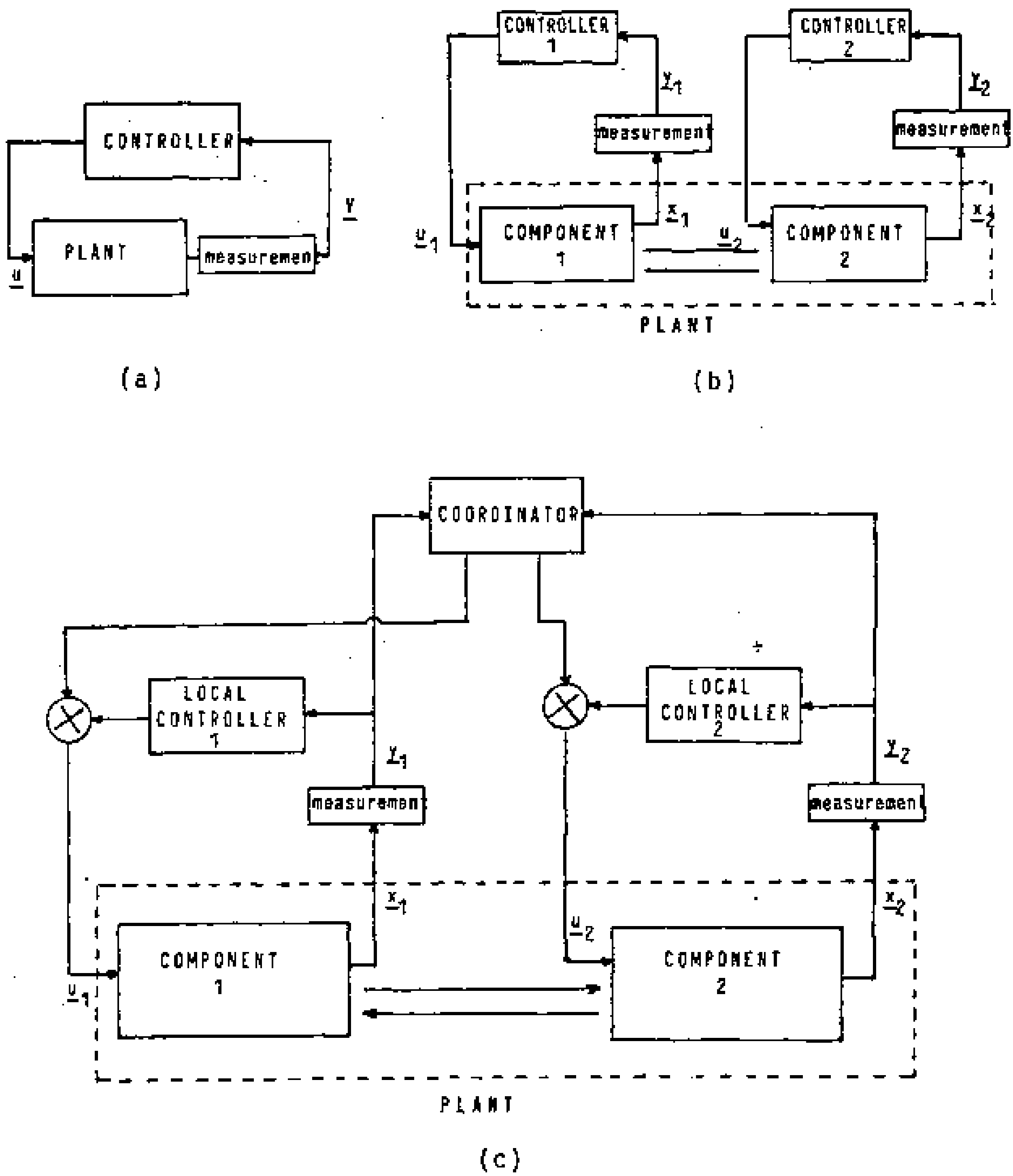

Figure II.2 - Control Structure Constraints.

where $\underline{x}$ represents plant states;

$\underline{y}$ represents measurable outputs;

$\underline{u}$ represents control input signals.

In Figure II.2, the case (a) shows the single structure constraint, and cases (b) and (c) illustrate multiple structure constraints where a decentralized and a coordinated control system are implemented, respectively.

The problem can be formulated as the following:

The previous section has established the optimal state feedback control as:

$$\underline{u}^*(t) = -R^{-1} B^T P \underline{x}(t) = F \underline{x}(t),$$

which results in the closed loop system:

$$\dot{\underline{x}}(t) = (A + B F) \underline{x}(t).$$

By taking the control structure into consideration for single constraint, the feedback control input is reduced to:

$$\underline{u}(t) = K \underline{y}(t)$$

with $$y = C \underline{x}$$

where $\underline{y}$ is the output vector of dimension L;

C is the output matrix of dimension L x N, specified by the control structure constraint.

The resulting closed loop system is:

$$\dot{\underline{x}}(t) = (A + B\ K\ C)\ \underline{x}(t)$$

For the multiple constraint system, consider Q sets of $\underline{u}_i$ (i = 1 ...Q) formed by the distinct set of components of $\underline{u}$, in which each $\underline{u}_i$ is generated using different sets of accessible outputs. Thus, $\underline{u}_i$ is the control input of the i^{th} control channel of dimension M_i and $\underline{y}_i$ is the corresponding output vector of dimension L_i, whose elements consist of accessible outputs for control channel i.

Therefore,

$$\underline{u}_i(t) = K_i\ \underline{y}_i(t) \qquad (i = 1, ...Q)$$

with

$$\underline{y}_i(t) = C_i\ \underline{x}(t)$$

where $\underline{u}_i$ is the control input of i^{th} control channel of dimension M_i;

$\underline{y}_i$ is the corresponding output vector of dimension L_i;

C_i is i^{th} output matrix of dimension L_i x N, specified by the control structure constraint.

The resulting closed loop system is:

$$\dot{\underline{x}}(t) = (A + \sum_{i=1}^{Q} B_i\ K_i\ C_i)\ \underline{x}(t)$$

where $B = [B_1\ \ B_2\ \ldots B_Q]$ is the compatible partition of the input matrix.

Several methods have been used to solve the problem of inaccessible states in linear quadratic control theory. Basically, solutions can be categorized in two different approaches: the first is based on the use of observers, which reconstruct the inaccessible states based on the outputs of the plant and then use these states to generate control inputs as with the original full state feedback. Luenberger [53] developed a deterministic scheme for reconstructing the inaccessible states; however, its performance depends strongly on an exact knowledge of the plant. For plants subject to Gaussian-type noise, a Kalman filter can be used to estimate optimally the states of the plant in a minimum variance sense [54]. Such an algorithm requires a priori knowledge of the noise characteristics. However, the use of observers increases the complexity of the control system, introducing additional order dynamics into the controller, and it may become impracticable or incovenient to implement if the order of the plant considered is large. It is believed that complex control systems are less reliable than simple ones. Furthermore, it has been demonstrated in practice that satisfactory plant performance can be achieved by controllers using only the available outputs. So, the matter of additional complexity has no guaranteed beneficial control effects.

One of the objectives considered in this thesis is the avoidance of on-line complexity in any of the proposed control systems. Several methods based on a different approach which involves feeding back only output variables using "best" gains generated off-line, have been developed. Davison and Goldberg [55] employed a "modal analysis" technique to determine the critical variables to be measured and used for feedback control.

Methods using dynamic feedback controllers can be used to compensate for the inaccessible states [56]; however, these methods also introduce additional order dynamics in the controller.

Levine and Athans [57] computed the optimal output feedback gains, which correspond to a suboptimal solution in the full state feedback optimal solution context. They have shown that the optimal solution is a function of the initial condition $\underline{x}(0)$. To eliminate such a dependence, the initial state was assumed to be a random variable uniformly distributed on a unit hypersphere of dimension N. The control problem then can be formulated as:

Find an optimal output gain K^o in the control law of the form

$$\underline{u}(t) = -K^o\, C\, \underline{x}(t)\,,$$

with $K^o\, C$ satisfying the control structure constraint specifications, i.e., $K^o \in \Omega$, which minimizes the cost function

$$J(K) = E\left[\int_0^\infty (\underline{x}^T\, Q\, \underline{x} + \underline{u}^T\, R\, \underline{u})\, dt\right].$$

The matrices Q and R correspond to the original quadratic cost function problem, and Ω denotes the set of all linear time invariant matrices (M x N) such that the control structure constraint is satisfied.

Defining

$$V = \int_0^\infty \emptyset^T(t, 0)\, (Q + C^T\, K^T\, R\, K\, C)\, \emptyset(t, 0)\, dt$$

where $\emptyset(t, 0)$ is the transition matrix.

The problem can be equivalently stated as Min ½ tr [V(K)].

It was shown that, for any K which results in negative closed loop eigenvalues, V is a symmetric and positive solution of the equation:

$$(A + B\,K\,C)\,V + V\,(A + B\,K\,C) + Q + C^T\,K^T\,R\,K\,C = 0\,. \qquad \text{(II.6)}$$

Also, the gradient of J, with respect to K is given by:

$$\frac{\partial J}{\partial K} = 2\,(R\,K\,C\,P\,C^T - B^T V\,P\,C^T)\,.$$

where P satisfies the Lyapunov equation.

$$(A + B\,K\,C)\,P + P\,(A + B\,K\,C)^T = -I. \qquad \text{(II.7)}$$

From the necessary condition for minimum, one gets:

$$K^o = R^{-1}\,B^T\,V\,P\,C^T\,(C\,P\,C^T)^{-1} \qquad \text{(II.8)}$$

Levine and Athans presented an algorithm which computes K^o by an iterative technique using Equations II.6, II.7 and II.8. However, it requires a stable initial guess for K, and its convergence to the optimal value is not guaranteed. Horisberger [58] has established the existence condition for K^o, providing that there exists an initial $K^o(0)$ such that $(A - B\,K^o(0)\,C)$ is assumptotically stable, and the Q matrix is positive definite. In the work of Horisberger and Belanger [58], and Choi and Sirisena [59] gradient technique based on the Fretcher-Power-Davison algorithm have been used to solve the optimization problem. Shapiro, Fredericks and

Rooney [60] extended the method to circumvent the stable initial guess required by the previous method.

The methods mentioned so far have not been developed for multiple control structure constraint problems. Kosut [61] derived two algorithms to find the best output gains which can be applied to multiple constraint situations. The first is based on minimum error excitation, in which the integrated feedback error signal (with respect to optimal state feedback) is minimized. The problem can be formulated as:

$$\underset{F}{\text{Minimize}}\ J = \tfrac{1}{2}\int_0^{\infty} \underline{q}(t)\ R\ \underline{q}(t)\ dt$$

with

$$\underline{q}(t) = (F - F^{*})\ \underline{x}(t)$$

where $\underline{q}(t)$ is the error excitation vector of dimension $(N \times 1)$;

R is a diagonal matrix;

F^{*} is the optimal state feedback gain matrix;

and F is an output feedback matrix $F = K\ C$ such that the control structure constraint is satisfied, i.e., $K \in \Omega$.

It was shown that the "best" output feedback matrix for single control constraint is expressed as:

$$F^{o} = F^{*}\ P\ C^{T}\ (C\ P\ C^{T})^{-1}\ C$$

with adjoint matrix P satisfying the equation

$$(A + B F^{*}) P + P (A + B F^{*})^{T} + I = 0.$$

And, for a multiple structure constraint, the i^{th} row of the best output matrix is expressed as:

$$F_i^{o} = F_i^{*} P C_i^{T} (C_i P C_i^{T})^{-1} C_i \qquad (i = 1, 2 \ldots.Q).$$

The other method is based on the "minimum norm" criteria, and it can be formulated as:

$$\underset{F \in \Omega}{\text{Minimize}} \left\| F - F^{*} \right\|,$$

and the solution was given by:

$$F^{o} = F^{*} C^{T} (C C^{T})^{-1} C \quad \text{for the single structure constraint,}$$

and

$$F_i^{o} = F_i^{*} C_i^{T} (C_i C_i^{T})^{-1} C_i \quad \text{for the multiple structure constraint.}$$

$$(i = 1, 2 \ldots.Q)$$

Note that the "minimum norm" solution is equivalent to an optimal state feedback where the terms involving the inaccessible states are deleted. Furthermore, the reader should note that neither of these methods ensures stability.

Bengtsson and Lindahl [62] developed a scheme which reduces a previously known optimal state feedback into a controller satisfying a predefined control structure constraint based on a maximum preservation of the eigenspace. The advantage of this method is that it can be applied to decentralized or hierarchical control structures and it serves as a useful tool for analysis of such systems.

Consider an optimal state feedback control statement:

$$\underline{u}^* = F^* \underline{x}$$

and the corresponding closed loop system

$$\dot{\underline{x}} = (A + B F^*) \underline{x} .$$

The objective is to find a structure constrained control law

$$\underline{u}_i = K_i \underline{y}_i \qquad (i = 1, \ldots.Q)$$

such that the corresponding closed loop system

$$\dot{\underline{x}} = (A + \sum_{i=1}^{Q} B_i K_i C_i) \underline{x}$$

has its dominant modes preserved.

The method is based on the following theorem:

Let $\Lambda = [\lambda_1, \lambda_2, \ldots \lambda_p]$ be a symmetric set of eigenvalues of the original optimal system $(A + B F^*)$ and let E be a real basis matrix for the correspondent eigenspace. Then if

$$K_i C_i E = F_i E$$

has solutions K_i^o (i = 1, 2 ....Q), then Λ is also the set of eigenvalues of

$$A + \sum_{i=1}^{Q} B_i K_i^o C_i \qquad \text{(II.9)}$$

Based on the above theorem, it is highly desirable to form the complete set of eigenvectors as the real basis matrix E and attempt to preserve all the original eigenvalues, but this is often not possible. In general, the number of eigenvalues that can be preserved is bounded by the number of outputs available.

It is easy to see that Equation II.9 consists of (Q x p) equations and $\sum_{i=1}^{Q} M_i L_i$ unknowns, consisting of K_i elements. The independent terms of $F_i E$ represent the projection of the input signal in the eigenvector, i.e., the element (i, j) of $F_i E$ is the part of i^{th} input signal generated by the projection of state vector in the j^{th}

selected eigenvector to be preserved. Therefore, generation of a true input signal would be obtained if the state were fully contained in that eigenvector.

One can note the similarity of this method to that of Kosut, in which the excitation error is minimized. Here the eigenspace is introduced to be considered the minimization.

Since, in general, preservation of all modes is not possible, good sense indicates that one should select the set of those dominant modes to be preserved. The obvious reason for this is that, after a short time, the fast modes will die out and the input signal will be generated by the persistant dominant modes only. Unfortunately, when such a selection is made, some of the remaining modes may become unstable; i.e., some of the remaining eigenvalues may move to the right half of the complex plane. At this point, one has to find all the critical modes and a least square solution can be applied.

Using the Equation II.9, the solution for K_i is given by:

$$K_i^o = F_i \; E \; (C_i \; E)^+$$

where + denotes pseudo inverse (see Appendix A)

For a consistent system, this approach results in a unique solution and the pseudo inverse is equivalent to the conventional inverse.

If the system is underconstrained, more than one solution exists, then one can find a unique solution which minimizes $\left\| K_i \; R_i \right\|$, where R_i is a $(L_i \times L_i)$ diagonal matrix adequately weighted. In this case, the solution is given by:

$$K_i^o = F_i \; E \; (R_i^{-1} \; C_i \; E)^+ \; R_i^{-1} \qquad \text{(II.10)}$$

If the system is overconstrained, no exact solution exists, and therefore, a least square approach will give a unique solution which minimizes $\left\| (K_i \; C_i \; E - F_i \; E) \; W \right\|$, where W is a positive definite $(p \times p)$ diagonal matrix adequately weighted. The solution is given by:

$$K_i^o = F_i \; E \; W \; (C_i \; E \; W)^+ \qquad \text{(II.11)}$$

A general solution for K_i^o can be obtained by combining the solutions in Equations II.10 and II.11. It results in:

$$K_i^o = F_i \; E \; W \; (R_i^{-1} \; C_i \; E \; W)^+ \; R_i^{-1} \qquad \text{(II.12)}$$

Note that each diagonal element in W expresses the weight imposed to preserve its respective mode in the least square sense, and an appropriate selection of of weighting elements in W is crucial for stability of the reduced system.

CHAPTER III

A PWR CONTROL SYSTEM DESIGN

III.1 - Control Problem Formulation and General Considerations

The control schemes presented in this thesis are proposed to control effectively the anticipated load changes of the PWR power plant described by the set of nonlinear differential equations derived in Chapter I. The state vector, including integral tracking errors, consists of 14 states as follows:

$$\underline{x}^T = [\rho_c, C, T_f, T_w, T_{w2}, T_p, T_m, P_s, M_f, C_v, u_1, \varphi_1, \varphi_2, \varphi_3].$$

where the new states φ_1, φ_2 and φ_3 are the integrated output error corresponding to T_w, M_f and C_v, respectively.

The control system is assumed to regulate the output vector $\underline{y} = [T_w, M_f, C_v]$, which consists of the average reactor coolant temperature T_w, water mass in the steam generator (water level) M_f and throttle valve opening (steam flow) C_v, into their corresponding set points. It does this by manipulating the input vector $\underline{u} = [u_R, W_{fw}, u_v]$

consisting of electrical signals for rod speed movement u_R, feedwater flow W_{fw} and electrical signal for throttle valve positioning u_v.

From plant operational requirements, it is suggested that T_w follow a set point T_{wref} which is a linearly increasing function with the load, as shown in Figure III.1. The water mass in the steam generator is regulated at a constant set point level (variable set point can also be used), and the throttle valve opening is regulated at set point positions which are assumed to be a linear function of corresponding load. The remaining plant variables are assumed to vary within their acceptable range.

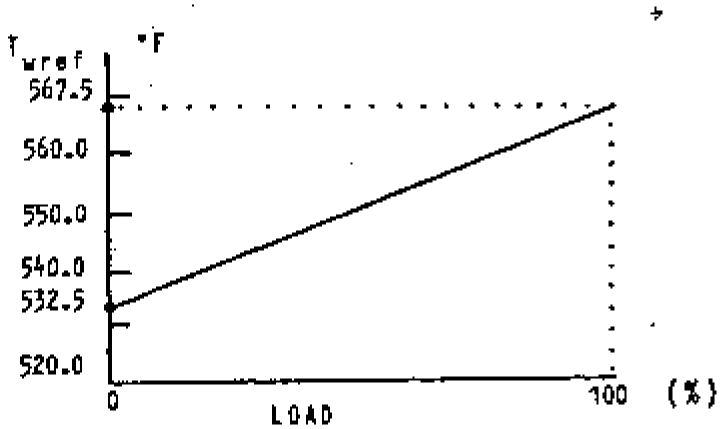

Figure III.1 - Coolant Average Temperature Program.

The magnitude of the rod speed, feedwater flow and valve positioning changes are limited by physical system considerations. Additionally, reactor safety imposes a more severe constraint which limits the rod withdrawn speed within a certain rate, and moreover the valve positioning time constant must consider the turbine dynamics which have not been considered in the plant model.

Basically, two control approaches have been adopted to solve the general problem. The first uses the control scheme based on the servo problem design described in Section II.2.3, and the second control approach is based on the model following design technique described in Section II.2.4. Control system variations with respect to their information structure, are also considered.

Based on simulations for a variety of conditions, the designer should then select the control system which incorporates minimum information transfer for a more reliable control system and reduced implementation cost without compromising the plant performance.

Since the design methods are based on the linearized plant equations around steady state operating conditions, it is expected that a different set of gains will be obtained for each operating power level. To overcome this problem, one can approximate by fixing a set of gains for each range of operating power level.

III.2 - Step Command Input Control System Design

The design procedure consists initially in finding the full state feedback control gains K and K_i in Equation II.5.a and then applying the "preserved mode" technique described in Section II.4 to the obtained solution to compute the desired control system corresponding to a prespecified control structure constraint.

The main concern in the first part of the design is to select suitable weighting matrices Q and R, such that when the input signal generated by Equation II.5.a is applied to the plant, it provides the plant with a satisfactory transient response. Unfortunately, there are no quantitative methods for finding adequate weighting matrices. However, in view of the cost function definition, the diagonal elements of Q_1 in Q express the penalty weight placed on the rate of change in the corresponding state, the diagonal elements of Q_2 in Q express the weight penalized on the corresponding error between the output and its desired set point, and the diagonal elements in R express the penalty placed on the rate of change of the correspondent control input variable. However, common practice suggests, as an initial guess, that the diagonal elements of Q be weighted at values which are approximately equal to the reciprocal of the operating range of corresponding state, and the elements of R should be weighted based on the control magnitude constraint in such a way that the resulting control signals do not exceed their corresponding saturation values.

Within the range of selected matrices, it was found that the closed loop eigenvalues were insensitive to some weighting elements. Such elements were then set to zero. After crude weighting matrices were found, some more detailed operational constraints were taken into consideration.

The rod speed is limited by placing an adequately large weight on R(1,1) for reactor safety reasons. Such a slow response may lead to an excessively low coolant temperature which can cause a reactor trip. The problem can be prevented by setting Q(13,13) and Q(14,14) to small values relative to Q(12,12) (see page 78 for state index reference). However, small Q(14,14) results in slow response for steam demand from the turbine which, in turn, degradates the plant load following performance. The remaining alternative relies on the capability of the steam generator to handle increased water deficit "inventory." One may conclude, based on a cost-benefit analysis, that an increased size of the steam generator may be desirable. Therefore, during normal maneuvering conditions, the cross limit equivalent feature can be provided by assigning the right weight balance among the elements corresponding to states 12, 13 and 14.

Preliminary simulations showed that the degree of anticipation of the control rod movement for a load following transient was sensible to the weight assigned for R(1,1).

The resulting nonzero weighting elements used for design are the following:

```
Q ( 1, 1 ) = 0.1
Q ( 8, 8 ) = 0.1E-2
Q ( 9, 9 ) = 0.1E-3              R ( 1, 1 ) = 0.1E8
Q (11, 11) = 0.1E1       and     R ( 2, 2 ) = 0.1E2
Q (12, 12) = 0.1E2               R ( 3, 3 ) = 0.1E6
Q (13, 13) = 0.2E-4
Q (14, 14) = 0.5E5
```

The optimal feedback gains were computed employing the diagonalization approach described in [30]. The resulting gains and the corresponding eigenvalues are given in Table III.2 and III.1, respectively.

The control structure constraint is considered in the second part of the design. Here, the full state feedback is reduced to an output feedback structure in an attempt to preserve the dominant modes of the original system into the specified reduced feedback control system. If such an objective can not be fully achieved, a least square solution given by Equation II.12 may be used and the expression is repeated here:

$$K_i^o = F_i\ E\ W\ (R_i^{-1}\ C_i\ E\ W)^+\ R_i^{-1}\ . \qquad (III.1)$$

Note that the output scaling matrix R_i has no effect on the solution if the system is underconstrained, which is a rare situation.

Table III.1 - System Poles*

OPEN LOOP	CLOSED LOOP
-0.7274E1	-0.7274E1
-0.2E1	-0.2E1
-0.1995E1 ± 0.5355	-0.1995E1 ± 0.5355
-0.7922	-0.7917
-0.50	-0.4885 ± 0.3383
-0.3112	-0.3088
-0.3651E-1 ± 0.3711E-1	-0.7796E-1 ± 0.7292
and five 0.0 (three	-0.4119E-1 ± 0.3320
from integral error state)	-0.3148E-1 ± 0.2180E-1

For a given information structure constraint specified by the matrix C_i, the problem is converted into one in which one must find an appropriate mode weighting matrix W such that the reduced set of gains, obtained by corresponding least square solutions, results in a stable and acceptable plant response.

* The pole a ± b denotes the complex conjugate pair where "a" is the real part and "b" is the imaginary part.

There are no closed methods for selecting the W matrix in Equation III.1 to provide a stable solution; however, search methods can be used. Experience has shown that, in most cases, the search is reduced to the elements in W corresponding to dominate modes only. A computer program providing interactive computation for such a procedure was used. Table III.3 shows adequate matrices W for each specific structure constraint.

Since we are interested in analyzing the control system performance with different information structure constraints, several cases have been considered. Each structure, with its corresponding gains, is listed in Table III.2 and, furthermore, each is case-coded for easy reference.

Figure III.2 illustrates the implementation diagram of the control system. Decentralized and/or central coordinator controllers are provided in this presentation according to the specific constraints considered.

Case I.A assumes that all states are accessible for the proposed control system. This case is referred to as a standard solution for comparison purposes. Case I.B considers a single structure constraint where a set of measurable states is used for the control system in a centralized fashion. Case I.C is constrained to fewer accessible states and structured with a distinct set of accessible states for each control channel. The case of

Table III.2 - Feedback Gains for Servo Problem

GAINS \ STATES		r	P_w	T_f	T_w	T_{out}	T_{in}	T_m	P_s	M_f	C_v	u_1	ξ_1	ξ_2	ξ_3
CASE I.A	K_1	-0.17	-0.54E-4	-0.17E-4	-0.19E-3	-0.25E-3	-0.26E-2	-0.56E-3	-0.35E-3	0.87E-5	-0.65E-3	-0.80E-1	-0.30E-3	0.27E-6	-0.14E-1
	K_2	0.15E2	0.72E-3	0.26E-1	0.67E-1	0.12	0.97	0.23	0.18	-0.44E-1	0.26E1	0.60E1	-0.15E-1	-0.12E-2	-0.4E2
	K_3	-0.32E-1	-0.87E-4	0.2E-3	0.50E-3	0.17E-3	0.39E-2	0.67E-3	0.21E-3	0.13E-3	-0.99	-0.16E-1	0.85E-3	0.76E-5	-0.56
CASE I.B	K_1	-	0.16E-4	-	-0.11E-1	-0.18E-2	0.14E-1	-	-0.96E-3	0.13E-4	-0.76E-1	0.28E-1	-0.43E-4	0.74E-6	-0.50E-1
	K_2	-	-0.55E-2	-	0.17E1	0.39E0	-0.13E1	-	0.27	-0.44E-1	0.12E2	-0.20E1	-0.4E-1	-0.12E-2	-0.36E2
	K_3	-	-0.73E-4	-	0.38E-2	0.79E-3	0.50E-4	-	0.37E-3	0.13E-3	-0.58	0.34E-2	0.89E-3	0.76E-5	-0.57
CASE I.C	K_1	-	0.21E-4	-	-0.17E-1	-	0.13E-1	-	-	-	0.11	-	-0.12E-3	-	-
	K_2	-	-	-	0.20E1	-	-	-	0.75E-1	-0.42E-1	-0.35E2	-	-	-0.11E-2	-0.47E2
	K_3	-	-	-	-0.43E-2	-	-	-	0.67E-3	0.19E-4	-0.27E-1	-	-	0.76E-6	-0.67E-1
CASE I.D	K_1	-	-0.32E-5	-	-0.75E-3	-	-	-	-	-	-	-	-0.98E-5	-	-
	K_2	-	-	-	-	-	-	-	0.34	-0.47E-1	0.33E2	-	-	-0.14E-2	-0.38E2
	K_3	-	-	-	-	-	-	-	0.39E-3	0.1E-3	-0.54	-	-	0.56E-5	-0.28
CASE I.E	K_1	-0.12	-0.56E-4	0.56E-4	-0.48E-4	0.0	-	-	-	-	-	-0.12	-0.32E-3	-	-
	K_2	-	-	-	-	-	0.42	0.12	0.12	-0.43E-1	0.33E1	-	-	-0.11E-2	-0.43E2
	K_3	-	-	-	-	-	-0.1E-2	-0.32E-3	-0.34E-3	0.15E-3	-0.98	-	-	0.06E-5	-0.56E0

Table III.3 - Matrix W Used (Diagonal Elements)

MODES	1	2	3	4	5	6	7	8	9	10	11	12	13	14
CASE I.B	0.1E-1	0.1E-1	0.1E-1	0.1E-1	0.1E-1	0.1E-1	0.1E-1	0.1E-1	0.1	0.1	0.1E1	0.1E1	0.1	0.1
CASE I.C	0.1E-1	0.1E-1	0.1E-1	0.1E-1	0.1E-1	0.1E-1	0.1E-1	0.1E-1	0.1	0.1	0.1	0.1	0.1	0.1
CASE I.D	0.1E-1	0.1E-1	0.1E-1	0.1E-1	0.1E-1	0.1	0.1	0.5E-1	0.1	0.1	0.1	0.1	0.5	0.5

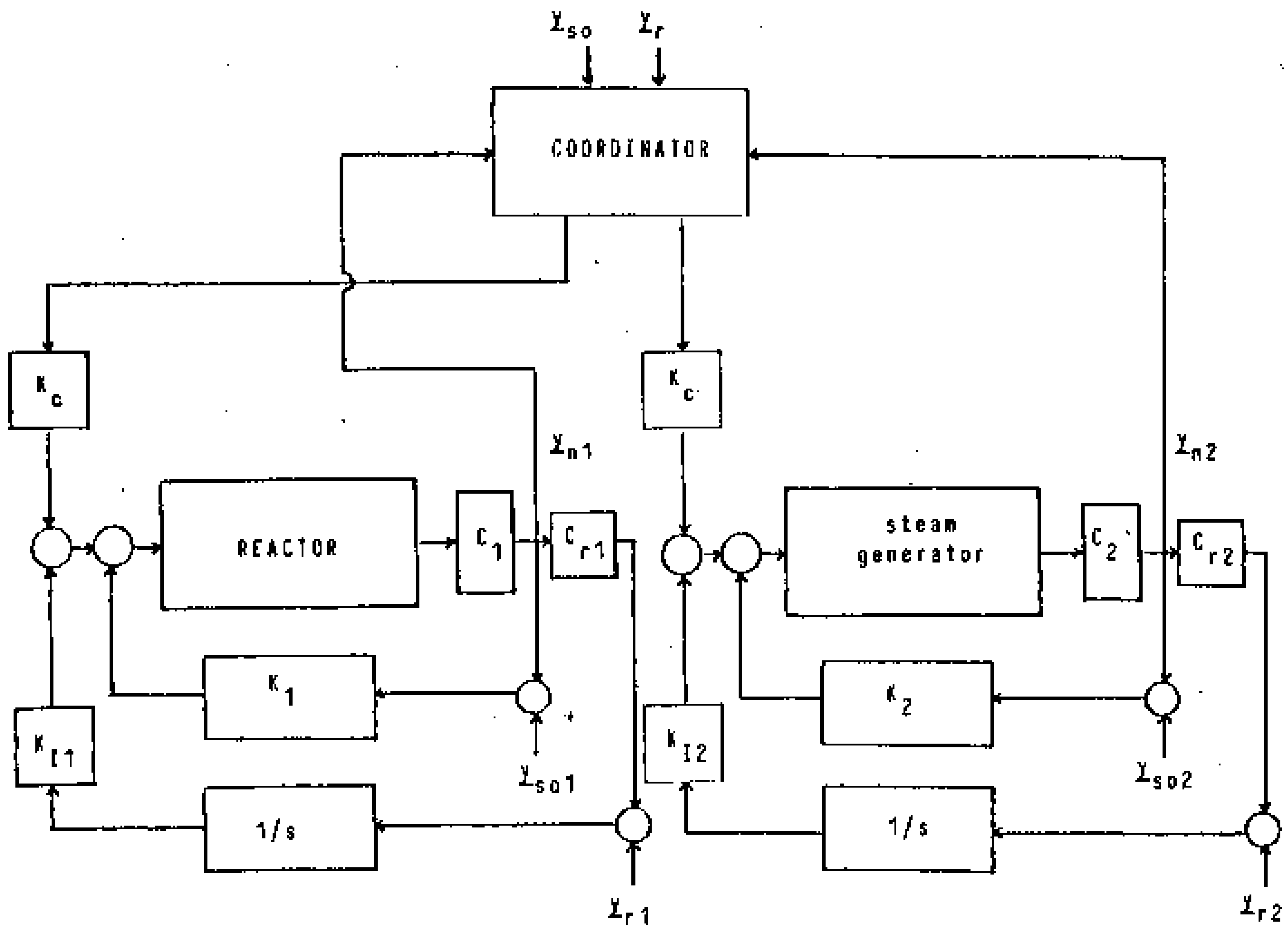

Figure III.2 - The Servo Control System Diagram a complete decentralization of information structure is represented in Case I.D. Finally, Case I.E presents the control system in which each major component controller was designed independent of the others.

III.3 - Model Following Control System Design

The design of the model following control system follows a similar procedure used for other servo design problems, except that here, a dynamic model is incorporated in the system to drive the plant set point and provide external feedfoward signals. The model consists of a fourth order linear system and the coefficients are selected such that the outputs respond according to a pre-stablished set point course. Table III.4 shows the coefficients of the model used for the design.

Table III.4

Model System Coefficients

AM (1, 1) = -0.04		
AM (1, 3) = -0.11		
AM (2, 2) = -0.01	BM (1, 1) = -0.072	CM (1, 1) = 0.3614
AM (2, 4) = -0.2	BM (4, 1) = 0.2	CM (2, 2) = 88.24
AM (3, 4) = 0.2		CM (3, 3) = 0.01
AM (4, 3) = -0.2		
AM (4, 4) = -0.4		

For a model following control system design, the model equations are incorporated in the plant, resulting in a 18th order system. The full state feedback gains in Equation II.5.b are computed using basically the same weighting matrices as used for the previous servo problem design. Here, the weight placed on the output state variables is used to penalize the deviations in the rate-of-changes between the respective plant and model outputs. The weight elements corresponding to tracking errors are adjusted such that the plant constraints are satisfied.

$$\underline{u}_n(t) = \underline{u}_n(0) + K\,(\underline{x}_n(t)-\underline{x}_{so}) + K_m\,\underline{x}_m + K_I \int_0^t (C\underline{x}_n(\tau)-C_m\underline{x}_m)\,d\tau \tag{II.5.2}$$

The consideration of control structure constraints proceeds in a manner similar to the servo problem. The resulting control gains for each structure constraint and full state feedback gains are given in Table III.5.

Case II.B assumes a single structure constraint and Case II.C assumes fewer but a practically chosen distinct sets of states. A decentralized information structure control system is established in Case II.D.

Figure III.3 illustrates the implementation diagram of the model following control system.

Table III.5 - Feedback Gains for Model Following Control System

GAINS \ STATES		r	P_w	T_f	T_w	T_{out}	T_{in}	T_m	P_s	M_f	C_v	u_1	φ_1	φ_2	φ_3
CASE II.A	K_1	-0.20	-0.61E-4	-0.53E-4	-0.39E-3	-0.38E-3	-0.42E-2	-0.88E-3	0.53E-3	0.10E-4	-0.95E-3	-0.94E-1	-0.59E-3	0.46E-6	-0.23E-1
	K_2	0.19E2	0.18E-2	0.34E-1	0.11	0.15	0.13E1	0.30	0.23	-0.52E-1	0.37E1	0.88E1	0.35E-1	-0.17E-2	-0.46E2
	K_3	0.53E-1	-0.56E-4	0.32E-3	0.14E-2	0.53E-3	0.95E-2	0.17E-2	0.69E-3	0.19E-3	-0.1E1	0.24E-1	0.24E-2	0.14E-4	-0.49
CASE II.B	K_1	-	0.35E-3	-	-0.16	-	-	-	0.15E-1	-0.73E-3	0.61E1	-	-0.99E-2	-0.57E-4	0.19E1
	K_2	-	-0.45E-1	-	0.21E2	-	-	-	-0.16E1	0.37E-1	-0.74E3	-	0.12E1	0.53E-2	-0.57E3
	K_3	-	-0.25E-3	-	0.96E-1	-	-	-	-0.73E-2	0.57E-3	-0.43E1	-	0.74E-2	0.44E-4	-0.15E1
CASE II.C	K_1	-	0.25E-4	-	-0.22E-1	-	0.15E-1	-	-	-	0.12	-	-0.21E-3	-0.13E-7	-0.53E-2
	K_2	-	-	-	0.34E1	-	-	-	0.28	-0.62E-1	0.57E2	-	-0.16	-0.25E-2	-0.15E2
	K_3	-	-	-	0.4E-2	-	-	-	-0.3E-3	0.25E-4	-0.92E-1	-	0.13E-3	0.12E-5	-0.57E-1

		X_{m1}	X_{m2}	X_{m3}	X_{m4}
CASE II.A	K_1	0.32E-2	-0.91E-3	0.4E-2	0.22E-2
	K_2	-0.54E-1	0.38E1	0.67E1	0.14E1
	K_3	-0.77E-2	-0.16E-1	-0.71E-2	0.11E-2
CASE II.B	K_1	0.3E-1	0.62E-1	0.57E-1	-0.58E-2
	K_2	-0.33E1	-0.34E1	0.26	0.24E1
	K_3	-0.22E-1	-0.49E-1	-0.36E-1	0.61E-2
CASE II.C	K_1	0.14E-2	-0.18E-4	0.30E-2	0.46E-2
	K_2	0.69	0.47E1	0.76E1	0.2E1
	K_3	-0.68E-3	-0.22E-2	0.7E-2	0.30E-2

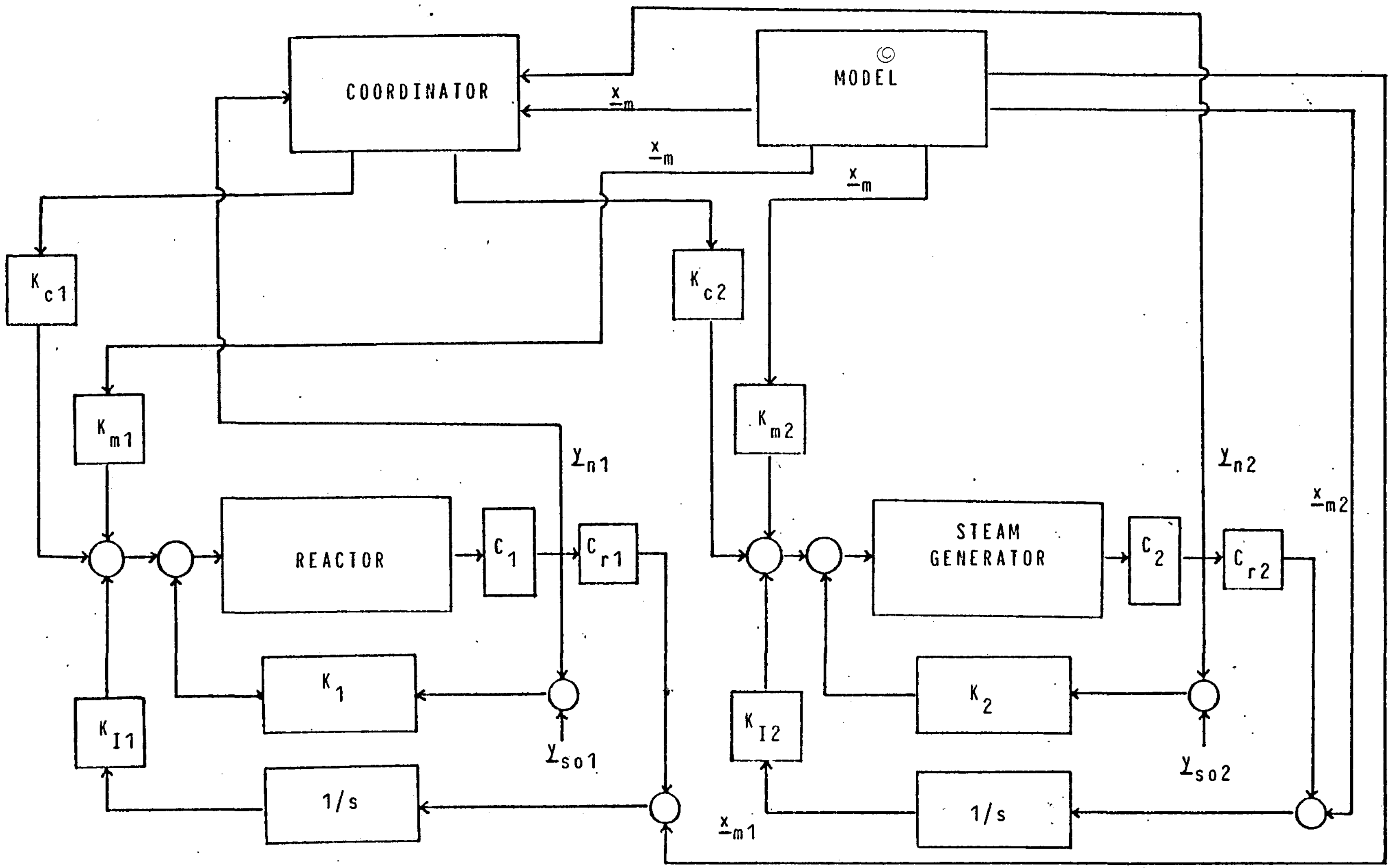

Figure III.3 - Model Following Control System Diagram

CHAPTER IV

PLANT SIMULATION: RESPONSES AND COMPARISON

Simulation tests were performed using the nonlinear model described in Chapter I.3.1. Two groups of control systems were considered. The control systems of group I were based on the servo problem presented in III.2, and the control systems of group II were based on the model following problem presented in III.3. The reader should use Table IV.1 as reference for the information structure assumed in each case-coded control system for the following figures in this Chapter.

The simulations performed to test the thesis concepts used the Runge Kutta integration method on the University's VAX 11/780 digital computer. All simulations assumed an initial steady state condition of 50% of power and a 10% load change.

Time responses of the following plant variables and their respective units are: power level in % of full power, average coolant temperature in °F, rod position in length-equivalent units according to the value of K_r, steam pressure in psia, water level in % of full level, throttle valve position in % of valve opening (propostional to the load), rod speed signal in units equivalent to the rate of rod position change, and feedwater flow in Kg/sec.

Figure IV.1 shows the plant response as derived from the simulations for a variety of information structures of the servo controller. Because of the absence of certain critical information provided for the control system,

Table IV.1 - Control Systems Case-Coded for Each Information Structure.

Group I - Servo Control Systems	
CASE I.A	$y_{1,2,3} = [\rho_r, P_w, T_f, T_w, T_{out}, T_{in}, T_m, P_s, M_f, C_v, u_1, \varphi_1, \varphi_2, \varphi_3]$
CASE I.B	$y_{1,2,3} = [P_w, T_w, T_{out}, T_{in}, P_s, M_f, C_v, u_1, \varphi_1, \varphi_2, \varphi_3]$
CASE I.C	$y_1 = [P_w, T_w, T_{in}, C_v, \varphi_1]$ $y_{2,3} = [T_w, P_s, M_f, C_v, \varphi_2, \varphi_3]$
CASE I.D	$y_1 = [P_w, T_w, \varphi_1]$ $y_{2,3} = [P_s, M_f, C_v, \varphi_2, \varphi_3]$
CASE I.E	$y_1 = [\rho_r, P_w, T_f, T_{out}, T_{in}, \varphi_1]$ $y_{2,3} = [T_{in}, T_m, P_s, M_f, C_v, \varphi_2, \varphi_3]$
Group II - Model Following Control Systems	
CASE II.A	$y_{1,2,3} = [\rho_r, P_w, T_f, T_w, T_{out}, T_{in}, T_m, P_s, M_f, C_v, u_1, \varphi_1, \varphi_2, \varphi_3, x_{m1}, x_{m2}, x_{m3}, x_{m4}]$
CASE II.B	$y_{1,2,3} = [P_w, T_w, P_s, M_f, C_v, \varphi_1, \varphi_2, \varphi_3, x_{m1}, x_{m2}, x_{m3}, x_{m4}]$
CASE II.C	$y_1 = [P_w, T_w, T_{in}, C_v, \varphi_1, \varphi_2, \varphi_3, x_{m1}, x_{m2}, x_{m3}, x_{m4}]$ $y_{2,3} = [T_w, P_s, M_f, C_v, \varphi_1, \varphi_2, \varphi_3, x_{m1}, x_{m2}, x_{m3}, x_{m4}]$

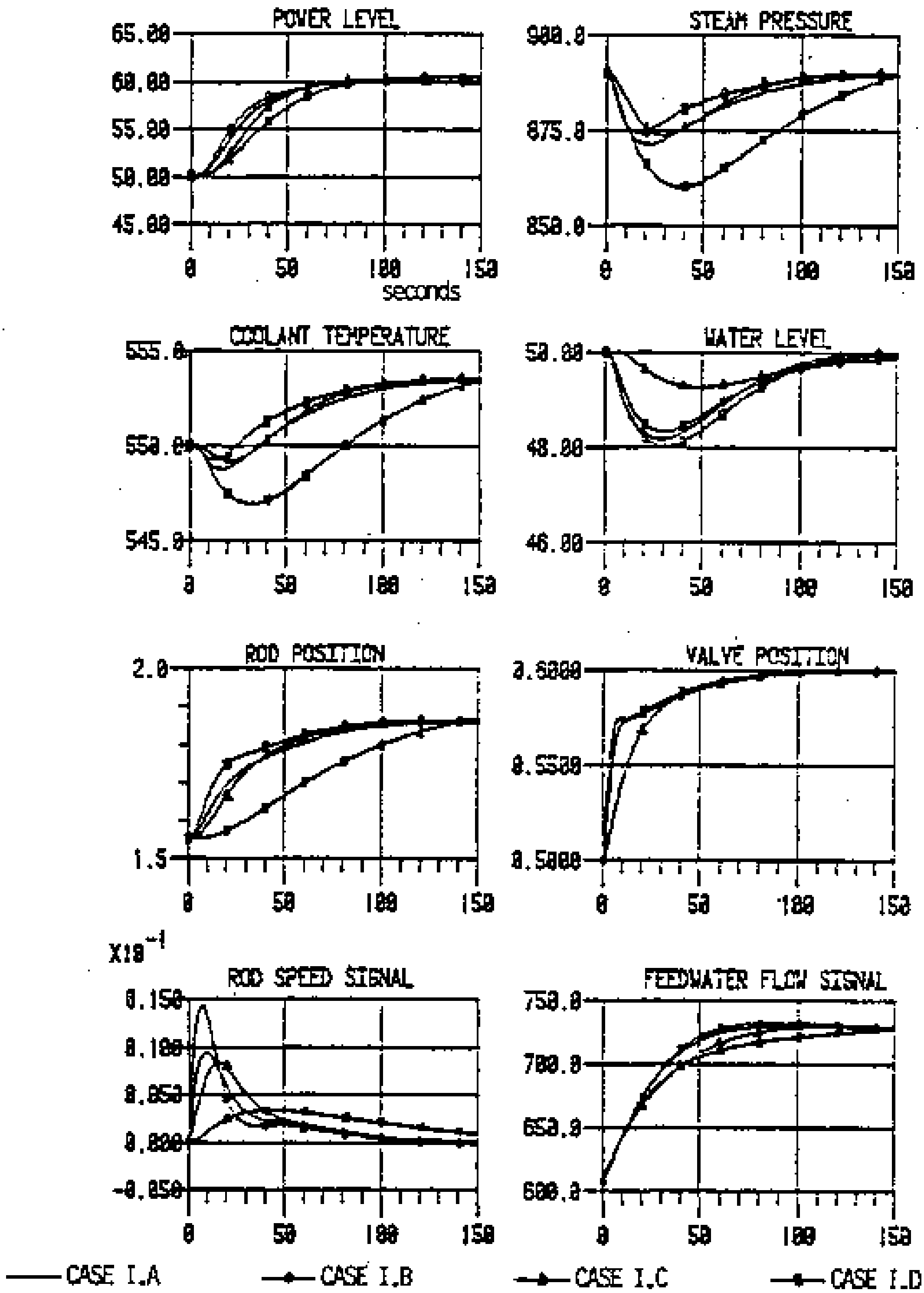

Figure IV.1 - Several Information Structures for Servo Control System.

plant performance degradation is noted for Case I.D in which a complete decentralization of information is assumed. The deficient coordination is characterized by retarded rod movement as shown in Figure IV.1. By adding a few additional plant information states for each localized controller, as considered in Case I.C (steam generator outlet temperature and turbine valve positioning for the reactor and reactor coolant temperature for the boiler), a performance similar to the full state feedback situation can be achieved. Although this case provides slower turbine control, less effort is placed on the steam generator water level control. In Case I.B, the faster reactor response is paid for by a required higher rod speed movement.

For the sake of illustration, Figure IV.2 shows a transient on the simulated plant controlled by a system designed in a decentralized fashion, Case I.E. The comparison is based on the fact that each subsystem is solved in such a way that the cost function for the overall system corresponds to the same cost function used in the centralized designs. The drawbacks presented here are that, the responses present overshoot characteristics and the rod speed input signal switches sign during the transient. These are clearly due to the lack of information, which should be provided in the design phase. In this particular case, it is mainly influenced by the

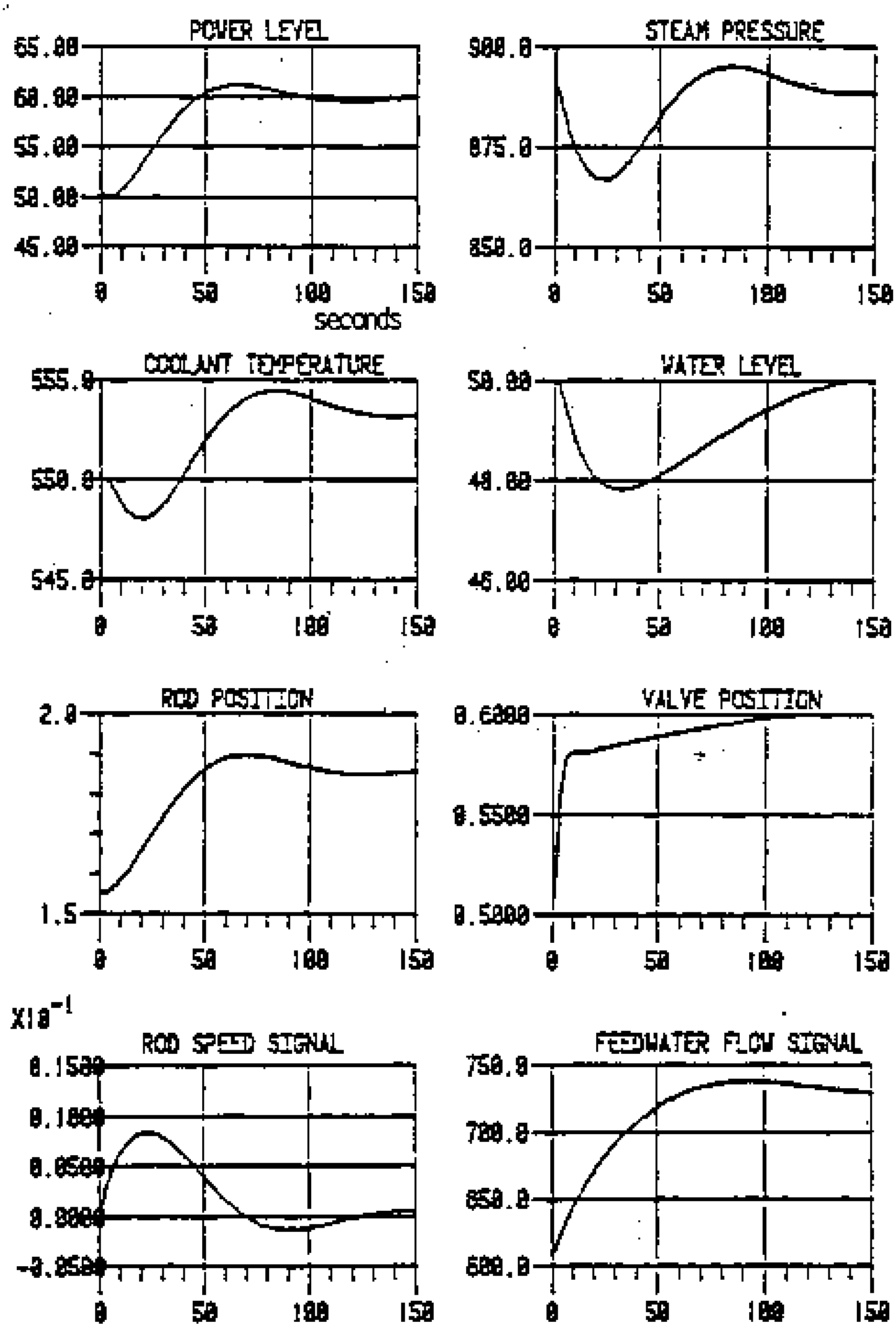

Figure IV.2 - Control System Based on Decentralized Design.

beneficial effect of the interconnection as the results of increased system reactivity produced by cooler coolant temperature, leaving the steam generator during a load increase transient.

Figure IV.3 presents a similar comparison for several information structures using the model following control system. After a short period, all cases present similar behavior. This period is the time required for the plant to track the model transient patterns. From a practical point of view, Case II.C represents a mode adequate control system because it requires minimum information exchange between the reactor and the boiler, and satisfactory responses can be obtained. This case assumes the same information structure as in the Case I.C, except that additional output tracking errors are provided.

The cross limit effect discussed in Chapter III is illustrated in Figure IV.4. Curve (a) corresponds to the transients of Case I.A. Curve (b) presents emphasis on the cross limit features in which the weight in Q(13,13) and Q(14,14) were reduced from 0.2E-4 and 0.5E5 to 0.5E-5 and 0.1E5, and Curve (c) disregards such a feature in which the weight in Q(12,12) was reduced from 1.0 to 0.1. Note that the ratio of the feedwater flow to the coolant temperature was considerably reduced for Case (c). Because of the heavier penalty placed on the coolant temperature error, anticipation of the rod movement following the load

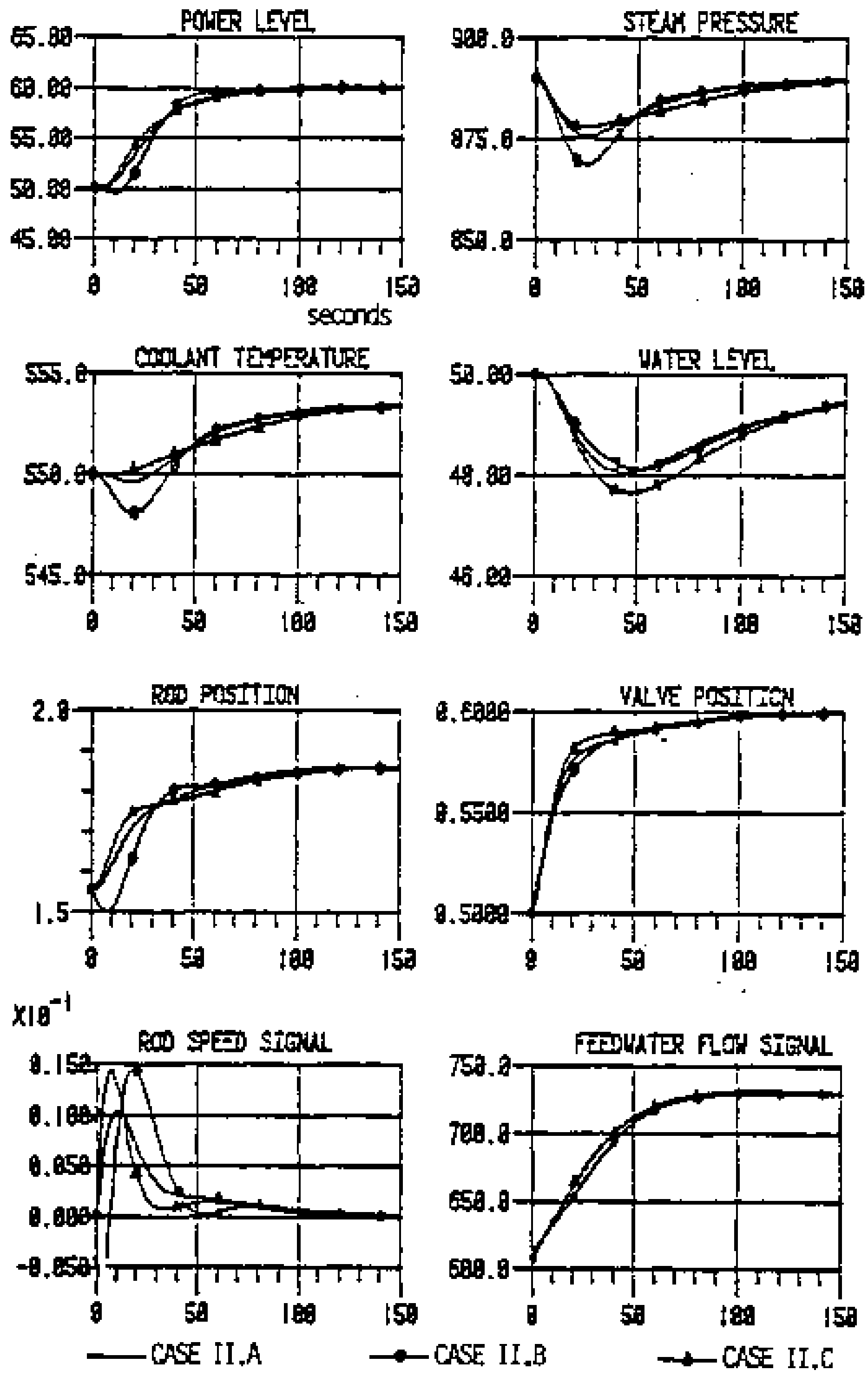

Figure IV.3 - Several Information Structures for Model Following Control System.

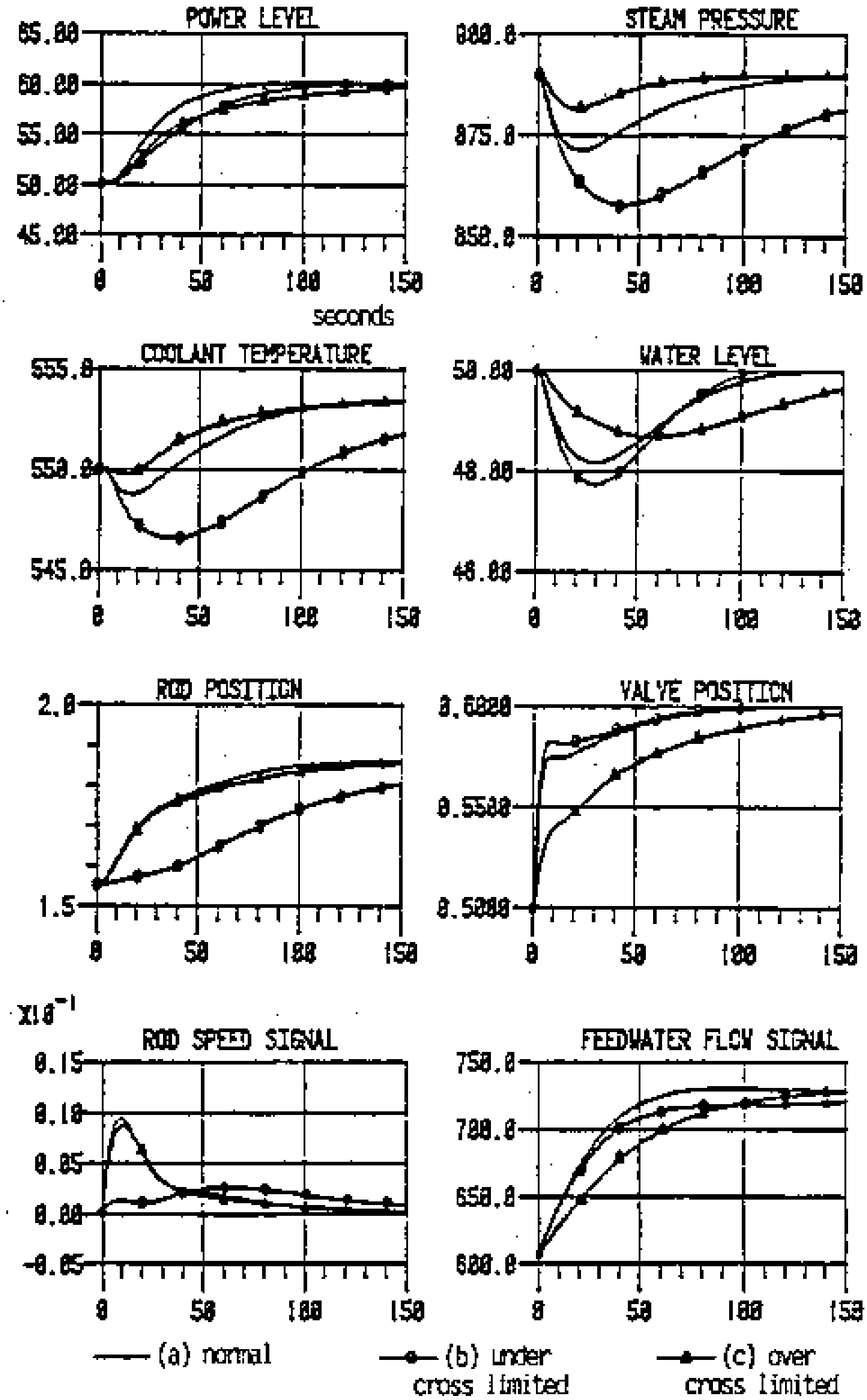

Figure IV.4 - Cross Limit Effect.

change was enhanced. The inverse argument applies for Case (b).

Figures IV.5, IV.6, IV.7 and IV.8 show the plant transients when the temperature reactivity coefficient value was increased by four times its original value. Figures IV.5 and IV.6 correspond to transients of a full state feedback control system for servo and model following design, respectively (Cases I.A and II.A). In these cases, for the system to be realizable, observers are needed to provide access to nonmeasurable states. Figures IV.7 and IV.8 correspond to the plant transients of simpler implementation controllers for servo and model following design, respectively (Cases I.C and II.C). To make the comparison more convenient, the nondisturbed plant transients are also shown in the plots. Because of different steady state conditions of the disturbed plant, the initial condition of the rod position is provided at a distinct value.

The above simulations show that the model following control system (Cases II.A and II.C) provides slightly better control than the servo controllers (Case I.A and I.C). Note that for model following, the output deviations relative to the original plant response (e.g. coolant temperature in Figures IV.6 or IV.8) reach almost zero error after 200 seconds elapsed time into the transients. No increase of maximum rod speed was required to achieve

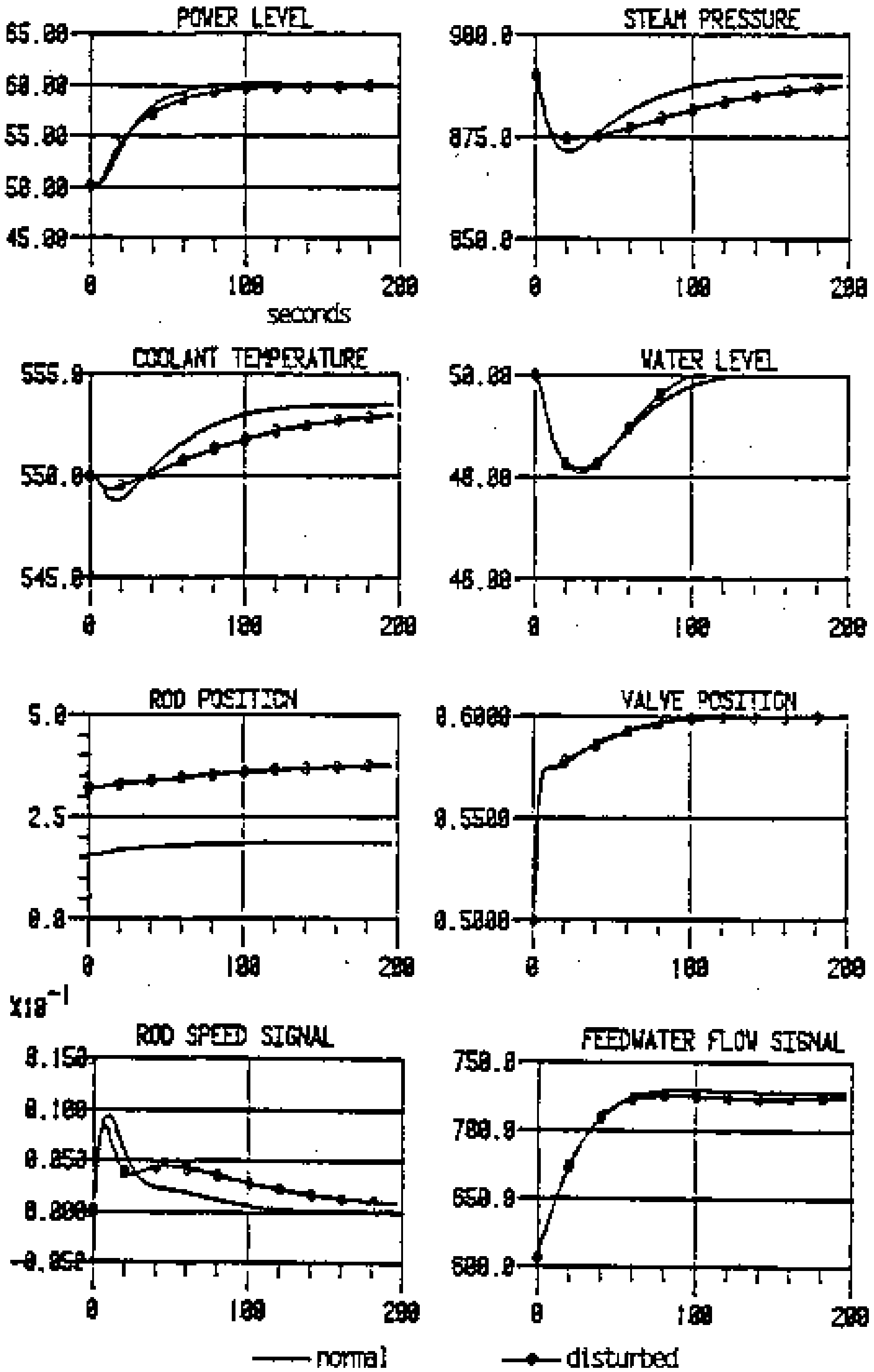

Figure IV.5 - Moderator Temperature Coefficient Disturbed for CASE I.A.

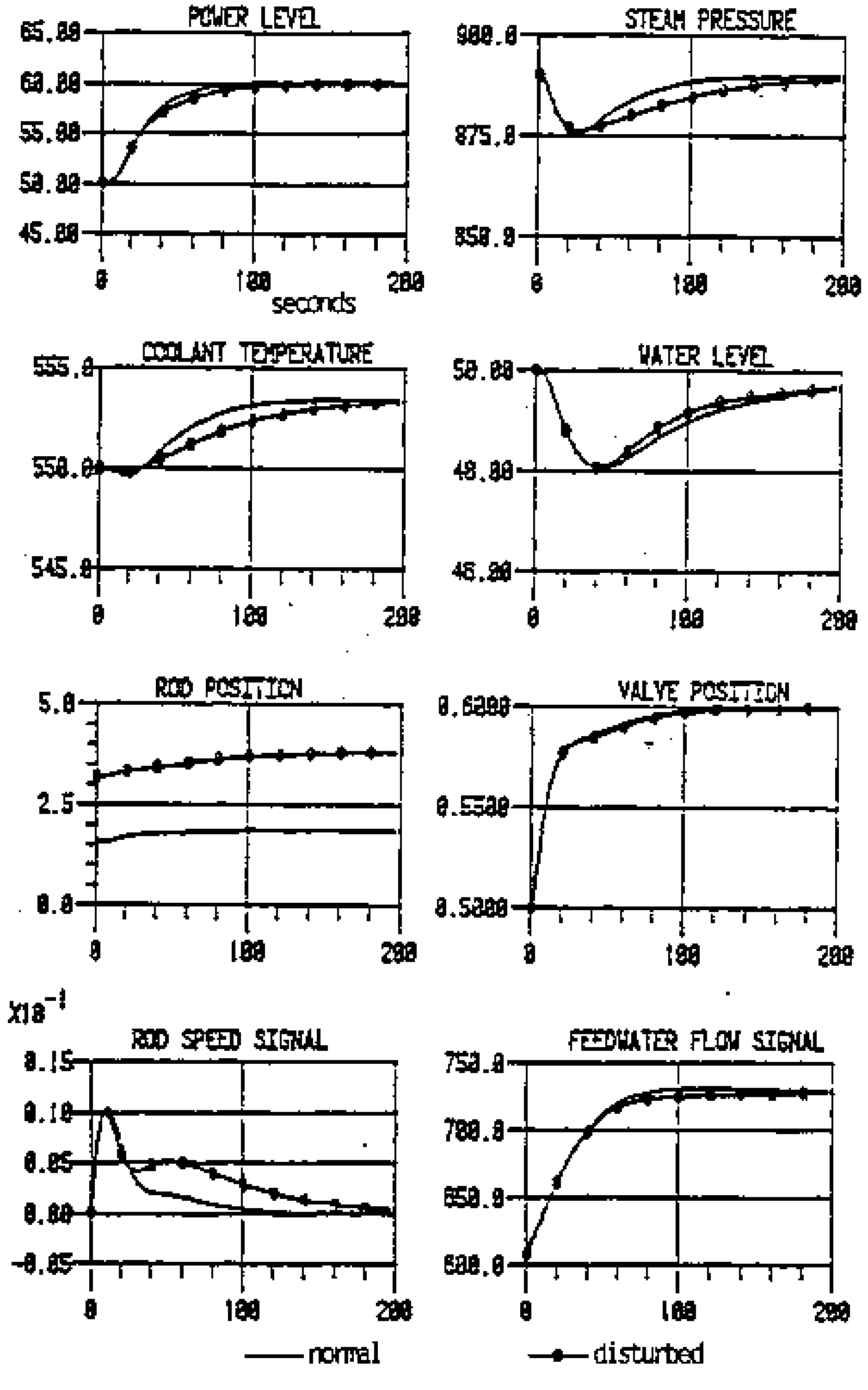

Figure IV.6 - Moderator Temperature Coefficient Disturbed for CASE II.A.

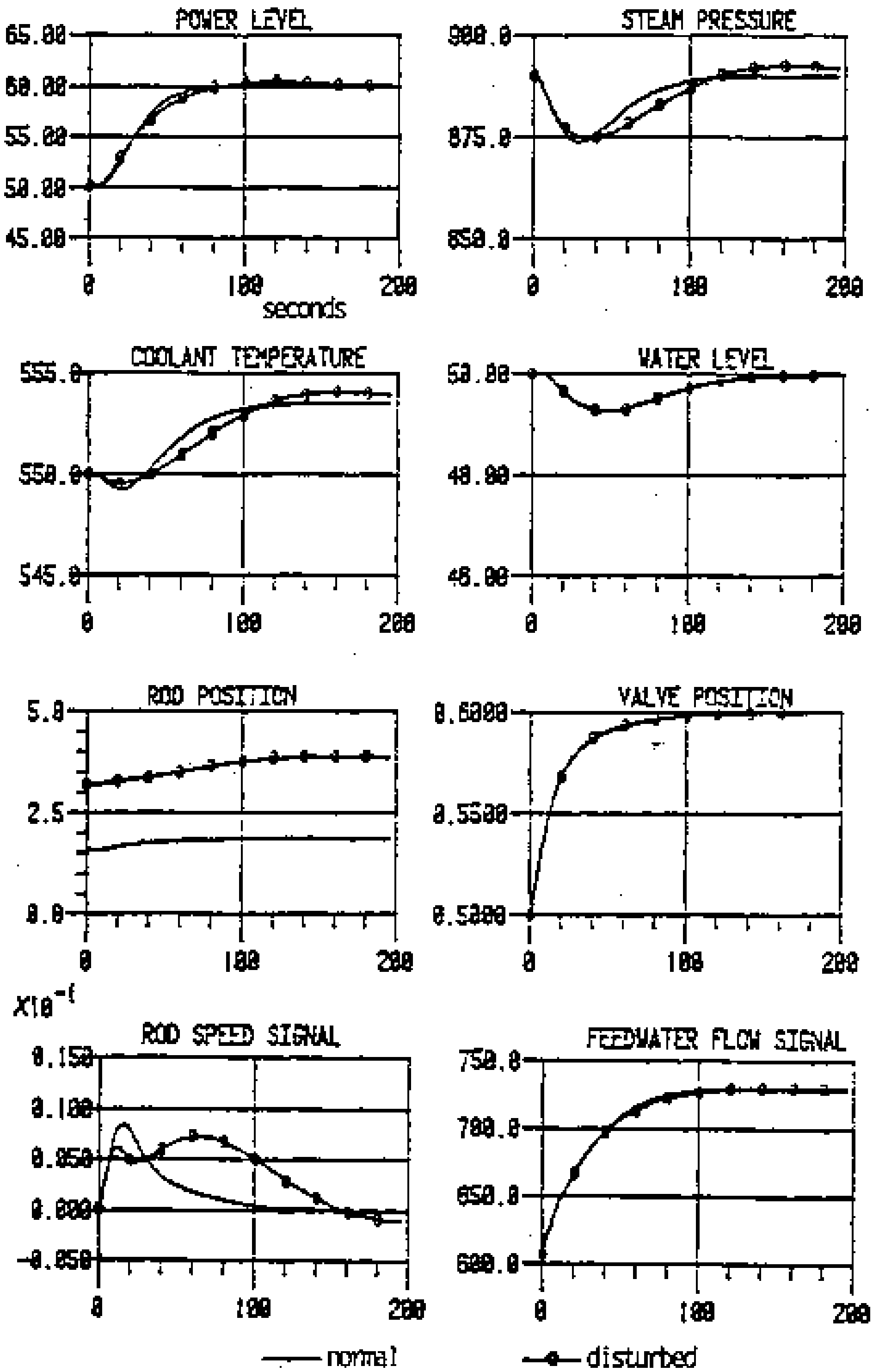

Figure IV.7 - Moderator Temperature Coefficient Disturbed for CASE I.C.

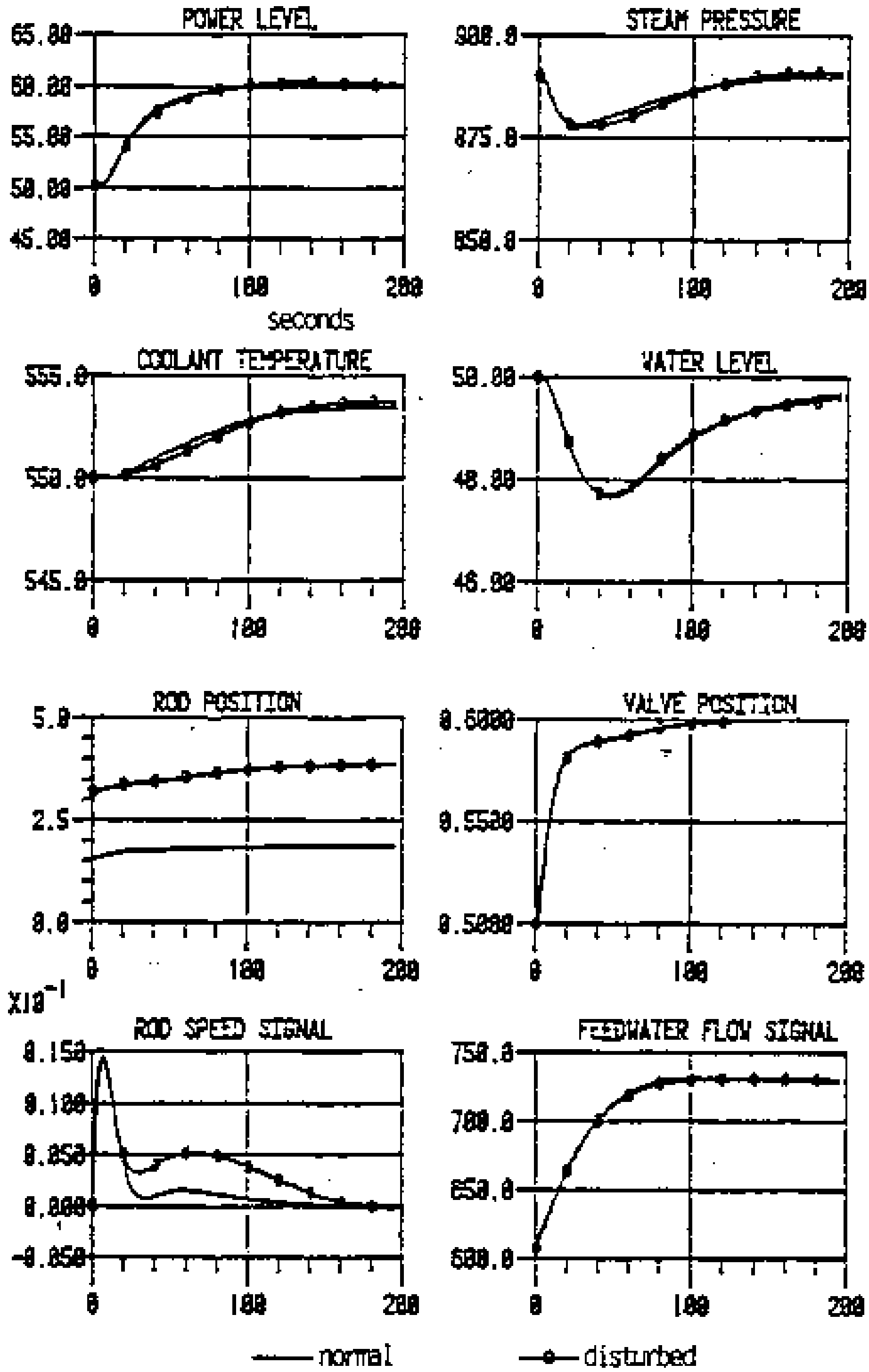

Figure IV.8 - Moderator Temperature Coefficient Disturbed for CASE II.C.

such a condition, although the total rod movement was increased to compensate for the disturbed reactivity coefficient. Such an example shows the robustness characteristic of the controller when this coefficient changes during the course of the core life.

Figure IV.9 illustrates the plant response for Cases I.C and II.C wherein the plant is subject to a ficticious external time-varying disturbance of type $w(t) = C(1 - e^{-\lambda t})$ in the power rate equation. This situation simulates a "power leakage" reactivity effects in the core. In this situation, both control systems provided a satisfactory control of their output variables.

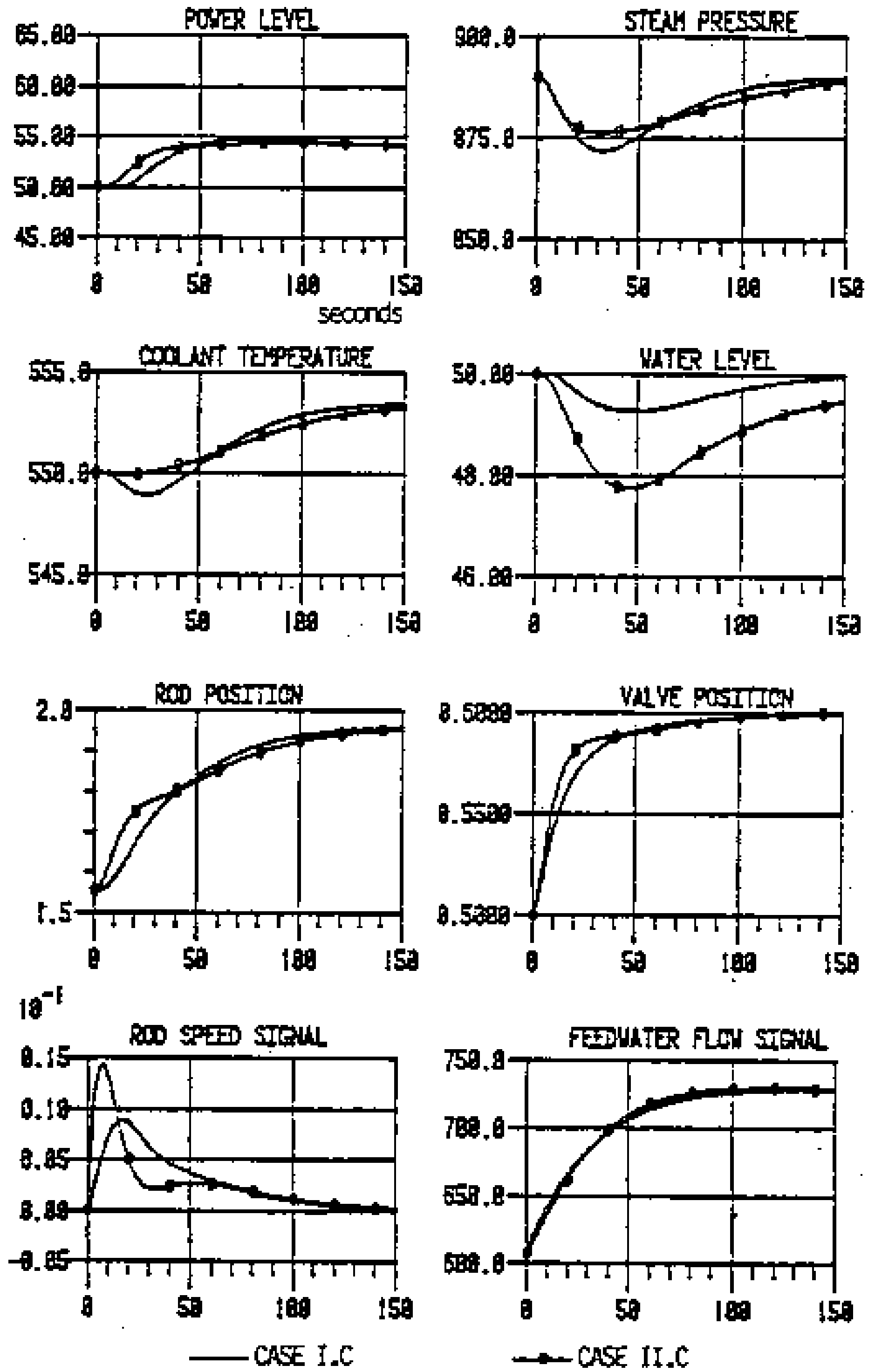

Figure IV.9 - CASES I.C and II.C Subject to External Disturbance.

CHAPTER V

SUMMARY AND CONCLUSIONS

This thesis involves the developing of design procedures directed towards the control of PWR nuclear power plants for load change transients using linear optimal control with quadratic cost functions. The objective is to find the optimal feedback gains and then an equivalent and reduced control solution using a mode preservation technique, which results in a simple and implementable control system.

Two control schemes have been investigated: the servo control approach and the model following control approach. Within each scheme, several control systems with different information structures have been proposed and evaluated. Based on the simulation results developed for this thesis, it was possible to select the controller which requires minimum information transfer among major components in the plant while satisfying the nuclear plant constraints (Cases I.C and II.C). It has been found that the use of a completely decentralized control system requiring only localized plant information considerably degrades the plant performance (see Figure IV.2).

Unlike previous work [19, 21], in which the feedforward input signal was obtained in an open loop fashion, the proposed control system approach generates the feedforward signal based on an on-line measurement of the plant states, permitting the system to handle plant parameter variations during the core life. In these situations, simulation tests showed that the control systems using model following provided slightly better performance than the servo controllers, as illustrated by transients shown in Figures IV.6 and IV.8 compared to Figures IV.5 and IV.7, respectively.

Furthermore, the model following control system proved to be more effective in ensuring standardized plant transients. Although the intended standardization was only partially successful because of delay due to the plant time response, it can be achieved after a relatively short period of time.

The cross limit features used in the Babcock and Wilcox Integrated Control system can be accomplished within the methodology of this design by balancing adequately the values of the elements corresponding to integral error states in the weighting matrix Q.

The rod maximum movement speed required for reactor safety and the permitted operational range of the water level deviation in the steam generator were the major plant constraints considered for the selection of the weighting matrices. The resulting throttle valve opening provided a satisfactory response characteristic for the turbine-generator control problem. Furthermore, examination of the elements of the weighting matrix provides an useful tool for sensitivity and plant design analysis. Using this approach, an adequate size of the steam generator can be estimated for the plant, so that it can perform a specified load following response as dictated by the needs of the power industry.

The nonlinear model presented in this thesis was derived based on theoretical principles. It is suggested, from a practical point of view, that the plant model be supplied with additional results of an identification scheme using input - output records of a specific power plant, so that the control system finally designed would be tailored to a particular situation.

One may also incorporate a time-varying model in the control system to accommodate adequately the plant parameter changes. This action would eliminate the bias presented by the open loop feature in the feedforward signal of the model following control system.

It may be stated, in conclusion, that the central objective of this thesis has been achieved; that is, a procedure based upon the ideas from the field of "modern control" has been developed, which allows rational and quantitative choices in reactor control system master design. The introduction of less expensive computers and the possible advantages of a distributed control system can now be explored with a greater sense of direction.

APPENDIX A

PSEUDO INVERSE (MOORE-PENROSE PSEUDO INVERSE)

This section presents some of the main definitions and properties of the pseudo inverse and establishes the solution of the least square minimization problem. Proofs of theorems and further details can be found in [63, 34].

DEFINITION 1: A matrix A^+ is defined as the pseudo inverse of matrix A if the following condition holds:

1. $A^+AA^+ = A^+$
2. $AA^+A = A$
3. A^+A and AA^+ are symmetrics

THEOREM 1: For any matrix A, there exists a pseudo inverse of A and it is unique.

Several basic properties of pseudo inverse are the following:

1. If A is square and nonsingular, then $A^+ = A^{-1}$;
2. If rows of A are linearly independent, then $A^+ = A^T (AA^T)^{-1}$, in particular, it is also called the left inverse of A;

3. If columns of A are linearly independent, then $A^+ = (A^TA)^{-1} A^T$, in particular, it is also called the right inverse of A.

THEOREM 2: Consider the equation $B X A = C$; the necessary and sufficient condition for the equation to have a solution is:

$$B B^+ C A^+ A = C$$

in which case the general solution is given by:

$$X = B^+ C A^+ + Y - B^+ B Y A A^+$$

where A,B,C and X are matrices with appropriate dimensions. Y is arbitrary.

DEFINITION 2: The matrix X_0 is a best approximate solution of an equation:

$X A = C$ if for all X, either

(1) $\|X A - C\| \geq \|X_0 A - C\|$

or

(2) $\|X A - C\| = \|X_0 A - C\|$ and $\|X\| \geq \|X_0\|$

THEOREM 3: The best approximate solution of equation $X A = C$ is:

$$X_0 = C A^+$$

where A^+ denotes the Moore-Penrose pseudo inverse.

In practical words, one may consider a linear equation $XA = C$, where A is (L x P), C is (M x P) and X is (M x L) dimensional matrices. For a consistent and full rank system, there results a unique and exact solution in which the pseudo inverse reduces to a conventional inverse and

$$\|X_0 A - C\| = 0 \quad \text{and minimum}$$

If the system is underconstrained, more than one exact solution exists in which the pseudo inverse reduces to the left inverse of A and

$$\|X\| \text{ is minimized}$$

If the system is overconstrained, no exact solution exists in which the pseudo inverse reduces to the right inverse of A and

$$\|XA - C\| \text{ is minimized}$$

Furthermore, one can consider a weighted minimization as follows:

For underconstrained problems, one introduces a diagonal positive defined matrix R of dimension (L x L) as:

$$XRR^{-1} A = C$$

using Theorem 3,

$$XR = C\ (R^{-1}\ A)^{+}$$

and a weighted solution in X is given by:

$$X_{o} = C\ (R^{-1}\ A)^{+}\ R^{-1} \qquad (A1)$$

For overconstrained problem, it introduces a diagonal positive definite matrix W of dimension (P x P) as:

$$X\ A\ W = C\ W$$

using Theorem 3, an error weighted solution in X is given by:

$$X_{o} = C\ W\ (AW)^{+} \qquad (A2)$$

The solutions A! and A2 can be combined to provide a generalized solution given by:

$$X_{o} = C\ W\ (R^{-1}\ AW)^{+}\ R^{-1}. \qquad (A3)$$

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